What’s Cool about Frege’s Logic

Cool Logic 8.3.2103

What is a Logic?

- A set of formulas?
- A system of rules of inference?
- An algebra?
- An empirical description of how people think?
- Frege: The most general rules of correct thinking

Characteristica Universalis

‘Thought is in essentials the same everywhere: it is not true that there are different kinds of laws of thought to suit the different kinds of objects thought about.’ (FA xv)

Frege's Peculiar Position

- One foot in the tradition:
- Logic as the art of judging righly

- Revolution in Logical Technique:
- Higher expressive power
- Predicates of higher arity
- Nested Quantifiers

The Revolution

- Traditional analysis: **S is P** (connection)
- Frege: **Fa** (function-argument analysis)
- !Negation: negation of whole content instead of separation

The Context Principle

‘never to ask for the meaning of a word in isolation, but only in the context of a proposition’ (FA: 22)

The Judgment Stroke

A reference to a judging subject is included in the notation

What kind of subject is this, individual or ideal?

Epistemological Interpretation

Claim:

- Logical Laws have to be understood as normative rules which guide rational inquiry in general

Arguments:

- The use of the judgement-stroke
- Logicist program motivated by problems concerning justification
- Objectivity can be interpreted as an epistemic notion (Sluga)

Objectivity

- Intersubjectivity (A common treasure)
- Normativity (*Prescriptive* Laws of Thinking)
- Independence

Independence

- ‘We must remind ourselves, *it* seems, that a proposition no more ceases to be true when I cease to think of it than the sun ceases to exist when I shut my eyes.’

- I understand objective to mean what is independent of our sensation, […] but not what is independent of the reason, for what are things independent of the reason? To answer that would be as much as to judge without judging, or to wash the fur without wetting it.

Rational Procedures

The word "white" ordinarily makes us think of a certain sensation, which is, of course, entirely subjective; but even in ordinary everyday speech, it often bears, I think, an objective sense. When we call snow white, we mean to refer to an objective quality which we recognize, in ordinary daylight, by a certain sensation. If the snow is being seen in a coloured light, we take that into account in our judgement and say, for instance, "It *appears* red at present, but it *is* white." Even a colour-blind man can speak of red and green, in spite of the fact that he does not distinguish between these colours in his sensations; he recognizes the distinction by the fact that others make it, or perhaps by making a physical experiment. […] It is in this way that I understand objective to mean what is independent of our sensation, […] but not what is independent of the reason, for what are things independent of the reason? To answer that would be as much as to judge without judging, or to wash the fur without wetting it.

Empirical Procedures

- Sensation (though not by itself and defeasible)
- Background knowledge
- Testimony
- Experiment (direct perception not necessary)
- Division of Labor
- Holistic Array (cf. Trägheitsgesetz)

Hierarchy of ‘Realms’

in order of generality:

Logic/arithmetic: Logical Deduction

Geometry: Geometrical Proofs

Empirical Knowledge: Diverse Procedures

The Textbook Objection

If mathematical objects Are entitites in an abstract realm then

a) How do they apply to reality

b) How can we get to know them (Benacerraf)

What is a Mathematical Object?

Semantic Characterization:

- Objects are the referents of proper names

- Selfsame Particulars

- Reidentifiable

The Context Principle (Application)

‘How, then, are numbers to be given to us, if we cannot have any ideas or intuitions of them? Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs.’

Reidentifiability and Definition

(Re-)identification through identity statements:

4 **is** the number of moons of Jupiter

Criterion for a Definition: Decidability of all identity statements

Definition (Hume's Principle)

The number which belongs to the concept F is the same as that which belongs to the concept G

'we can never [...] decide by means of our definitions whether any concept has the number JULIUS CAESAR belonging to it, or whether that same familiar conqueror of Gaul is a number or is not.' (FA 68)

Definition (2)

The Number which belongs to the concept *F* is the extension (*Umfang*) of the concept 'equinumerous to the concept F'

Eternal Bliss

‘On this view of numbers the charm of work on arithmetic and analysis is, it seems to me, easily accounted for. We might say, indeed, almost in the well-known words: the reason's proper study *is* itself. In arithmetic we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects given directly to our reason and, as its nearest kin, utterly transparent to it. (FA: 115)