On the dialogical approach to semantics

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Dialogues and Games:
Historical Roots and Contemporary Models

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Outline

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Background

- Paul Lorenzen (1915 – 1994):
  - *Einführung in die operative Logik und Mathematik* (1955)
    - Background whose modification led to the dialogical approach.
  - “Logik und Agon” (1958/1960)
    - First paper suggesting to analyze meaning of logical constants in terms of certain sorts of two-player games.
  - “Ein dialogisches Konstruktivitätskriterium” (1959/1961)
    - Proposes to clarify *den vagen Begriff den ‘Konstruktivität’* via the notion of dialogue.

Calculi

- Aims to formulate a 'general theory of calculi'.
- A *calculus* is a collection of 'rewriting rules' of the form

$$A_1, \ldots, A_n \Rightarrow A_{n+1},$$

where each A_i is a string over an alphabet consisting of 'atomic figures' and 'variables'.

Example

Consider the following calculus:

- $\mathcal{K}$: Three rules: $\Rightarrow a$ and $\Rightarrow b$ and $x, y \Rightarrow xy$.

In $\mathcal{K}$ precisely the non-empty strings over the alphabet $\{a, b\}$ are derivable.

Proto-logic and admissibility

- Proto-logic: theory about deducing figures in calculi. Its specific goal to study the admissibility of rules.
- Rule R is *admissible* (zulässig) in calculus $\mathcal{K}$ if everything derivable in $\mathcal{K} + R$ is already derivable in $\mathcal{K}$.
  - The term 'admissible' is actually coined by Lorenzen.
- Admissibility is interpreted operationally:
  - R is admissible in $\mathcal{K}$ if there is an *elimination procedure* (Eliminationsverfahren) which, applied to any production of a string in $\mathcal{K} + R$, yields a production of this string in $\mathcal{K}$.
- Proto-logic seen as conceptually prior to logic.
  - Logical operators interpreted with reference to proto-logic.

View on logic

- The meaning of implication is explicated via admissibility:
    - Relative to calculus $\mathcal{K}$, the meaning of sentence $(A \rightarrow B)$ is that the rule $A \Longrightarrow B$ is admissible in $\mathcal{K}$, that is, there is an elimination procedure from $\mathcal{K} + (A \Longrightarrow B)$ to $\mathcal{K}$.
    - More specifically, a hierarchy of calculi is postulated. $(A \rightarrow B)$ is taken to be derivable in calculus of level $n + 1$ if the rule $A \Longrightarrow B$ is admissible in calculus of level n.
    - The more nestings of $\rightarrow$, the more levels of calculi.
    - The notion of elimination procedure reminiscent the notion of procedure made use of in the BHK-semantics. (The connection to admissibility is proper to L.)

View on logic (cont.)

- Proper ('eigentlich') logical operators: $\wedge, \vee, \exists$
  - Interpreted in terms of calculus-internal symbol manipulation rules.
  - These 'introduction rules' available:
    - $A, B \Rightarrow (A \wedge B)$,
    - $A \Rightarrow (A \vee B)$, $\quad B \Rightarrow (A \vee B)$,
    - $A \Rightarrow \exists x A$,
  - The corresponding elimination rules will be admissible (given certain general assumptions regulating the calculi).
- Improper ('uneigentlich') logical operators: $\rightarrow, \neg, \forall$.
  - Interpreted in terms of an associated (meta-theoretical) procedure; lead to the introduction of meta-calculi.

Towards dialogues

- Lorenzen's game-theoretical ideas (from 1958 on) emerge from his work on operative logic.
- Apparently an important motivation in this development: to get rid of the hierarchy of meta-calculi.
- If a calculus $\mathcal{K}$ should be viewed as a game, it would be a one-player game (Solospiel).
- Instead of a hierarchy of meta-calculi, there will be just one two-player game (dialogue), played relative to a calculus $\mathcal{K}$.
  - Arguments about atomic statements would be 'settled' by the underlying calculus: in the relevant calculi the relation '— is a derivation of — in $\mathcal{K}$' is recursive.
  - Actually dialogues can be defined without presupposing such underlying calculi.

Towards dialogues (cont.)

- There will be game rules explicating how the players may act with respect to the logical operators. All operators have such dialogue rules — in this respect they are on a par.
- The operators receive their meaning from the actions they permit in dialogues (defense, attack).
- The distinction proper/improper is retained. Now Lorenzen identifies the improper operators ($\rightarrow, \neg, \forall$) as the *conditional* (bedingte) ones. These are the operators that require two players for their interpretation.[1]
- The idea of elimination procedure that was used to interpret improper operators in operative logic will now appear in a new form: in the notion of strategy.

[1] For clarification, cf. the discussion on 'particle rules' below.

Outline

Games and plays

- A game is specified by laying down game rules and winning conditions.
- A play of the game is any sequence of positions generated in accordance with the game rules.
- Game rules indicate the following:
  - The initial position.
  - Whether a given play generated can be further extended.
    - If it can, the rules specify which player must make a move, and which actions are available to the player.
- In a two-player zero-sum game, the winning conditions specify — for all plays not further extendible — which player wins the play. The one who does not win, loses.

Strategies

- Observe that *terminal plays* (plays not further extendible) are won or lost — not games.
- A strategy of player X in a game is a set of instructions (a function) which yields for every play at which it is X's turn to move an action which complies with the game rules.
- X's strategy σ is winning, if against every sequence of moves by the adversary, making moves according to σ leads to a terminal play won by X.

Outline

The setting

- Dialogues are (two-player zero-sum) games.
- The two players: **P** (or Proponent), **O** (or Opponent).
- The game rules will be referred to as dialogue rules:
  - particle rules (Partikelregeln),
  - structural rules (Rahmenregeln)

Kuno Lorenz [1967].

- Basic ideas:
  - Dialogue rules provide the meanings of logical operators.
  - Notions like truth and validity are ‘meta-theoretical notions’ that emerge on the level of winning strategies only.

Particle rules (attack-defense rules)

- Moves (actions) in dialogues often termed ‘utterances’.
- Particle rules incorporate the idea that utterances induce commitments.
- The rules are normative: They
  - tell how such commitments may be tested, i.e., how the corresponding utterances may be attacked;
  - specify the utterer’s obligations triggered by a given test, i.e., indicate how one may defend one’s utterance against a given attack.

Particle rules (cont.)

Let X and Y be distinct players.

Note: proper *vs.* improper operators

($\wedge$):

utterance	$X : (A \wedge B)$
attack	$Y : ?_L$ or $Y : ?_R$
defense	$X : A$ *resp.* $X : B$

($\vee$):

utterance	$X : (A \vee B)$
attack	$Y : ?_\vee$
defense	$X : A$ or $X : B$

($\rightarrow$):

utterance	$X : (A \rightarrow B)$
attack	$Y : A$
defense	$X : B$

($\neg$):

utterance	$X : \neg A$
attack	$Y : A$
defense	$X :$ —

($\forall$):

utterance	$X : \forall x A$
attack	$Y : ?_{\mathrm{t}}$
defense	$X : A[x/\mathrm{t}]$

($\exists$):

utterance	$X : \exists x A$
attack	$Y : ?_\exists$
defense	$X : A[x/\mathrm{t}]$

- Meant to provide the ‘core meaning’ of logical operators.
- Regulate the actions of the players on the ‘local level’.

Structural rules

- Complement the particle rules so that it is determined in detail how dialogues can be conducted.
- Taken jointly the dialogue rules define, for all (first-order) sentences A, the dialogue $\mathcal{D}(A)$ about A.
- The choice of structural rules affects the dialogical meaning associated with logical operators.
- It also affects the 'semantic attributes' being characterized:

A has the attribute α iff

There is a winning strategy for player **P** *in $\mathcal{D}(A)$.*

Here α may, depending on the case, be for example 'materially true', 'intuitionistically valid' or 'classically valid'.

Structural rules for formal dialogues $\mathcal{D}(A)$

(1) **Starting rule:** Initially **P** utters A (if possible). Then **O** and **P** each choose a natural number n resp. m (termed their repetition ranks). Thereafter the players move alternately, each move being an attack or a defense.

(2) **Repetition rule:** In the course of the dialogue, **O** (**P**) may attack or defend any single (token of an) utterance at most n (resp. m) times.

(3) **Winning rule:** Whoever cannot move has *lost* and his or her adversary has *won*.

(4) **Formal rule:** Player **P** may not utter an atomic sentence unless it has already been uttered by **O**.

Formal dialogues (cont.)

(5a) **Intuitionistic rule:** Each player may attack any complex sentence uttered by the adversary, or respond to *the last attack to which no defense has yet been presented*. That is, the move that has been attacked last must be defended first. Consequently one cannot in general postpone a defense very much without losing the possibility of the defense. Also, no revised defenses possible.

(5b) **Classical rule:** Each player may attack any complex sentence uttered by the adversary, or respond to *any* attack, including those that have already been defended. Consequently, one can postpone responses to attacks indefinitely without losing the possibility of the defense. Also, revising old defenses is possible.

Example

Consider playing the classical dialogue $\mathcal{D}_c(A \vee \neg A)$:

O			P	
			$A \vee \neg A$	0
1	$n := 1$		$m := 2$	2
3	$?_\vee$	0	$\neg A$	4
5	A	4	—	
			A (revising the defense against move 3)	6

Then think of playing the intuitionistic dialogue $\mathcal{D}_{int}(A \vee \neg A)$:

O			P	
			$A \vee \neg A$	0
1	$n := 1$		$m := 2$	2
3	$?_\vee$	0	$\neg A$	4
5	A	4	—	

Outline

Meaning is a matter of dialogue rules

- Characteristically, the dialogical approach suggests that meanings of logical operators are given by laying down the dialogue rules — specifying the play level of dialogues.
- Different rules — different argumentative practices.

  Different practices typically yield different meanings to the logical operators. Cf. intuitionistic and classical logic.
- The ‘core meaning’ (particle rules) of logical operators remains constant when varying the structural rules.
  - Incidentally, this is one way of attempting to make sense of ‘logical pluralism’ [Rahman et al.]

Outline

“Strategic notions” and winning strategies

- Until now nothing has been said of ‘semantic attributes’ such as truth, falsity, validity or refutability.
- In the dialogical approach such notions are defined via the notion of winning strategy: e.g.,
  - A is classically valid if there is a winning strategy for **P** in the formal dialogue $\mathcal{D}_c(A)$.
  - A is intuitionistically valid if there is a winning strategy for **P** in the formal dialogue $\mathcal{D}_{int}(A)$.
  - A is materially true if there is a winning strategy for **P** in the material dialogue $\mathcal{D}_{mat}(A)$. Definition
- Semantic attributes serve to describe global properties of a dialogue. They are viewed as metatheoretical notions.

Division of labor

- The level of plays: meaning
  - No reference to strategic notions.
- The level of strategies: semantic attributes
  - In a formal dialogue, the existence of a w.s. for **P** marks validity. A specific w.s. for **P** serves to prove validity.
- **Note:** Meaning relative to the type of dialogue considered.
  - Expectable when classical formal dialogues and intuitionistic formal dialogues are compared.
  - *Less expectable* when material dialogues and classical formal dialogues are compared.

The justification of dialogue rules?

- The viability of dialogical semantics depends on whether we accept dialogue rules as constitutive of meaning.
- On what basis can we do so?
- Dialogicians would claim that:
  - with dialogue rules we have reached the semantic rock-bottom; they cannot be justified with reference to anything more basic.
  - meaning is a matter of linguistic practices and dialogues are a reasonable theoretical regimentation of such practices — which in any case are about commitments created by utterances.

The justification of dialogue rules? (cont.)

Someone not accepting the dialogical approach at the outset would like to see why certain rules are chosen.

- Particle rules are supposed to explicate commitments.
- Do not commitments typically presuppose a semantic attribute *relative to which* they are commitments: committed to A being true, provable, refutable, etc. ?
- In the dialogical approach, semantic attributes are meant to emerge on the strategic level. So it is crucial that they are not presupposed already by the play level.
- Particle rules look like rules about commitments.
- The dialogician must either insist that the rules come first and they give rise to the notion of commitment; or insist that commitments do not presuppose semantic attributes.

Outline

Proof-conditional semantics

- Basic notions: proof (object), constructive procedure.
  - Basic notions in dialogues: types of moves.
- Meanings of logical operators explicated in terms of the notion of proof.
  - In dialogues: explicated in terms of possible moves.
- Lays down how proofs of complex sentences are related to proofs of certain syntactically less complex sentences.
  - In dialogues: How utterances are related to syntactically less complex utterances in terms of attacks and defenses.

Proof-conditional semantics (cont.)

- Already the basic semantic notion is of strategic character
  - being provable $\hat{=}$ the existence of a w.s. for **P**
  - a proof object $\hat{=}$ a w.s. for **P**;

    constructive procedures (when applied to proof objects) would correspond to some sort of higher-order strategies.
  - no counterpart to the play level.
- The corresponding semantic maneuver in dialogues would be to suggest that meanings are defined in terms of winning strategies.
- But the framework of dialogues introduces the distinction *play level / strategic level*. The ground level of plays finds its theoretical use as the locus of meaning constitution.

Truth-conditional semantics

- In typical formulations of truth-conditional semantics, truth is taken as a basic notion.
- Now, truth — like proof — is a ‘strategic notion.’
- So, also truth-conditional semantics attempts to explicate the meanings of logical operators using strategic notions.
- For the dialogician this would mean making meaning a metatheoretical issue. While in the dialogical approach issues of truth, validity etc. are viewed as such, questions of meaning are not.

Outline

The *Eigenart* of the dialogical approach

- The dialogical approach proposes an original account of semantics of logical operators.
  - Locates meaning in the play level.
- Truth-conditional and proof-conditional approaches operate with 'strategic notions' (truth, proof).
  - They do not recognize a more fundamental level of meaning constitution.
  - Dialogues, again, propose an analysis of these notions.

Plausibility

- The tenability of the dialogical viewpoint depends on how one succeeds in arguing for the status of dialogue rules.
- If they can be motivated or explicated or understood only with reference to some strategy-level notion, we are running in circles.
- This is one of the kinds of philosophical issues that the philosophically relevant game-based approach to logic that was the topic of this talk must come to grips with.

Material dialogues

- The particle rules remain intact.
- The factual truth-value of each atomic sentence assumed to be given. (In practice: a model is assumed to be given.)
- Structural rules modified as follows:
  - Repetition ranks are allowed to be infinite ordinal numbers if the domain of the model is infinite.
  - The winning rule: whoever utters a false atomic sentence, or cannot move, has lost while the adversary has won.
  - Material dialogues have no formal rule.
  - As a matter of fact, it makes no difference whether the intuitionistic rule (5a) or the classical rule (5b) is adopted.

Back

Improper *vs.* proper operators

The distinction between improper ($\rightarrow, \neg, \forall$) and proper ($\vee, \wedge, \exists$) operators appears as follows in the particle rules:

- Suppose X is the player who utters a sentence and Y the one who attacks this utterance.
- Then it is precisely when the attack pertains to an improper operator that the attack induces a 'commitment' on Y:
  - $A \rightarrow B$: player Y utters the antecedent A;
  - $\neg A$: player Y utters the negated sentence A;
  - $\forall x A$: player Y introduces a constant symbol t.
- With the proper operators Y does not get 'committed' to anything of the sort, she just reminds X of his 'commitments'.
- In the 'dialogue tableaus' two sides would not be needed, were it not for the improper operators.