

# SOME FUNNY CASES OF IMPLICATION

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Cool logic

## INDICATIVE CONDITIONALS

(1) If I agree with you, then we **will** both be wrong.

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

### MATERIAL IMPLICATION

FALSE ANTECEDENT

TRUE CONSEQUENT

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NOT MATERIAL IMPLICATION  
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UPWARD MONOTONIC

NO MORE MONOTONICITY

### MORE CONDITIONALS

# FALSE ANTECEDENT

$$p \models \bar{p} \rightarrow q$$

(2) The butler did it; hence, if he didn't, the gardener did.

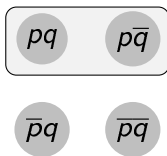


FIGURE 1:  $p$

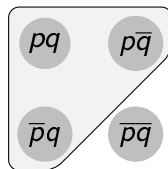


FIGURE 2:  $\bar{p} \rightarrow q$

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# TRUE CONSEQUENT

$$q \models p \rightarrow q$$

- (3) You are sitting here in this classroom. Hence, if you were killed by a bomb this morning, then you are sitting here in this classroom.

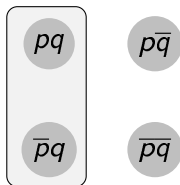


FIGURE 3:  $q$

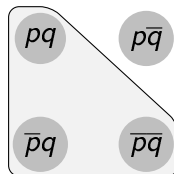


FIGURE 4:  $p \rightarrow q$

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# CONTRAPOSITION

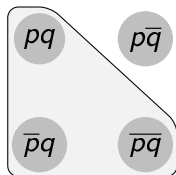


FIGURE 5:  $p \rightarrow q$

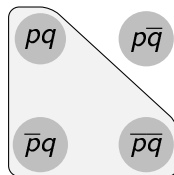


FIGURE 6:  $\bar{q} \rightarrow \bar{p}$

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CONTRAPOSITION:  $p \rightarrow q \equiv \bar{q} \rightarrow \bar{p}$

- (4)
- a. Yog and Zog play chess according to normal rules, but with the special conditions that Yog has white 9 out of 10 times, and there are no draws. Up to now, there have been a hundred games. When Yog had white, he won 80 out of 90. And when he had black, he lost 10 out of 10. Suppose Yog and Zog played one of the hundred games last night and we don't yet know what its outcome was. In such a situation we might utter:
  - b. If Yog had white, he won. 8/9
  - c. If Yog lost, he had black. 1/2

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## BROMBERGER ON: $\overline{p} \rightarrow \overline{q}$

- (5) It's not the case that if there's a god, then we are free to do whatever we like.

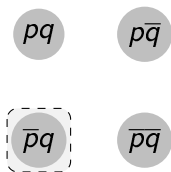


FIGURE 7:  $\overline{p} \rightarrow \overline{q}$

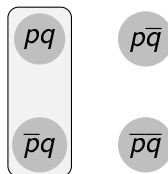


FIGURE 8:  $p$

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# STRENGTHENING THE ANTECEDENT

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## STRENGTHENING THE ANTECEDENT

- (6) a. If Indiana Jones uses his gun, then he wins the knife fight.
- b. If Indiana Jones uses his gun and he is out of bullets, then he wins the knife fight.



# STRENGTHENING THE ANTECEDENT

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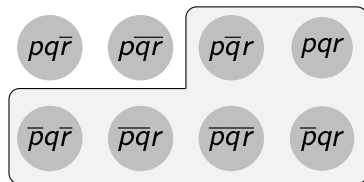


FIGURE 9:  $p \rightarrow r$

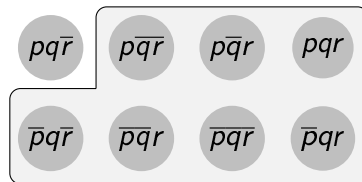


FIGURE 10:  $(p \rightarrow q) \rightarrow r$

# TRANSITIVE LUCK

SOME FUNNY  
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## LOTTERY

- (7)
- a. If I win a million dollars, I will leave my job.
  - b. If I leave my job, I will lose my apartment.
  - c. Hence, If I win a million dollars, I will lose my apartment.

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## REPRESENTATION

- (8)
- a.  $p \rightarrow q$
  - b.  $q \rightarrow r$
  - c.  $p \rightarrow r$

# TRANSITIVE LUCK

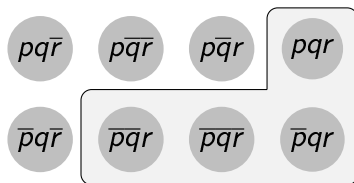


FIGURE 11:  $p \rightarrow q \wedge q \rightarrow r$

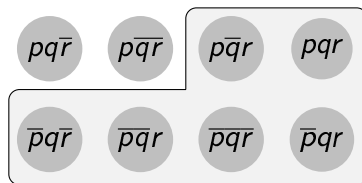


FIGURE 12:  $p \rightarrow r$

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# SPLITTING RESPONSIBILITY

SOME FUNNY  
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## CONJUNCTION TO DISJUNCTION

- (9)     a.    If we release the safety and press the button, we can nuke our neighbours.  
          b.    Hence. If we release the safety, we can nuke our neighbours or if we press the button, we can nuke our neighbours.

## REPRESENTATION

$$(10) \quad (p \wedge q) \rightarrow r \models (p \rightarrow r) \vee (q \rightarrow r).$$

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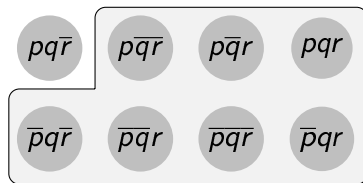


FIGURE 14:  $(p \rightarrow r) \vee (q \rightarrow r)$

## RAMSEY (1931) P.15A

- (11) "If two people are arguing "If  $p$  will  $q$ ?" and are both in doubt as to  $p$ , they are adding  $p$  hypothetically to their stock of knowledge and arguing on that basis about  $q$ ; so that in a sense "If  $p$ ,  $q$ " and "if  $p$ ,  $\bar{q}$ " are contradictories."

# SUPPOSITIONAL CONDITIONALS

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## SUPPOSITIONAL CONDITIONALS

Suppose the antecedent, does the consequent follow?

EDGINGTON (2003)

- (12) ...the *raison d'être* of indicative conditionals depends on the fact that we are uncertain of many things. God doesn't have any use for thoughts of the "If Harry didn't do it . . . " or "If it rains tomorrow . . . " kind.

# STANDARD MODAL LOGIC

SOME FUNNY  
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## QUANTIFICATION OVER POSSIBLE WORLDS

- ▶ Necessity is universal quantification.
- ▶ Possibility is existential quantification.

## KRATZER: THE MODAL BASE

- ▶ Deontic modals quantify over a set of worlds that satisfies a body of law or principles.
- ▶ Worlds need to be accessible from the evaluation world.

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# KRATZER CONDITIONALS

## CONDITIONAL

- (13) a. If I agree with you, then we will both be wrong.  
b.  $p \rightarrow q$

## RESTRICTOR APPROACH

- (14) a. The antecedent of a conditionals restricts the possible worlds against which the modal in the consequent is evaluated.  
b.  $(\Box q)^p$

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# UPWARD MONOTONICITY IN KRATZER SEMANTICS

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## UPWARD MONOTONICITY OF QUANTIFIERS

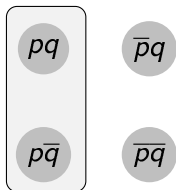
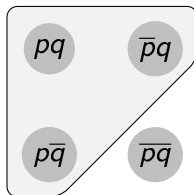
If  $\forall x\phi$  and  $\phi \models \psi$  then  $\forall x\psi$

## INHERITANCE (CARIANI 2012)

If  $\phi \models \psi$  then  $\Box\phi \models \Box\psi$

## ROSS'S PUZZLE

- (15) a. You must mail that letter.  
b. You must mail that letter or wrestle a walrus.

FIGURE 15:  $p$ FIGURE 16:  $p \vee q$ 

## MATERIAL IMPLICATION

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## ALTERNATIVE MODAL LOGICS

# PROF. PROCRASTINATE

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## PROF. PROCRASTINATE

- (16)     a.    Prof. Procrastinate ought to accept and write the book review.  
          b.    Prof. Procrastinate ought not to accept.

## REPRESENTATION

- (17)     a.     $\Box(p \wedge q)$   
          b.     $\Box\bar{p}$   
          c.     $\Box p$

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# CONDITIONALS

## CATCH 22

- (18) If soldiers take your property, then soldiers have permission to take your property.

## REPRESENTATION

- (19)  $p \rightarrow \Diamond p$

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## MODALS

- (20)
- a. You must eat your vegetables.
  - b. You may skip the broccoli.

## ANDERSONIAN MODALS

- (21)
- a.  $\Box\varphi =_{df} \overline{\varphi} \rightarrow s$
  - b.  $\Diamond\varphi =_{df} \varphi \rightarrow \overline{s}$

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## ETERNAL DAMNATION

- (22)
- Jack jaywalked.
  - Hence. Jack must kill everyone.

## REPRESENTATION

- (23)
- $p \wedge s \models s$
  - $s \models \bar{q} \rightarrow s$

# IT LOOKS LIKE A DUCK...

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## RELEVANT IMPLICATION

- (24)
- a.  $\Box p =_{df} \bar{p} \Rightarrow s$
  - b.  $\Diamond p =_{df} p \Rightarrow \bar{s}$



# MONOTONICITY PUZZLES

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BUT...

$$(25) \quad \vdash (p \Rightarrow q) \Rightarrow (\Box p \Rightarrow \Box q)$$

## ASHER, BONEVAC, BARKER, CARIANI, LASSITER

Give up monotonicity.

## MONOTONIC INFERENCES

- (26)
- a. Soldiers may neither rape nor pillage.
  - b. Soldiers may not rape.

## STATE OF THE ART

## SOME FUNNY CASES OF IMPLICATION

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# BISCUIT CONDITIONALS

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## BISCUIT CONDITIONALS

(27) If you're hungry, there are biscuits on the table.

# THANK YOU

Thank you for listening  
The End

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