Imperfect-Information Games in Computing

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Normal vs extensive form

- Normal-form game for n players:

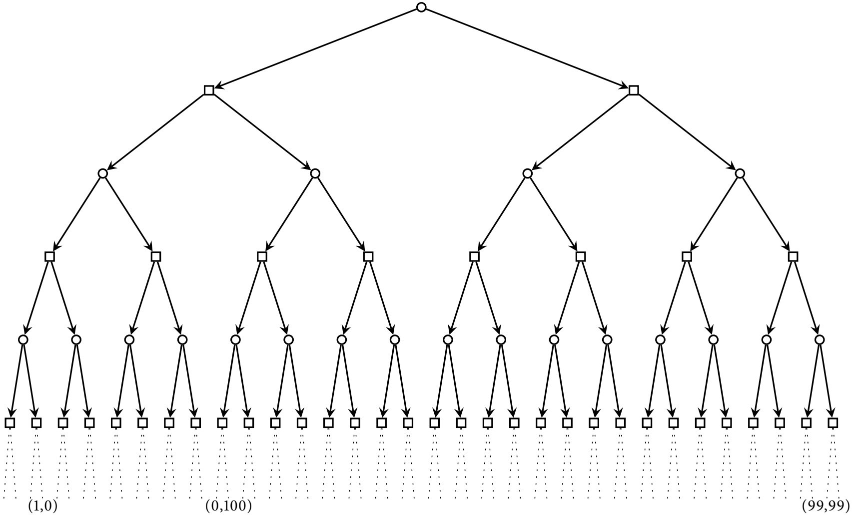
$$\Gamma = \big((S^i)_{i<n}, (u^i)_{i<n} \big)$$

Each player picks a strategy $s^i \in S^i$:

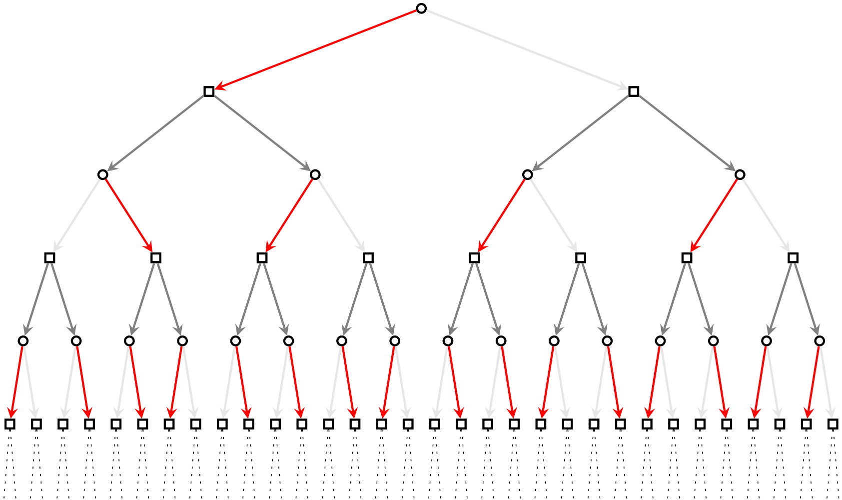
$$\textbf{outcome } s = (s^0, s^1, \dots, s^{n-1}) \quad \blacktriangleright \quad \textbf{payoff } u^i(s).$$

- Extensive form: **structure** in strategies.

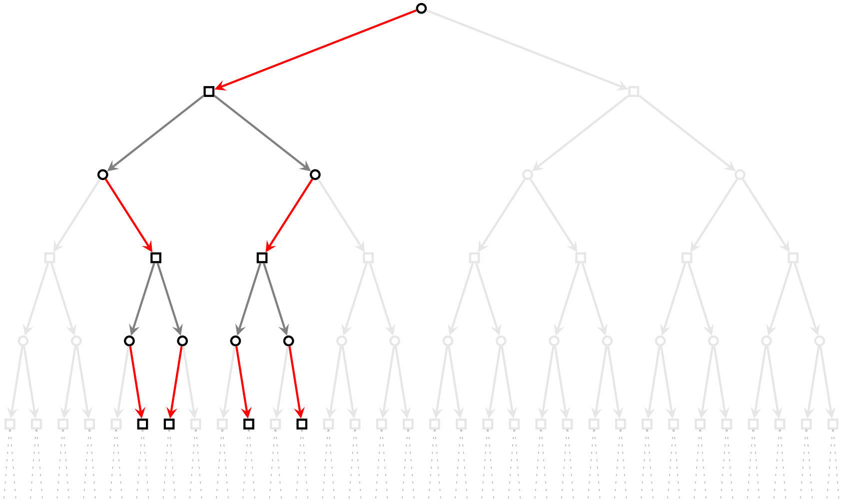
Perfect information: transition structure



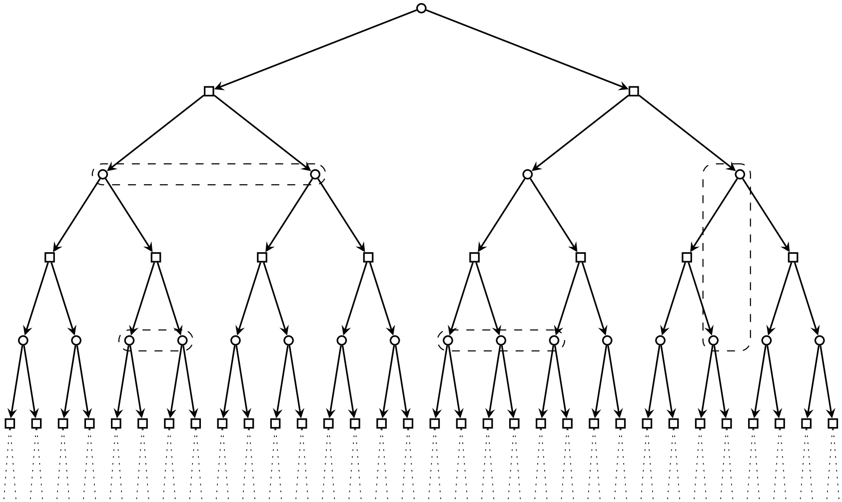
Strategy with perfect information



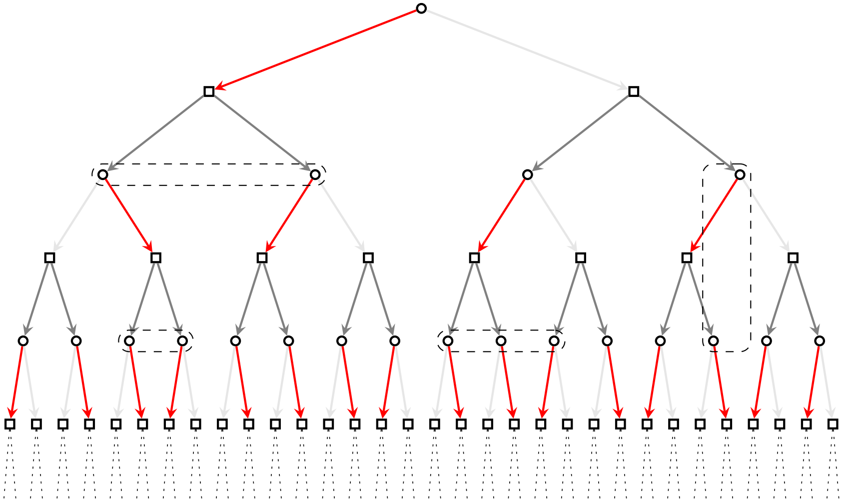
Pruning



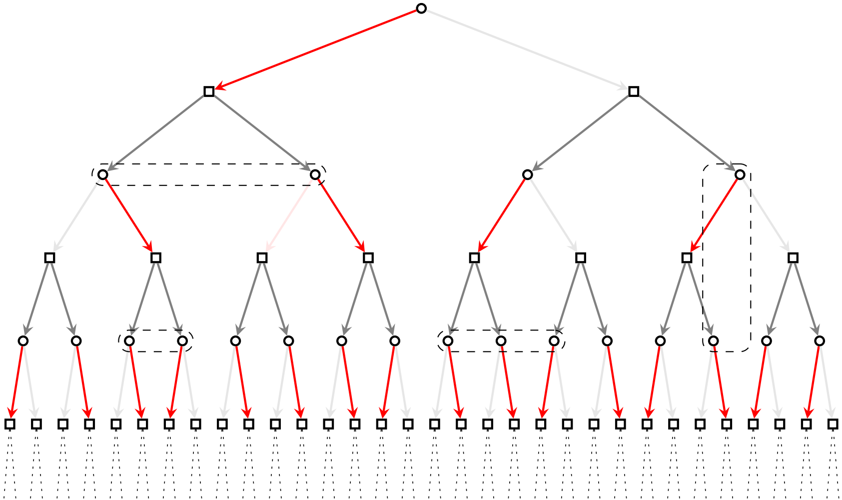
Imperfect information: + information structure



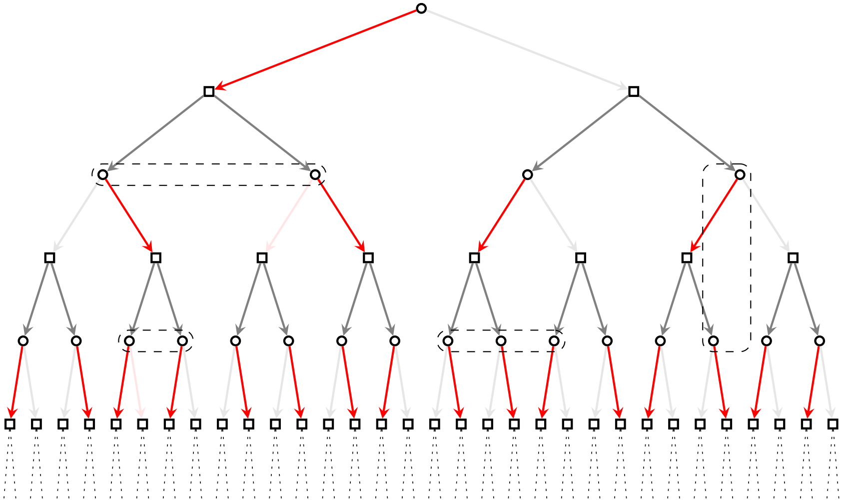
Strategy with imperfect information



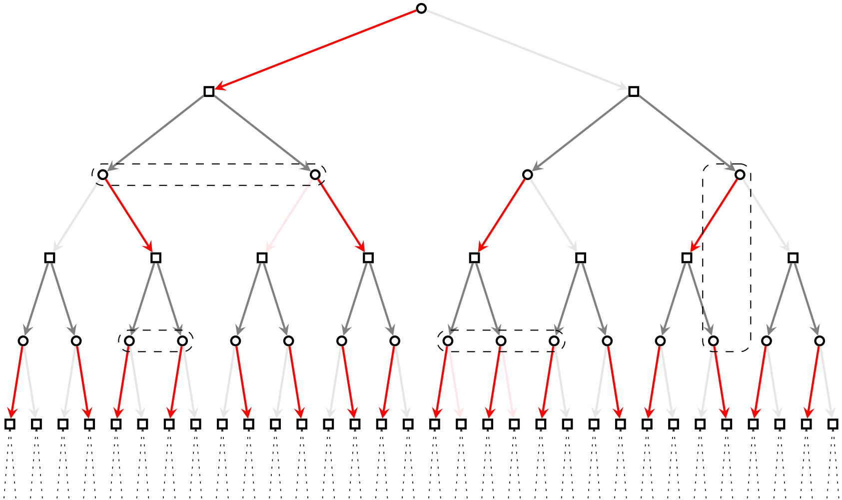
Strategy with imperfect information ▸



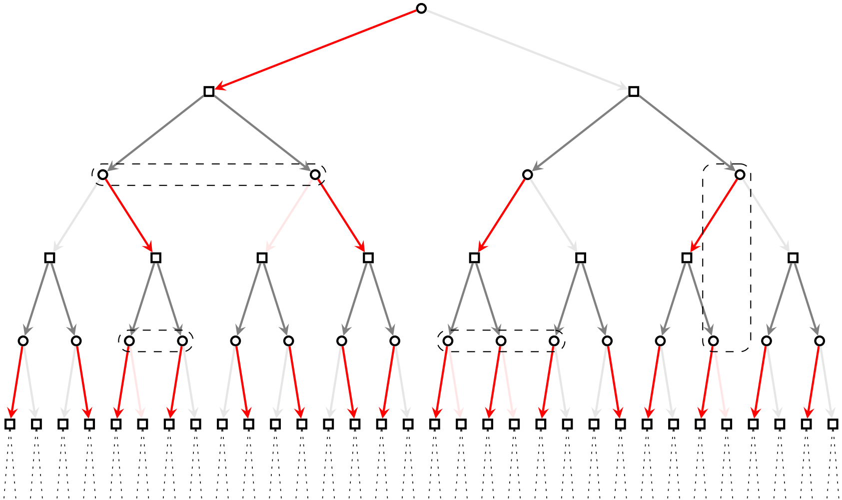
Strategy with imperfect information ▸▸



Imperfect-information strategy ▸▸▸



Strategy with imperfect information ▸▸▸▸



Characterisation

- Imperfect information:

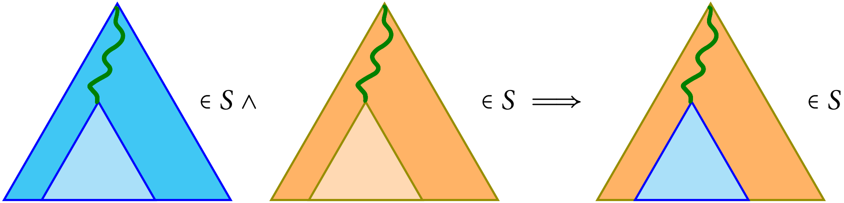
  strategy: information set $\mapsto$ action

- Perfect information:

  all information sets are singletons

Another characterisation

The perfect-information property of a strategy set:



Imperfect information is about available ***sets of strategies***

Games in Computation

Wishlist:

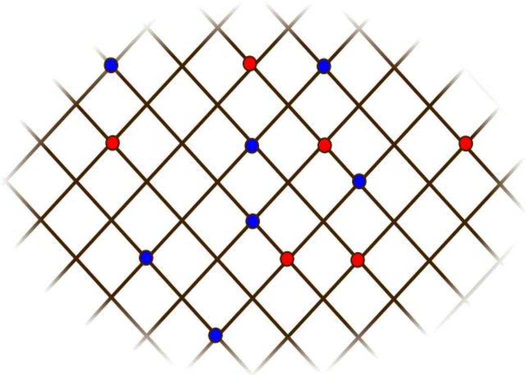
- model plays of infinite duration
- capture uncertainty about
  - order of moves
  - initial state
  - implementation details

+ compositionalilty and tractable complexity

Why and how

- Automata theory
  - nondeterminism
  - synchronisation/homing sequences
- Controller synthesis
  - Plant, supervisor, observable/controllable event
  - Safety conditions
- Distributed computation
  - private/public variables, alternation, scheduling
  - Acceptance: reachability conditions

Verification: reactive dynamics



System: actions ↑, ↓ ▸ ◂ Environment: observations •, •, ∘

Winning conditions

- Reachability: observe a good event
- Safety: never observe a bad one
- Büchi: something good over and over again

- Parity: observation priorities — least one seen infinitely often is even
  - nested Reachability & Safety
- ω-regular: finite-state monitor is satisfied

Transform a reactive model into a **game**.

Parity games with imperfect information

Parity games are **generic** for ω-regular specifications.

Strategy: $(\text{Observations})^* \to \text{Actions}$.

Questions:

- **decide** whether the system can ensure a win
- **construct** a winning strategy

Assumptions here:

• strictly turn based • observable winning condition • sur win

Classical solution [Reif84]

Powerset construction:

- keeps track of what the system can distinguish from memory
- yields game with perfect information (over the powerset)

Winning positions and strategy can be transferred back and forth.

Corollary: Memoryless determinacy / perfect-information

- **Finite-memory determinacy** / imperfect information

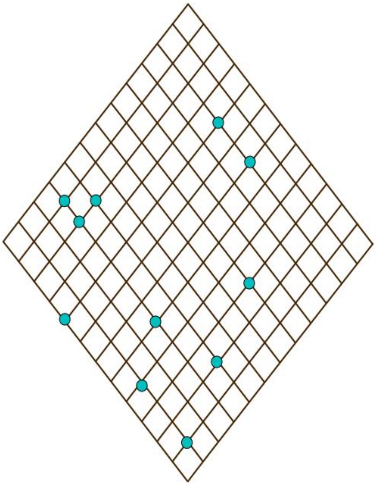
Complexity

- Problem is **Exptime**-hard
- Exponential **memory** might be needed.

- However, the information-set construction
  - uses **exponential** space
  - has **no on-the-fly** solution
  - is independent of **objective**

Can we do better?

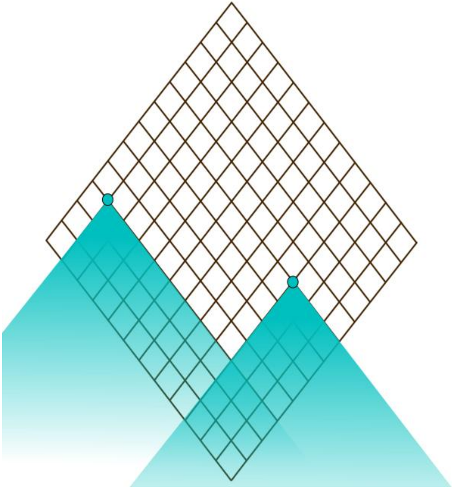
Antichains



- Interesting sets

  have a particular structure

Antichains



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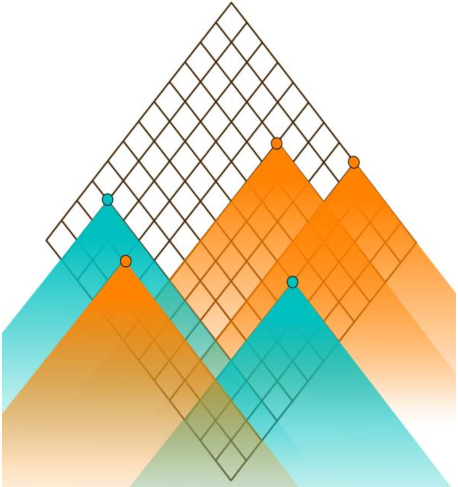
  - **downwards-closed**

Antichains



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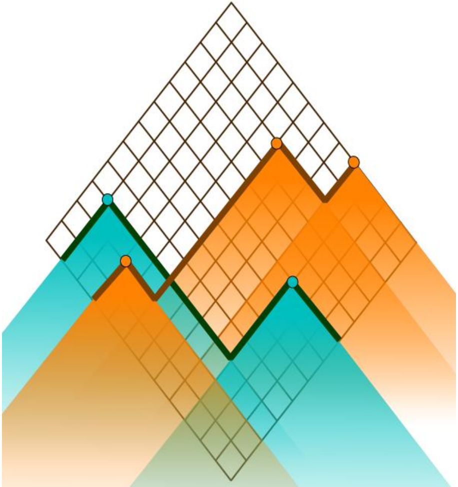
- Interesting operations

  preserve it:

  - CPre, $\cup$, $\cap$, **iteration**

CPre(X) = $\{Y$:
System can force the play into $X\}$.

Antichains



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Algorithm with antichains / imperfect information

[CDHR07]

- Evaluates characterisation of winning positions

  as a μ-calculus formula

  over the lattice of antichains

- Strategy **construction** more tricky, but works.

Three players = Trouble

Two players, with common winning condition $W \subseteq V^*$ (all finite!)
the third one is indifferent.

Q: Are there strategies (s^1, s^2) such, that for all s^3, the outcome is in W.

Folk argument. This problem is undecidable.

Reduction from Halting Problem

Imperfect information + the third player can enforce coordination on

the n-th configuration of a Turing Machine $M = (Q, \Sigma, q_0, \delta)$

in round n.

Ingredients:

- Actions: $\Sigma \cup Q \cup \{\blacktriangleright, \blacksquare\}^2$,
- Observations: $\{\blacktriangleright, \blacksquare\}$
- Mask: Player i sees $\blacksquare$ iff in component i; otherwise $\blacktriangleright$;

Idea

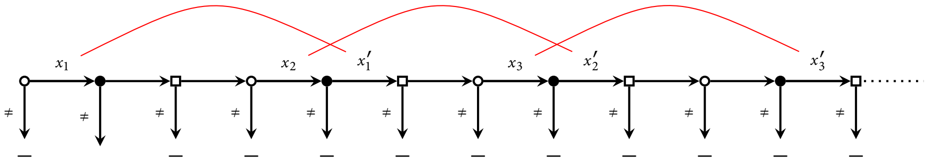
The actions of the two protagonists can produce descriptions (x, x') of machine configurations.

The transition structure can enforce that either

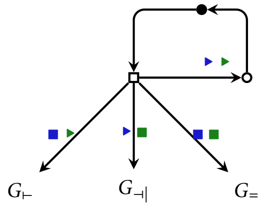
$(G_{\vdash})$ $x \vdash x'$,

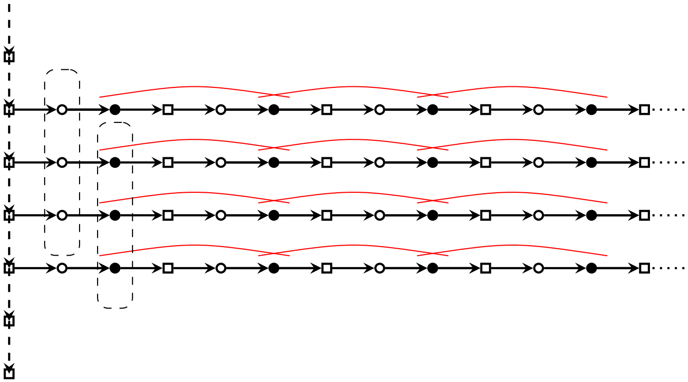
$(G_{\dashv})$ $x \dashv x'$, or

$(G_{=})$ $x = x'$



Construction





Undecidability

- any distributed winning strategy (s^1, s^2) must produce the n-th configuration of M after observing $\blacktriangleright^n \blacksquare$

– defined only, if machine never halts.

Conclusion The distributed control problem is in general undecidable.

Outlook

- Interactive vs Distributed
  - player aggregation
  - inductive solution concepts
- Specific kinds of imperfect information
  - uncertainty about initial state
  - concurrency
- Abstracion, interface theories