Dialogical Properties of Obligationes

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What is an *obligatio*?

Definition

An *obligatio* is a turn-based disputation between two agents, the Opponent and the Respondent, where the Opponent puts forward a sequence of propositions, and the Respondent is obligated to follow certain rules in his responses to the Opponent's propositions.

The Respondent has three (four) actions: Concede or accept; deny or reject; doubt(; and draw distinctions).

Recent research on *obligationes*.

- The origin of *obligationes* is unclear, as is their purpose.

  Uckelman, S.L., 2008. "What is the point of *obligationes*?", talk presented at Leeds Medieval Congress, July 2008: http://staff.science.uva.nl/~suckelma/latex/leeds-slides.pdf

- First treatises edited in the early 1960s; most research dates from late 1970s and later.

- Few treatises currently translated out of Latin; not very accessible to non-medievalists.

Authors who wrote on *obligationes*.

- William of Sherwood (1190–1249).
- Nicholas of Paris (fl. 1250).
- Walter Burley (*or* Burleigh) (c. 1275–1344).
- Roger Swyneshed (d. 1365).
- Richard Kilvington (d. 1361).
- William Ockham (c. 1285–1347).
- Robert Fland (c. 1350).
- John of Holland (1360s).
- Richard Brinkley (temp. 1365–1370).
- Richard Lavenham (d. 1399).
- Ralph Strode (d. 1387).
- Peter of Candia (late 14th C).
- Peter of Mantua (d. 1399).
- Paul of Venice (c. 1369–1429).

Obligationes according to Burley (1).

Burley defines the general goal of an *obligatio* as follows:

> *The opponent's job is to use language in a way that makes the respondent grant impossible things that he need not grant because of the positum. The respondent's job, on the other hand, is to maintain the positum in such a way that any impossibility seems to follow not because of him but rather because of the positum.*

$\Rightarrow$ the goal is consistency, not logical truth or validity.

References

Burley, W., *Tractatus de obligationibus*, in R. Green, ed., *An introduction to the logical treatise 'de obligationibus'* 2 (1963).

Burley, W., *Obligations (selections)*, in N. Kretzmann & E. Stump, eds., *The Cambridge translations of medieval philosophical texts* 1: logic and the philosophy of language (Cambridge University Press, 1988): 369–412.

Obligationes according to Burley (2).

General Rule 1 Everything following from an *obligatum* must be granted
  (where '*obligatum*' is interpreted as what has been granted or
  what must necessarily be granted).

General Rule 2 Everything incompatible with the *obligatum* must be
  denied.

General Rule 3 One must reply to what is irrelevant in accordance with its
  own quality.

Definition

A proposition is *irrelevant* or *impertinent* if neither it nor its negation
follows from the set of propositions which have already been conceded
(which includes the negations of propositions which have been denied).

An example *positio.*

Suppose φ does not imply $\neg\psi$ and φ is known to be false.

Opponent	Respondent	Notes
I posit φ.	I admit it.	$\Phi_0 = \{\varphi\}$.
$\neg\varphi \vee \psi$.	I concede it.	Either φ implies ψ, then the sentence is relevant and follows from Φ_0; or it doesn't, then it's irrelevant and true (since φ is false); $\Phi_1 = \{\varphi, \neg\varphi \vee \psi\}$.
ψ	I concede it.	Pertinent, follows from Φ_1.

(This example shows how, given a *positum* which is false, but not inconsistent, the Opponent can force the Respondent to concede any other consistent proposition.)

Dialogical games.

1. Two players (Opponent and Proponent)
2. Set of possible/allowed/legal moves
3. Set of strategies, of which a subset is designated as winning

Characteristics of *obligationes*.

1. Two players (Opponent and Respondent)
2. Set of possible/allowed/legal moves
3. Concept of winning

Characteristics of *obligationes*.

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Characteristics of *obligationes*.

1. Two players (Opponent and Respondent)
2. Set of possible/allowed/legal moves ⇒ Set of required moves.
3. Concept of winning ⇒ winning strategy (?)

Are *obligationes* (dialogical) games?

- Meanings of connectives
- Closed- vs. open-world models
- Winning conditions
- Goal of the games

Meanings of connectives.

- In Lorenzen's dialogical logic, the logical constants gain their meaning via their attack and defense rules.
- In *obligationes*, the truth conditions for the logical constants must be known in advance, otherwise the Respondent cannot make any inferences.

Closed- vs. open-world models.

- Dialogical logic works only in *closed-world* situations:
  - ▶ The Proponent is not allowed to assert any atom which the Opponent has not already asserted.
  - ▶ One feature of a successful strategy for the Proponent will be to force the Opponent into asserting as many atom as possible.
  - ▶ The Opponent wants to assert as few atoms as possible, and in particular, he never has any incentive to assert any atom which does not occur in the formula which is under discussion.
- *Obligationes* are *open-world* dialogues:
  - ▶ It can be advantageous to the Opponent to introduce formulas into the disputation which involve proposition letters not present in the *positum*.
  - ▶ Not only is this possible, but it's often necessary in order to trap the Respondent into responding badly.

Winning conditions (1).

- In dialogical logic (as presented in [RK]) infinite-length games are allowed, but finite termination conditions can be given (i.e., a fixed-point is reached), and hence winning conditions can be easily defined.

- Because *obligationes* are open-world, games can be infinite without reaching a fixed point. But in practice, the Opponent will always call "*Cedat tempus*" after a finite amount of time has elapsed.

References:
[RK] Rahman, S. & L. Keiff, "On how to be a dialogician", in D. Vanderken, *Logic, thought, and action*: 359–408.

Winning conditions (2).

So how to define winning conditions?

- Opponent has won if, when he calls "*Cedat tempus*", the Respondent has conceded a contradiction, or has both conceded and denied the same proposition.
- Under what conditions has Respondent won?
  - ▶ Just because the Respondent has not conceded a contradiction after a finite amount number of steps is no guarantee that *positum* is consistent. It is always possible that the Opponent has not yet introduced new atoms which will be the cause of the Respondent's downfall.
  - ▶ Paul of Venice: Respondent wins if the Opponent gives up before Respondent has conceded a contradiction.

Goal of the games.

- Dialogical logic: prove/demonstrate logical validity of formulas.
  - ▸ Validity or invalidity is only known after game has terminated
- *Obligationes*: truth-value of initial statement should be known in advance.
  - ▸ *Obligationes* as proofs of possibility or consistency [DN].

References:
Dutilh Novaes, C. *Formalizations après la Lettre*, Ph.D. thesis, Universiteit Leiden (2005).

An alternative approach

King explains the apparent "content-freeness" of obligational disputations by point out that

> they operate at a higher level of logical generality than that at which substantive debate occurs. If this is correct, then actual obligational moves—perhaps even recognized as such—are the vehicle whereby real argument takes place

and thus *obligationes* provide a "meta-methodology" for reasoning.

Reference:
King, P. "Opposing and responding: comments on Paul Spade", preprint, 2004,
http://individual.utoronto.ca/pking/presentations/Spade_Comments.pdf

Abstract dialogue systems (1)

Definition

An *abstract dialogue system* contains the following elements:

- A *topic language* $\mathcal{L}_t$, closed under classical negation.
- A *communication language* $\mathcal{L}_c$. We denote the set of *dialogues*, that is, the set sequences of $\mathcal{L}_c$, by $M^{\leq\infty}$, and the set of finite sequences of $\mathcal{L}_c$ by $M^{<\infty}$. For a dialogue $d = m_0, \ldots, m_n, \ldots$, the subsequence $m_0, \ldots, m_i$ is denoted d_i.
- A *dialogue purpose* or *goal*.
- A set $\mathcal{A}$ of *agents* (participants) and a set $\mathcal{R}$ of *roles* that the participants can occupy. Each participant a has a (possibly empty) *belief base* $\Sigma_a \subseteq \mathcal{P}(\mathcal{L}_t)$ and a (possibly empty) *commitment set* $C_a(d_n) \subseteq \mathcal{L}_t$.
- A *logic L* for $\mathcal{L}_t$.

Abstract dialogue systems (2)

Definition (con't)

- A *context* $K \subseteq \mathcal{L}_t$, representing the (shared, consistent, and unchanging) knowledge of the agents specified at the outset.

- A set E of *effect rules* $\mathrm{C}_a(d_n) : M^{<\infty} \to \mathcal{P}(\mathcal{L}_t)$ for $\mathcal{L}_c$, specifying how utterances $\varphi \in \mathcal{L}_c$ in the dialogue affect the commitment stores of the agents.

- A *protocol* P for $\mathcal{L}_c$, specifying the legal moves of the dialogue, which is a function from the context and a non-empty $D \subseteq M^{<\infty}$ to $\mathcal{P}(\mathcal{L}_c)$, satisfying the requirement that if $d \in D$ and $m \in \mathrm{P}(d)$, then $d, m \in D$. The elements of D are called *legal finite dialogues*, and $\mathrm{P}(d)$ is the set of moves allowed after move d. At any stage, if $\mathrm{P}(d) = \emptyset$, then the dialogue has *terminated*. A protocol will often be accompanied by a *turn-taking* function $T : D \to \mathcal{P}(\mathcal{A})$, which takes a finite dialogue d_n and specifies who governs move m_{n+1}, and *termination* conditions, which specify when $\mathrm{P}(d) = \emptyset$.

- A set of *outcome rules* O.

Properties of protocols

Definition

- A protocol has *public semantics* iff the set of legal moves is always independent from the agents' belief bases.
- A protocol is *context-independent* iff the set of legal moves and the outcome is always independent of the context, that is, $P(K, d) = P(\emptyset, d)$.
- A protocol is *fully deterministic* iff P always returns a singleton or the empty set.
- A protocol is *unique-move* iff the turn shifts after each move; it is *multiple-move* otherwise.

Protocols which are not fully deterministic are *permissive*.

Approaching *obligationes* as ADSs

- Two designated roles: Opp (Opponent) and Res (Respondent).
- Dialogue purpose: consistency (in Burley-style *positio*).
- Topic language $\mathcal{L}_t$ = the communication language $\mathcal{L}_c$.
- Turn-taking protocol is unique-move: $T(\emptyset) = \text{Opp}$, $T(d_n) = \text{Opp}$ if n is odd, and $T(d_n) = \text{Res}$ if n is even.
- Protocol P: Opp's moves are not constrained, Res's moves must be made in reaction to the move of Opp at the previous stage.
- Effect rules E: similarly constrained.
- Outcome rules: If Res realizes the goal, then he wins. If Opp realizes the goal, then he wins.

The underlying logic: Dynamic Epistemic Logic

Definition

(Actions of Res). Let φ_n be a proposition put forward by Opp. The possible actions of Res (designated Act) are:

$$
\begin{array}{ll}
\textbf{concede}: & [\varphi_n?]\top \\
\textbf{deny}: & [\neg\varphi_n?]\top \\
\textbf{doubt}: & [\top?]\top
\end{array}
$$

With the following truth conditions:

$$\mathfrak{M}, w \vDash [\varphi?]\psi \quad \text{iff} \quad \forall v \in \mathfrak{M} \upharpoonright \varphi, v \vDash \psi$$

Note

The last clause in this definition is equivalent to saying "I don't know"; $[\top]\top$ will always be valid.

Knowledge bases

Definition

Given an epistemic model $\mathfrak{M}$, the *knowledge bases of* Opp *and* Res are defined as follows:

$$
\begin{aligned}
\Sigma_{\mathsf{Opp}}^{\mathfrak{M}} &:= \{\varphi \colon \mathfrak{M}, w^* \vDash K_{\mathsf{Opp}}\varphi\} \\
\Sigma_{\mathsf{Res}}^{\mathfrak{M}} &:= \{\varphi \colon \mathfrak{M}, w^* \vDash K_{\mathsf{Res}}\varphi\}
\end{aligned}
$$

Protocols in *obligationes*

> **Definition**
>
> Let α be a designated formula representing "*cedat tempus*". The *uniform protocol* P_u is invariant over all contexts and is defined for a finite dialogue d_m:
>
> $$\mathrm{P}_u(\emptyset) = \mathcal{L}_c$$
>
> if $m_n = \alpha$ $\mathrm{P}_u(d_n) = \emptyset$
>
> otherwise, if n is odd, $\mathrm{P}_u(d_n) = \mathcal{L}_c$
>
> and if n is even, $\mathrm{P}_u(d_n) = \{[m_n?]\top, [\neg m_n?]\top, [\top?]\top\}$

Formation of the commitment sets

Definition

The rules governing the commitment sets $\mathrm{C_{Opp}}$ and $\mathrm{C_{Res}}$ are defined as follows:

$$
\begin{array}{ll}
\text{for all } n & \mathrm{C_{Opp}}(d_n) = \emptyset \\
\text{if } n \text{ is even} & \mathrm{C_{Res}}(d_n) = \mathrm{C_{Res}}(d_{n-1}) \\
\text{if } n \text{ is odd} & \mathrm{C_{Res}}(d_n) = \mathrm{C_{Res}}(d_{n-1}) \cup \{m_n\}
\end{array}
$$

Protocol specifying required moves

Definition (Burley's protocol)

Let Γ_n be the sequence of Res's move in a dialogue d_n. For a DEL model $\mathfrak{M}$ and context K, $\mathrm{P}^{\mathsf{Bur}}(K, \emptyset) = P_u(\emptyset)$ and if n is odd, $\mathrm{P}^{\mathsf{Bur}}(K, d_n) = P_u(d_n)$. For n even,

- For $d_0 = m_0 =$ the *positum*,

$$\mathrm{P}^{\mathsf{Bur}}(K, d_0) = \begin{cases} \textbf{concede}{:}m_0 & \text{iff } \exists w \in W, \mathfrak{M}, w \models m_0 \\ \textbf{deny}{:}m_0 & \text{iff } \forall w \in W, \mathfrak{M}, w \nvDash m_0 \end{cases}$$

- For $d_n, n > 0$:

  If $\mathfrak{M} \upharpoonright \Gamma_n \models m_n$: $\mathrm{P}^{\mathsf{Bur}}(K, d_n) = \textbf{concede}{:}m_n$

  If $\mathfrak{M} \upharpoonright \Gamma_n \models \neg m_n$: $\mathrm{P}^{\mathsf{Bur}}(K, d_n) = \textbf{deny}{:}m_n$

  Otherwise:

  If $\mathfrak{M}, w^* \models K_{\mathsf{Res}} m_n$: $\mathrm{P}^{\mathsf{Bur}}(K, d_n) = \textbf{concede}{:}m_n$

  If $\mathfrak{M}, w^* \models K_{\mathsf{Res}} \neg m_n$: $\mathrm{P}^{\mathsf{Bur}}(K, d_n) = \textbf{deny}{:}m_n$

  If $\mathfrak{M}, w^* \models \neg(K_{\mathsf{Res}} m_n \vee K_{\mathsf{Res}} \neg m_n)$: $\mathrm{P}^{\mathsf{Bur}}(K, d_n) = \textbf{doubt}{:}m_n$

Two outcome rules

Definition (Local winning)

If $m_n = \alpha$, then Opp *wins* if $\mathfrak{M} \restriction \Gamma_n = \langle \emptyset, \{\sim_a^{\mathfrak{M}, \Gamma_n} : a \in A\}, V^{\mathfrak{M}, \Gamma_n} \rangle$ and Res wins otherwise.

Definition (Global winning)

Opp *wins* if there is some n such that
$\mathfrak{M} \restriction \Gamma_n = \langle \emptyset, \{\sim_a^{\mathfrak{M}, \Gamma_n} : a \in A\}, V^{\mathfrak{M}, \Gamma_n} \rangle$. Res wins otherwise.

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- While superficially *obligationes* look like games, on close inspection the analogy falls apart.

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- They have more in common with abstract dialogue frameworks as developed in argumentation theory and AI.
- Viewing them this way provides support for King's analysis of obligational rules as working at the meta-level.

Thank You