Winning Strategies in Two-Player Games with Partial Information

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- As usual, game graphs are non-terminating.

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$$\pi \sim_i \pi' \Longrightarrow f(\pi) = f(\pi')$$

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2^{nd} Extend $\sim_i^V$ to $\sim_i$.

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  $\pi \overleftarrow{\sim}_i \pi'$ iff $\overleftarrow{\pi} \sim_i \overleftarrow{\pi}'$ where
  $\overleftarrow{\pi}$ is obtained from π by deleting all moves $u \to v$ from π such that $u \in V_{1-i}$ and $u \sim_i^V v$.
  (Asynchronous case.)

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Thus, we can ignore the partial information of player 1 here!

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Idea:
Turn a game with partial information into a game with full information such that the existence of winning strategies for player 0 is preserved.

$\rightsquigarrow$ Powerset Construction

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- Given a Büchi automaton $\mathcal{B}$ with $L(\mathcal{B}) = W_0$, one can construct a Büchi automaton $\overline{\mathcal{B}}$ with $L(\overline{\mathcal{B}}) = \overline{W}_0$. ($\omega$-regular languages are closed under complementation.)

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Theorem

- *The strategy problem for ω-regular games with partial information is decidable.*
- *Finite memory strategies can be synthesized.*

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Asynchronous Case

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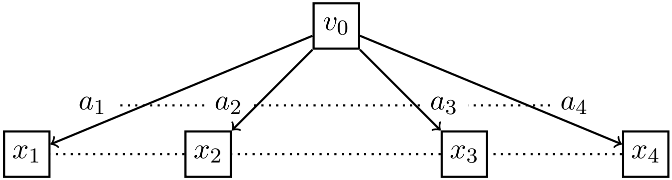

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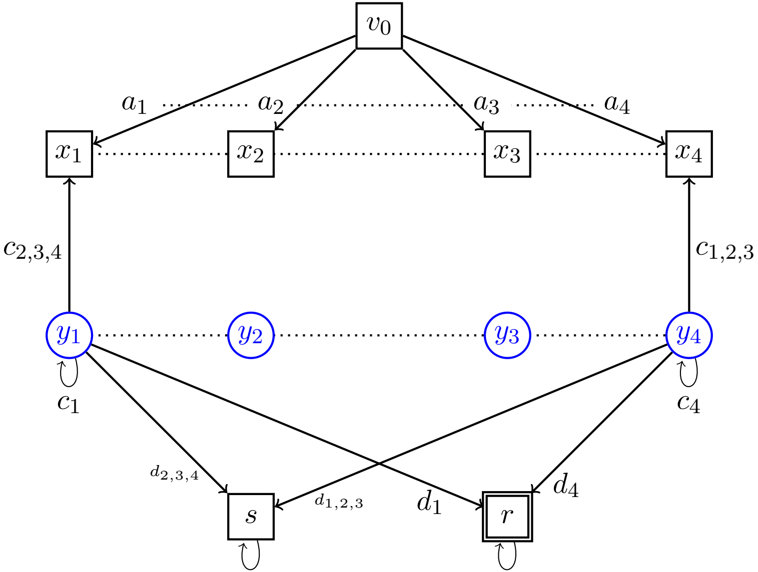

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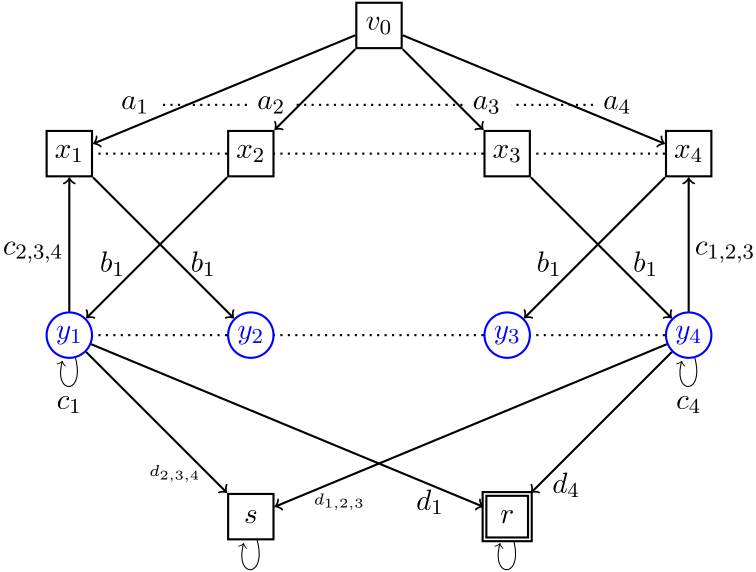

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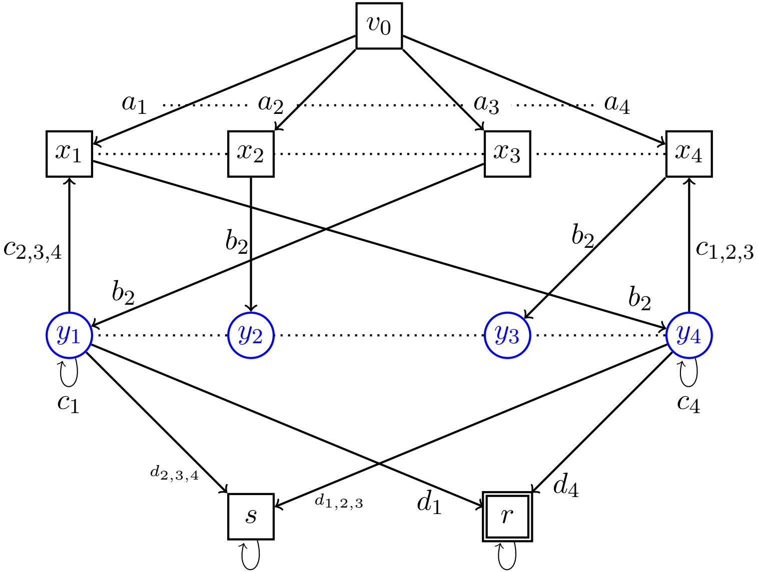

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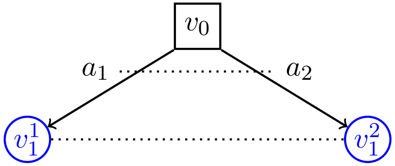

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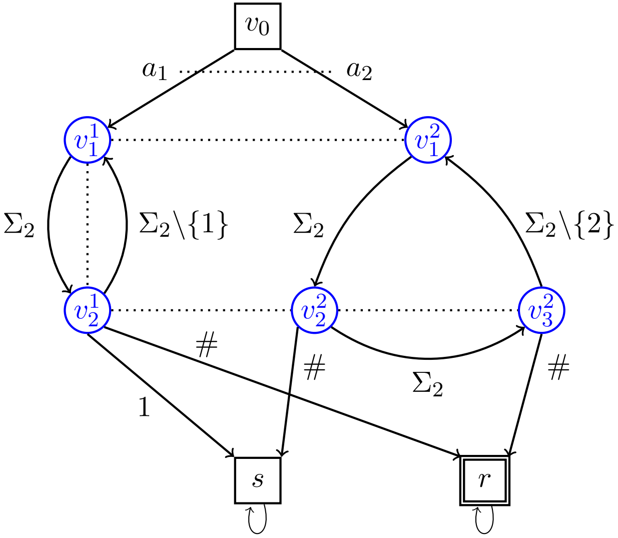

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Alternating tree automaton:

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$\leadsto$ Three player game with partial information.

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If $\mathcal{A}$ is universal, then the game is a two-player game with partial information!

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(2) Restrict the strategies of player 0 to information based strategies.

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- The "narrowing" of a deterministic automaton is universal.

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