What is practice-based philosophy of logic?
A case-study: uses of formal languages in logic

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- Practice-based philosophy of mathematics: going beyond the ‘foundations of mathematics’ program.

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- The logic that philosophers of logic talk about is all too often the logic of several decades ago, when (mathematical) logic was almost exclusively concerned with the foundations of mathematics.
- First-order logic still often seen as the quintessential logical system.

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- It **need not replace** traditional philosophy of logic.

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- The **social** level of logic as a collective, **public** enterprise; it involves networks of people who communicate with each other and whose work builds on previous work (cumulative enterprise).
- Logicians share specific (social) conventions on how work in logic ought to be done.
- The **individual** level of logic as a cognitive enterprise; even though the social aspect is fundamental for the creative process, ultimately thinking remains an individual, **private** matter.

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- Actual practices are not always (necessarily) 'right'.

Methodology for practice-based philosophy of logic

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- Once data is gathered, we can proceed with the ‘usual’ philosophical methodology (problematic itself…).
- For the individual level of logic as a cognitive enterprise, a promising approach seems to be to take into account findings from cognitive science.
- For the collective level of logic as a social enterprise, data-gathering seems a delicate matter. Serious sociological methodology would have to be employed. (Surveys?)

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- **Reflective equilibrium** between practices and theory: ideally, the dialogue should go both ways.

A case study: uses of formal languages in logic

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- It is important to go beyond simply accepting that ‘this is how it is done’. Does it really make a difference for logical investigations? Is it necessary for logic as a discipline?
- One cannot deny the substantial changes that logical practices underwent since it became customary to do logic with formal languages; this phenomenon requires an explanation.

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- And what role(s) do they play?
- Social dimension: languages used for communication among logicians.
- Cognitive dimension: does the manipulation of formal languages as such contribute to insight in logic?

The languages of logicians

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- This can be observed in particular in oral contexts.
- But research in logic with the use of formal languages is significantly different from research in logic without them (as history shows).

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- As such, they involve predominantly (but perhaps not fundamentally) our **visual** capacities.
- Historically, they came into being only after a very long process going through the use of schematic letters and the development of languages for mathematics (algebra in particular), spanning over many centuries and two continents (Staal 2006).

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- Formal languages emerged from a different kind of technology (written languages), initially in order to increase precision and objectivity in science – **expressive** function.
- But they turned out to have an **operative** dimension that is arguably more decisive for work in logic than expressivity.

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- Operative writing: “a medium for representing a realm of cognitive phenomena […] a tool for operating *hands-on* with these phenomena in order to solve problems or to prove theories pertaining to this cognitive realm.”
- Writing not having this operative dimension could be referred to as (‘merely’) representational or expressive writing.

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- Few authors have emphasized their calculative, operative advantages (Leibniz is the notable exception).
- Perhaps a tension between what logicians say about their practices and the practices themselves?

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- Expressive role: increased precision.
- Iconic role: they function like graphs, diagrams.
- Operative, calculative role: a ‘paper-and-pencil’ import that seems to play an important role in how logicians reason and arrive at new results.

The expressive role of formal languages

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- Risk of expressive loss.

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- This suggests possible cognitive connections between doing logic and our visual faculties (experiments?).

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- Formal languages: a hands-on tool for discovery in logic.
- What are the features of formal languages that allow them to perform this operative function?

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- But in science we are asked to reason deductively, as deduction allows for a much higher degree of certainty and indefeasibility.
- We need devices that help us counter our usual reasoning tendencies, and formal languages are among such devices.

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- Good for everyday life, terrible for science: belief-bias leads to conservativeness, it hinders discovery and the advancement of knowledge.
- In deductive reasoning, belief-bias is to be suppressed: no external information is allowed to ‘sneak in’, and thus prior beliefs should not interfere in the process.

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[In a formal system] rules of inference are laid down which allow one to pass from the axioms to new formulas and thus to deduce more and more propositions, the outstanding feature of the rules of inference being that they are purely *formal*, i.e. refer only to the *outward structure* of the formulas, not to their meanings, *so that* they could be applied by someone who knew nothing about mathematics, or by a machine. (Gödel 1995, 45)

Conclusions

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Conclusions

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- This phenomenon is related to the fact that they are a technology with built-in mechanisms for the suppression of ‘natural’ reasoning patters, which are conservative and seek to confirm prior belief.
- These mechanisms reside in the operative (‘paper-and-pencil’) nature of formal languages, but their original purpose was essentially expressive.