Second Philosophy, Pluralism and the Multiverse

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Prologue

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- Change from her old views from "Realism in Mathematics" which were hardcore realist.

Second Philosophy

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- Practice first.

Characterising the Second Philosopher

"Imagine a simple inquirer who sets out to discover what the world is like, the range of what there is and its various properties and behaviors. She begins with her ordinary perceptual beliefs, gradually develops more sophisticated methods of observation and experimentation, of theory construction and testing, and so on; shes idealized to the extent that shes equally at home in all the various empirical investigations, from physics, chemistry, and astronomy to botany, psychology, and anthropology." Maddy, p. 38.

Characterising the Second Philosopher

"She believes that ordinary physical objects are made up of atoms, that plants live and grow by photosynthesis, that humans use language to describe the world to one another, that social groups tend to behave in certain ways, and so on. She also believes that she and her fellow inquirers are engaged in a highly fallible, but partly and potentially successful exploration of the world, and like anything else, she looks into the matter of how and why the methods she and others use in their inquiries work when they do and dont work when they dont; in these ways, she gradually improves her methods as she goes." Maddy, p. 39.

Four Examples of Set Theory in Practice

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- Dedekind's introduction of sets gave them as new objects to serve a wide array of mathematical goals such as rigorous definitions of real number and continuity and a foundation of arithmetic.

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- Finally, Maddy turns to the contemporary case of determinacy, in particular $AD^{L(\mathbb{R})}$ (asserting the determinacy of all sets of reals in the smallest inner model containing all the real numbers) which has fallen into favour because it is fruitful, is implied by large cardinal axioms and implied by almost all sufficiently strong mathematical theories.

A New Discipline

"This study of actual set-theoretic methods also confirms the Second Philosopher's initial impression that this is an inquiry governed by norms distinct from familiar observation, theory-formation and testing: for example, she isnt accustomed to embracing new entities to increase her expressive powers (as in Cantor) or to encourage definitions of a certain desirable kind (as in Dedekind), or to rejecting a theory because it produces less interesting consequences (as with the alternative to determinacy's theory of projective sets that results from $V = L$)." Maddy, p. 53.

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- Compare to Robust Realism.

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"Set theory is the activity of developing a theory of sets that will effectively serve a concrete and ever-evolving range of mathematical purposes. Such a Second Philosopher would see no reason to think that sets exist or that set-theoretic claims are true—her well-developed methods of confirming existence and truth aren't even in play here—but she does regard set theory, and pure mathematics with it, as a spectacularly successful enterprise, unlike any other." Maddy, p. 89.

The Continuum Hypothesis

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- Robust Realist: True or false thanks to the abstract objects.
- Second Philosopher: "Her analysis is simpler: 'CH or not-CH' is a theorem, established by her best methods as a fact about V; therefore CH is either true or false there." Maddy, p. 61.

Philosophical Positions: Gotta Catch 'em All

"...if the new methods seem a bit odd, but still of-a-piece with the old, then she concludes that she's made a surprising discovery, that the world includes abstracta as well as concreta. If, on the other hand, she regards the new methods and would-be objects as sharply discontinuous with what came before, she has no grounds for thinking pure mathematics is true, so she concludes that this new practice—valuable as it is—isn't in the business of developing a body of truths." Maddy, pp. 101-102.

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Objective Reality

So the Thin Realist may assert that the fruitful sets exist and the resulting set theory is a body of truths, while the Arealist may think that mathematics does not even enter the realms of truth and existence, but both are rooted in Second Philosophy, thus prioritise practice over philosophy, and the methods of set-theoretic practice ultimately follow the direction of mathematical depth, fruitfulness and effectiveness.

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- Why not go Intuitionistic on it? e.g. IZF, HA, smooth infinitesimal analysis Or paraconsistent??? e.g. rescue naïve comprehension.
- "A plurality or multiplicity of approaches to central questions of *truth* and *proof* is simply an observable fact." Hellman & Bell, p. 65.

Objective Reality Bites Back

- "...that constructive mathematics is of value *even to those who reject the philosophical assumptions of its founders*. It provides a clear and precise insight into difficulties that numerical analysts and others who are interested in obtaining quantitative information about solutions of equations face on a regular basis." Davies, p. 253.

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- "I interpolate that one does not have to be an intuitionist or a dialetheist to take intuitionist or paraconsistent mathematics to be legitimate. It suffices that these are interesting *mathematical* enterprises." Priest, p. 2.

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- Different logics, but talking about different things.

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- Hamkins proposes an opposing view, the *multiverse view*, which holds that there isn't one concept of set but many distinct concepts of sets, each with a corresponding set-theoretic universe in which it is instantiated.
- "Our most powerful set-theoretic tools, such as forcing, ultrapowers and canonical inner models, are most naturally and directly understood as methods of constructing alternative set-theoretic universes." Hamkins, p. 3.

Thanks