

# On the role of practices in logical disagreements

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# 1

# Introduction

## 1.1 “Practice-based” philosophy of science

- Principle:  
the study of the practices of scientists, rather than a focus on the finite outputs of the scientists
- The **contingency / inevitability** problem in the philosophy of science
- The particular position of logic, between philosophy and mathematics

## 1.2 Philosophizing “practice” in logic

- A possible equivocation
  - (a) Logic in practice  
A logical theory about how we actually think in practice  
(feasible logic, dynamic epistemic logic, non-monotonic logic, etc.)
  - (b) Practice in logic  
An experimental practice of logical theories  
(institutions, background formation, cooperation in logic)

The core problem: (b), and not (a)!

(see Dutilh-Novaes’ social vs. individual, cognitive aspect of logic)

Or: does a difference in logical practice entail a difference in logical theory?

## 1.2 Philosophizing “practice” in logic

- The background of a practice-based philosophy of logic: 2 lines of thought

The **practical** line (dynamic production)

(1) What do logicians do?

How do they conduct their research? (including the decision procedures)

(2) Why do they practice logic in a certain way, rather than another one?

The **theoretical** line (static product)

(3) What is logic?

(4) What is logic about?

How does any answer to (1)-(2) contribute to an answer to (3)-(4)?

Current trend: from **what** to **how** ... from **how** to **what**, eventually?

## 1.2 Philosophizing “practice” in logic

- The epistemological difficulty: **practice** and **facts** in logic

Practice in natural science: experience, and facts  
which counterpart in formal, non-natural sciences?

Does logical practice assume the occurrence of “logical facts”?  
logical fact is a non-sense, in the Wittgenstein tradition of logic

The logical fact:

- obviousness (philosophy of logic)
- naturalness (natural deduction)
- pure intuition (mathematical logic)
- linguistic intuition (philosophical logic)

A practice of “counter-facts”: the role of paradoxes (“negative facts”)

## 2

## Examples



## 2.1 Why do Achilles and the Tortoise disagree?

### **Premises**

(A) Things that are equal to the same are equal to each other

(B) The two sides of this Triangle are things that are equal to the same

### **Conclusion**

(Z) The two sides of this Triangle are equal to each other

## 2.1 Why do Achilles and the Tortoise disagree?

(1)		(2)		(3)	
A		A		A	
B	→	B	→	B	→ ...
∴ Z		C : (1) is valid		C : (1) is valid	
		∴ Z		D : (2) is valid	
				∴ Z	

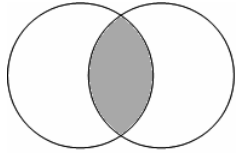
## 2.2 Do “A propositions” have existential import?

John Venn (1834-1923) vs. Lewis Carroll (1832-1898)

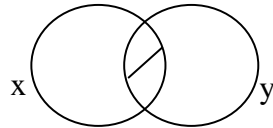
- In “common” use, **universals** do assert the **existence** of their subject
- In the domain of symbolic logic, the choice is not a question of who’s **right** or **wrong**, but merely a question of **convenience**

## Venn's symbolism

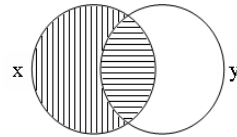
No  $x$  is  $y$



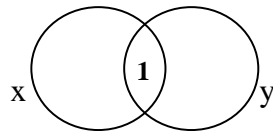
Some  $x$  is  $y$



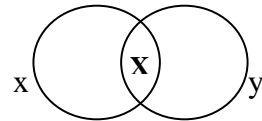
1883



1894

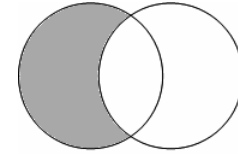


1894



1896

All  $x$  are  $y$



$$xy = 0$$

$$xy \neq 0 \text{ or } xy > 0$$

$$xy' = 0$$

## Carroll's symbolism

No  $x$  is  $y$

0	

$xy_0$

Some  $x$  is  $y$

I	

$xy_1$

All  $x$  are  $y$

I	0

$x_1 \dagger xy'_0 \quad (x_1y'_0)$

Simplicity



Practicality

Venn

Carroll

Existential Import

No

Yes

## 2.3 Disagreement between logical systems: logical pluralism

- As an aside: about disagreement in the **decision procedures**  
See Aberdein's talk: logic as a consequence vs. deductive system  
A motivated, or arbitrary choice?

Example: **algebraic** vs. **relational** (possible world) semantics

A methodological change: for sake of efficiency, simplicity?

An institutional change: Kripke's modal semantics in the fashion?

- A point for methodology:

*Reductio ad absurdum*, a shorter process than matrices

Dugundji's theorem (no characteristic finite matrix for Lewis' systems **S1-S5**)

- A point for institutions:

Philosophers are more attracted by "possible worlds" than the mathematical matrix theory

## 2.3 Disagreement between logical systems: logical pluralism

- An answer to (2): **Logical pluralism**

How can a difference in logical practice alter a set of logical theorems?

- Two sorts of logical pluralism (Haack (1978))

- **Local** pluralism

“different logical systems are applicable to (i.e. correct with respect to) different areas of discourse (...)” (p. 223)

- **Global** pluralism

“the global pluralist denies either that the classical and deviant logician are really using ‘valid’/’logically true’ in the same sense” (p. 223)

- A sample of how (1)-(2) contribute to an answer to (3)-(4) ...

**What** they do in philosophical logic: depends upon **why** they do so



## 2.3 Disagreement between logical systems: logical pluralism

- Local pluralism: disagreement about the sentential variables

**classical** vs. **quantum** logic

$$\models_{\text{CL}} A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C) \quad \text{Distributivity}$$

B: proposition about the position of a particle

C: proposition about the momentum of a particle

We cannot have both  $(A \wedge B)$  and  $(A \wedge C)$ , in **QL**

- Global pluralism: disagreement about the logical constants

**classical** vs. **intuitionist** logic

$$\models_{\text{CL}} A \vee \neg A, \models_{\text{CL}} \neg\neg A \rightarrow A$$

$\neg$ : absurdity, proved impossibility

$$\neg A =_{\text{df}} \Box \sim A$$

**classical** vs. **paraconsistent** logic

$$\models_{\text{CL}} (A \wedge \neg A) \rightarrow B$$

$\neg$ : falsifiability, possible falsity

$$\neg A =_{\text{df}} \Diamond \sim A$$

## 2.3 Disagreement between logical systems: logical pluralism

**Question (2):** Why are there such disagreements between logics?

**Answer 1:** the **social** status of logicians

Mathematicians in intuitionist logic: Brouwer, Kolmogorov

Science theorists in paraconsistent logic: Jaśkowski, Mares, Batens

Computer scientists in linear logic: Girard

Linguists, metaphysicians in modal logic: Montague, Kripke, Plantinga

**Problem:** Quine was a mathematician, as most of classical logicians

**Answer 2:** the **philosophical** formation of logicians

Constructivists vs. Platonists, Anti-realists vs. Realists

**Problem:** not every logician feels philosophical troubles in practice ...

## 2.3 Disagreement between logical systems: logical pluralism

One motivated/non-arbitrary practice: **philosophical logic**

A treatment of **paradoxes**: from plausibly correct premises to patently incorrect consequences

- Revising, solving-game logical systems = **Understanding** what's wrong?  
(see Aberdein's talk: non-classical logics vs. non-deductive schemes)

Examples:

- The Liar Paradox: how can one sentence be both true and false?
- Aristotle's sea-battle case: does Bivalence entail determinism?
- Fitch's Paradox: is every truth knowable and known?
- Moore's Paradox: how can sentences be both absurd and consistent?
- Sorite's Paradox: how to go from a grain to a heap?

Logical practice: finding an **appropriate** logical system among several ones

## 2.3 Disagreement between logical systems: logical pluralism

Problem with paradoxes: **operational** vs. **representational** analyses

- **Operational** aspect of logic (O)

The way theorems are deduced from operational methods (inference rules)

A treatment of paradoxes through their **inference rules**

- **Representational** aspect of logic (R)

The way a statement is formalized and regimented within a formal language

A treatment of paradoxes through their **logical form**

- Purpose: showing how a surprising result is so, or not so much

A “dummy” paradox: Logical Omniscience (a technical by-product)

Preconceptions: pluralist, **descriptive** logics vs. monist, **normative** logics

## 2.3 Disagreement between logical systems: logical pluralism

- Liar Paradox: (O) type theories, many-valuedness  
(R) dialetheism, illocutionary logic
- Sea-battle case : (O) many-valuedness  
(R) the scope of necessity
- Fitch's Paradox: (O) non-classical rules (which inference rule to cancel?)  
(R) formalization of "being true" (realist vs. antirealist)
- Moore's Paradox: (O) modal logic, non-classical logics  
(R) statements vs. sentences, illocutionary logic
- Sorites' Paradox: (O) many-valued logics  
(R) discrete space (logics) vs. continuous space  
(infinitesimal calculus)

## 2.3 Disagreement between logical systems: logical pluralism

What is the most shocking: a paradox, or its treatment?

Some weaken the logic in the argument still further. This is like tuning down the volume on your radio so as not to hear the bad news. You will not hear much good news either. Other remedies leave the logic untouched, but weaken the verificationist principle itself. This is like censoring the news: you hear things loud and clear, but they may not be so interesting.

(J. van Benthem, “What we may come to know” (2004))

A technical, operational approach of logical paradoxes:

- it helps to give rise to new logical products for **expected** solutions

A philosophical, representational approach of logical paradoxes:

- a right formulation of the problem is better than a vacuous solution

# 3

# Conclusion

### 3.1 An aporia: the “experimenter” regression

- A circularity with any scientific expert’s report

First: the correctness of an established fact requires an experimenter’s report

Now: the credibility of an experimenter is based on facts he already reported

Therefore: the correctness of an expert’s report is established by the expert.

- In philosophical logic

The “logical fact”: the established theorem (within a relative system)

The “logical counter-fact”: the logical paradox (from a relative system)

How to establish an agreement about a logical paradox?



## 3.2 A way out: a feed-back relation in practice

- An ideal : scientific “insight” in a cooperative way

One technician: the logician (**how** to construct some logical system)

One expert: the philosopher (**why** to construct some logical system)

Expected results: a two-fold requirement

- efficiency, or fruitfulness (technical practice): creativity of logic
- relevance (reflective practice): appropriateness of logic

- A social division of labour?

Who’s the so-called “philosopher”, as an expert ?

- A professional teacher with specified subject-matters
- The member of a research department with oriented problems
- The philosopher and the logician are often one and the same

## 3.2 A way out: a feed-back relation in practice

A **rational** conclusion about practice : contingency logical practice

How are we right to be surprised by a “practical turn” in logic?

- Logic is **internally** necessary, logics are **externally** contingent
- philosophical logic: a contingent choice of necessary truths
- relative necessity of logics, to be chosen for social or professional options

Toward an “**irrational**” ...

- logics serve as a knowledge ground while resulting from social learning
- procedure decisions and inference rules proceed like habits in science

... or even “gossiping” view of logical practice?

- Quine was opposed to modal logic because he disliked his most famous champion: Kripke (a plausible “little story” within the history of logic)

THANK YOU ...