What's a Theory to Do?
Classicality with the Purpose of Capturing

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Opening remarks

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"But the more difficult the task is, the greater would be the merit of accomplishing what such excellent thinkers—to mention the most illustrious only—as Frege, Russell and Hilbert have tried in vain: namely, **to avoid the logical paradoxes without infringing classical logic.**" – Kurt Grelling, "The Logical Paradoxes"

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- Cobreros et al. do not make a compelling case for the classicality of their logic
- That's okay; STTT still meets Leitgeb's "real" criteria.
- How to decide between largely classical theories of truth which offer different treatments of paradoxical arguments?

Strict-Tolerant Transparent Truth

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STTT is a first-order logic with a transparent truth predicate T and a quotation device $\langle\ \rangle$.

Definition

A truth predicate is **transparent** iff, where φ is some sentence in the language, all occurrences of $T\langle\varphi\rangle$ and φ are intersubstitutable *salva veritate* in all extensional contexts.

- Nice properties: validates all T-biconditionals, represents truth as a predicate which respects compositionality, no type restrictions.
- The fact that STTT's consequence relation is not transitive plays an important role in accounting for paradoxes.

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$$v_g^{\mathcal{M}}(Pt_1, \ldots, t_n) = \begin{cases} 1 & \text{if } (g(t_1), \ldots, g(t_n)) \in P \text{ in } \mathcal{M} \\ 0 & \text{otherwise} \end{cases}$$

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$$v_g^{\mathcal{M}}(\forall x\varphi) = \min\{v_{g[x\mapsto a]}(\varphi) \mid \text{for all } a \text{ in } \mathcal{M}\}$$

Kripke-Kleene models and $\mathcal{L}^+$

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- STTT's full language, $\mathcal{L}^+$:
  1. $\langle\varphi\rangle$ names φ.
  2. Valuations of φ and $T\langle\varphi\rangle$ agree on all models.
  3. Reference to sentences of $\mathcal{L}^+$ within $\mathcal{L}^+$ made possible by arithmetizing $\mathcal{L}^+$'s syntax using Gödel numbering and Peano arithmetic.

Validity

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Definition

STTT consequence: $\Gamma \not\models^{STTT} \Delta$ iff there is a KK model whereby every member of Γ gets truth value 1 and every member of Δ gets value 0.

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Ripley result 1: $\Gamma \vDash^{CL} \Delta$ iff $\Gamma \vDash^{ST} \Delta$
Ripley result 2: If $\Gamma \vDash^{CL} \Delta$, then $\Gamma^\star \vDash^{STTT} \Delta^\star$ for any uniform substitution $\star$ on the full language $\mathcal{L}^+$.

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- STTT's notion of consequence is **not transitive**.
- We have **a Liar sentence** λ, which says in $\mathcal{L}^+$ that $\neg T\langle\lambda\rangle$.
- Assume that, for some sentences φ and ψ, there is a KK model $\mathcal{M}$ on which $v_{\mathcal{M}}(\varphi) = 1$ and $v_{\mathcal{M}}(\psi) = 0$. Then $\varphi \models^{STTT} \lambda$, since no KK model makes $v(\varphi) = 1$ and $v(\lambda) = 0$; and $\lambda \models^{STTT} \psi$, since no KK model makes $v(\lambda) = 1$ and $v(\psi) = 0$. But note that $\varphi \not\models^{STTT} \psi$, because our $\mathcal{M}$ is a countermodel; for $v_{\mathcal{M}}(\varphi) = 1$ and $v_{\mathcal{M}}(\psi) = 0$.

Non-transitive consequence relation (Cont'd)

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- The only **counterexamples to generalized transitivity** will be of the following form: the arguments from Γ, φ to Δ and from Γ to φ, Δ will be STTT-valid, but the argument from Γ to Δ will fail because there is a KK model $\mathcal{M}$ where all $\gamma \in \Gamma$ and $\delta \in \Delta$ are such that $v_{\mathcal{M}}(\gamma) = 1$ and $v_{\mathcal{M}}(\delta) = 0$, but $v_{\mathcal{M}}(\varphi) = \frac{1}{2}$.

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- This means *any counterexample to generalized transitivity in STTT hinges on the cut-formula φ being equivalent to the Liar sentence λ.*

An STTT-invalid liar argument

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$$\text{Explosion}\;\dfrac{\vee\text{E, 1}\;\dfrac{\wedge\,\text{I}\;\dfrac{\text{Def. }\lambda\;\dfrac{\text{Transparency}\;\dfrac{[T\langle\lambda\rangle]^{(1)}}{\lambda}}{\neg T\langle\lambda\rangle}}{T\langle\lambda\rangle \wedge \neg T\langle\lambda\rangle} \qquad \wedge\,\text{I}\;\dfrac{\text{Def. }\lambda\;\dfrac{\text{Transparency}\;\dfrac{[\neg T\langle\lambda\rangle]^{(1)}}{\lambda}}{T\langle\lambda\rangle}}{T\langle\lambda\rangle \wedge \neg T\langle\lambda\rangle} \qquad \text{LEM}\;\dfrac{\top}{T\langle\lambda\rangle \vee \neg T\langle\lambda\rangle}}{T\langle\lambda\rangle \wedge \neg T\langle\lambda\rangle}}{\bot}$$

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STTT-valid proof steps:

$$\text{Explosion}\ \dfrac{T\langle\lambda\rangle\wedge\neg T\langle\lambda\rangle}{\bot}\ (1) \qquad\qquad \text{LEM}\ \dfrac{\top}{T\langle\lambda\rangle\vee\neg T\langle\lambda\rangle}\ (2)$$

Paradoxes

"All formulable paradoxes will have treatments like the liar (...) somewhere in the derivation of the troublesome conclusion, if every individual step is valid, there will be an illicit use of transitivity. The descent from 1 to 0 will not happen all at once, but it will happen bit by bit instead." (Cobreros 13)

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Does STTT preserve classical logic?

- Cobreros et al. fail to make a compelling case for the classicality of their logic.
- This isn't a big deal.

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- Not necessarily just a terminological issue; it has a "philosophical core".
- Can STTT be said to preserve classical logic if it lacks generalized transitivity?
- Maybe not, if it is *weaker* than classical logic (because it lacks a metainference.)

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- A logic is not weaker because it loses a metainference.
- S5 is a strenghtening of S4; S5 validates $\Diamond p \supset \Box\Diamond p$, while S4 does not.
- Consider the metainference: "If $\vDash \Diamond p \supset \Box\Diamond p$, then $\vDash \bot$." S4's consequence relation is closed under this rule; S5's is not.

Losing generalized transitivity

"If STTT gives up something important about T-free classical logic, it cannot be because it fails some metainferences that hold for T-free classical logic; any way at all of extending classical logic will do that. It must rather be because there is something important about the *particular* metainferences in question (...) In the case of STTT, we reckon the focus should rest on (generalized) transitivity." (Cobreros 10)

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- No longer talking about whether STTT is classical, or weaker than classical logic, but whether it violates an *important* metainference.
    - Kind of importance?
    - Relevance of this new question?
- Cobreros et al. never settle the new issue they raise anyway.

Since their discussion about the (un)importance of generalized transitivity stops here, there is a critical lacuna in Cobreros et al.'s argument for STTT's classicality on philosophical grounds.

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- Leitgeb.
    - (Evidence that they expect the same things from a theory of truth.)

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- Two are of interest:
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- Two are of interest:
  - "The outer and the inner logic should coincide": $A \vDash B$ iff $T\langle A\rangle \vDash T\langle B\rangle$
  - "The outer logic should be classical"
- Why?

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- Because it *is* the standard theory; "the principle of minimal mutilation tells us to be as conservative as possible."
- "It is presupposed by standard mathematics, by (at least) huge parts of science, and by much philosophical reasoning."
- So, we want our logic to be classical because it "fits" our inferential practices.

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- Is generalized transitivity important?
- The ability to reason transitively is "a hallmark of rational inference"(Hinzen 131)

Classicality with the purpose of capturing

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- Transitivity in paradoxical circumstances?

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- Transitivity in paradoxical circumstances?
- A theory of truth could only tell us what counts as a valid or an invalid argument here by ignoring the very facts it was supposed to describe.

Classicality with the purpose of capturing

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Since STTT behaves just like classical logic outside of paradoxical circumstances, Leitgeb has no reason to prefer classical logic over STTT.

Closing remarks

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- To what criteria should we refer when choosing among logics which do an equally good (bad?) job at capturing the way in which we reason?