Fairness and Optimality in Matching

Summer School on COMSOC - Amsterdam

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Matchings: some examples

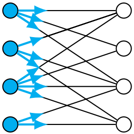
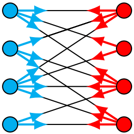
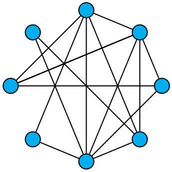
- College admissions
- Job market
- Housing market
- Kidney exchange
- Schedule design / task assignment
- Residents / hospitals assignment
- Dating apps
- Groups for working projects
- ...

	Monday	Tuesday	Wednesday	Thursday	Friday
8.00 - 10.00					
10.00 - 12.00					
14.00 - 16.00					
16.00 - 18.00					

Matching under preferences

Focus on one-to-one matchings
→ Matching from graph theory: a subset of disjoint edges in a graph

⇒ Evaluation of the matching via preferences

Bipartite graph		Non-bipartite graph
One-sided preferences	Two-sided preferences	
		
House allocation	Marriage setting	Roommate setting

K. Bettina, D. F. Manlove, and F. Rossi. Matching under Preferences. In *Handbook of Computational Social Choice*, chapter 14, Cambridge University Press, 2016

Matching framework

- Set N of n agents
  - Marriage setting: $N = M \cup W$ with $|M| = |W|$
- Set O of n objects (houses)
- Each agent $i \in N$ has strict ordinal preferences (linear order) over P_i:

House allocation	Marriage setting	Roommate setting
$P_i = O$	$P_i = M$ if $i \in W$ $P_i = W$ if $i \in M$	$P_i = N \setminus \{i\}$

$\Rightarrow$ Solution: assignment σ such that $\sigma(i) \in P_i$ for each $i \in N$ and $\sigma(i) \neq \sigma(j)$ for every agents $i \neq j$

- Assumptions:
  - No indifference or unacceptabilities in the preferences
  - Each agent must be matched

Desirable properties

- Stability: search for a solution which is immune to perturbations from agents
- Optimality: search for a solution which maximizes the global satisfaction of agents
- Fairness: search for a solution which equally treats agents

$\Rightarrow$ How can they be satisfied in matchings?
$\rightarrow$ Preference restrictions

$\Rightarrow$How do they fit together?

Outline

Structured preferences

Stable matchings

Optimal matchings

Fair matchings

Outline

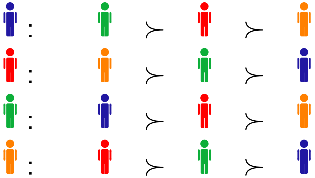
Structured preferences

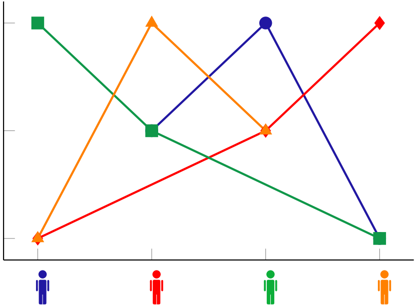
Stable matchings

Optimal matchings

Fair matchings

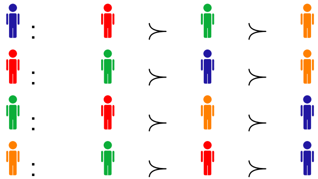
Single-peaked (SP) preferences

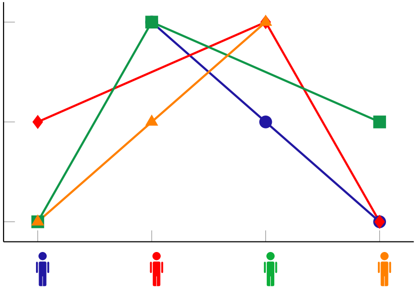


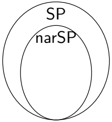


D. Black. On the rationale of group decision-making, *Journal of Political Economy*, 1948

Single-peaked and narcissistic (narSP) preferences

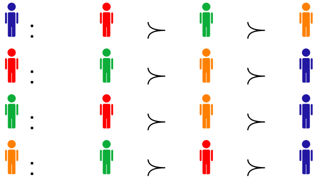


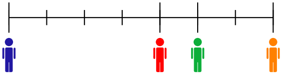


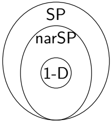


J. Bartholdi III and M. A. Trick. Stable matching with preferences derived from a psychological model, *Operations Research Letters*, 1986

1-Euclidean preferences

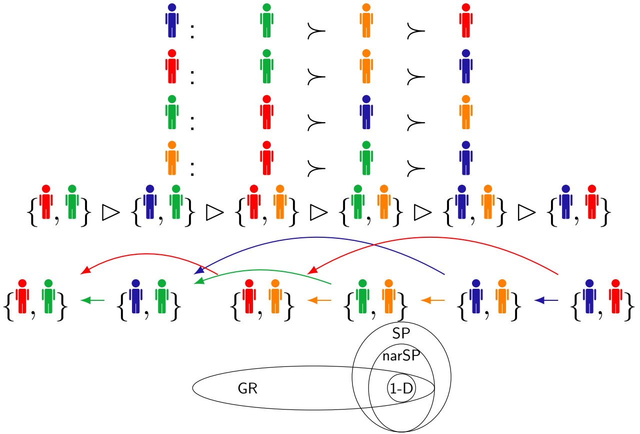






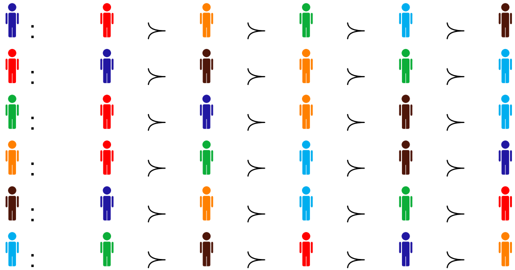
C. H. Coombs. Psychological scaling without a unit of measurement, *Psychological review*, 1950

Globally-ranked (GR) preferences



D. J. Abraham, A. Levavi, D. F. Manlove, and G. O'Malley. The stable roommates problem with globally-ranked pairs, *Internet Mathematics*, 2008

Iteratively mutual best preferences



A. Abizada. Exchange-stability in roommate problems, *Review of Economic Design*, 2019

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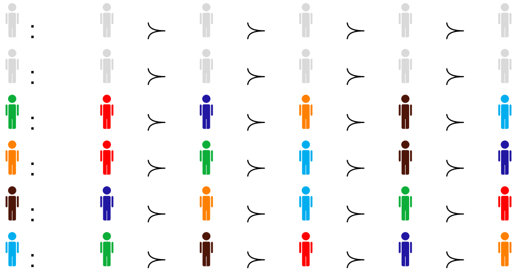

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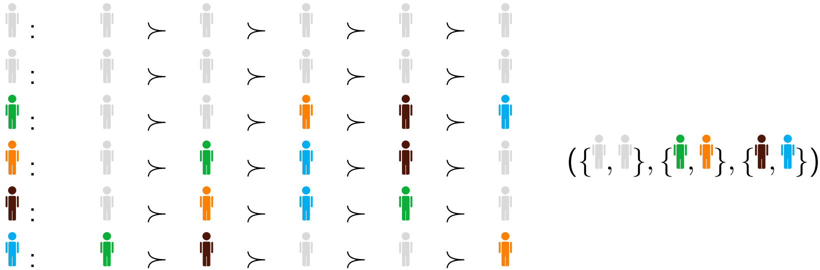

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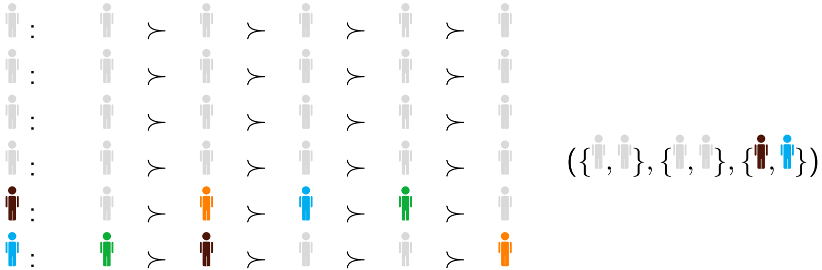

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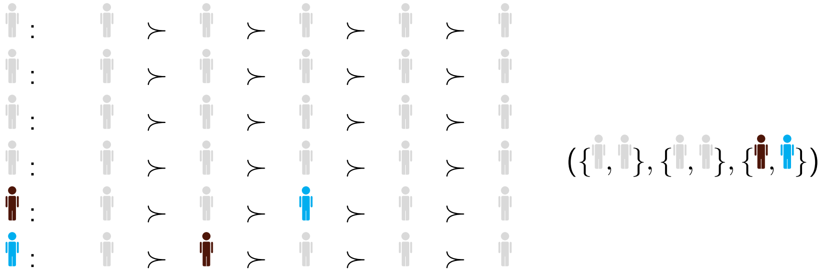

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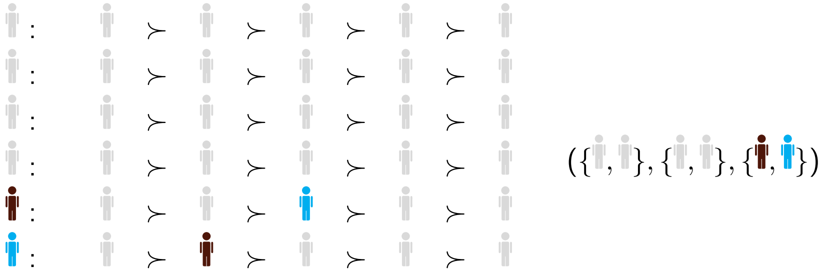

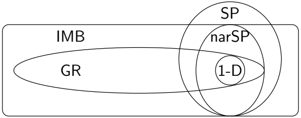

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Outline

Structured preferences

Stable matchings

Optimal matchings

Fair matchings

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- Swap-stable matchings

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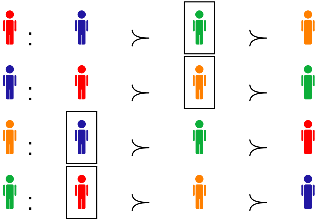
Fair matchings

Stability w.r.t. blocking pairs

Blocking pair: a pair of agents who prefer to be matched together than with their current partner



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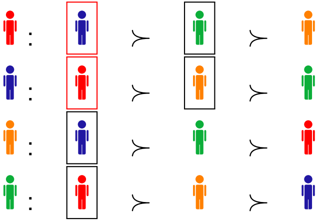

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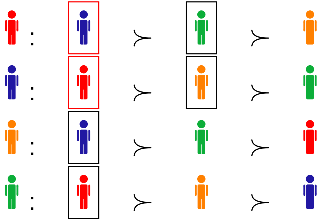


BP-stable matching: a matching with no blocking pair

→ Meaningful only in marriage and roommate settings

The stable marriage problem

There always exists a BP-stable marriage matching and we can find one in polynomial time

Deferred-acceptance algorithm Example

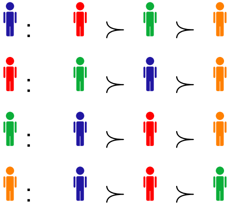
- The available men iteratively propose to their most preferred woman
- The women iteratively accept their best received proposal

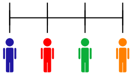
$\Rightarrow$ always terminates in a quadratic number of steps and outputs a BP-stable marriage matching

D. Gale, and L. S. Shapley. College Admissions and the Stability of Marriage, *The American Mathematical Monthly*, 1962

The stable roommate problem

- A BP-stable roommate matching does not always exist, even under single-peaked preferences





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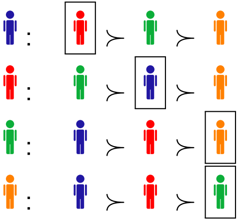

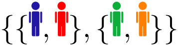

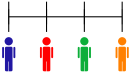

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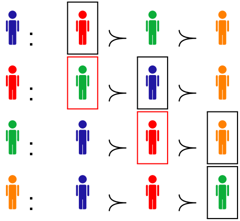

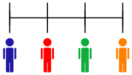

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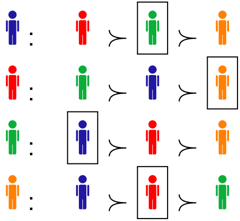

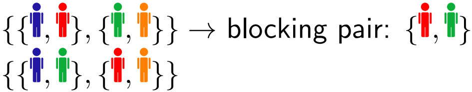

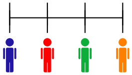

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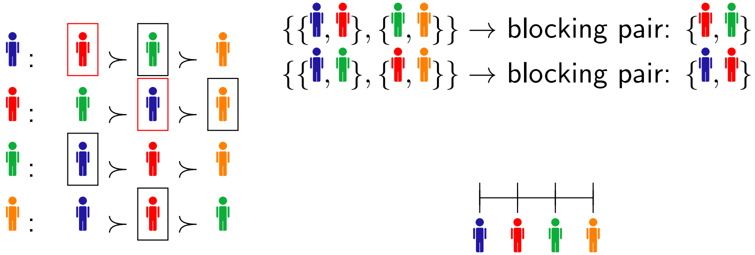

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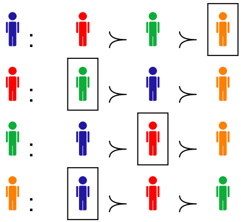

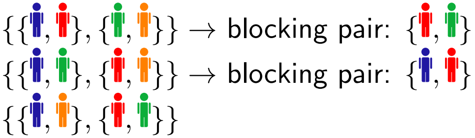

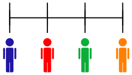

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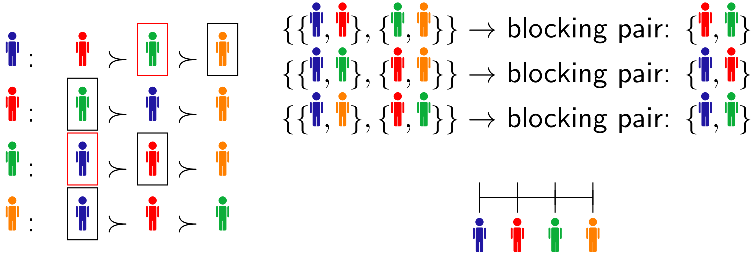

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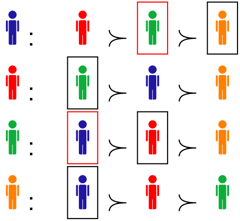
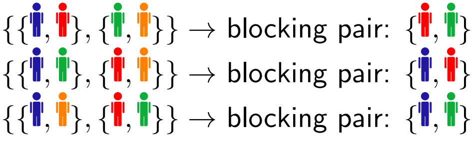
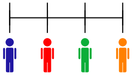




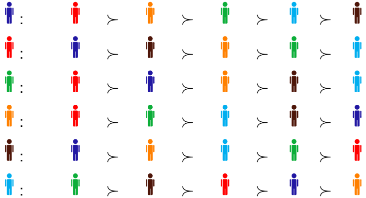


- Checking the existence of a BP-stable roommate matching and constructing one (if it exists) can be done in polynomial time

R. W. Irving. An Efficient Algorithm for the "Stable Roommates" Problem, *Journal of Algorithms*, 1985

Restricted roommate setting

- There always exists a BP-stable roommate matching under:
  - IMB preferences



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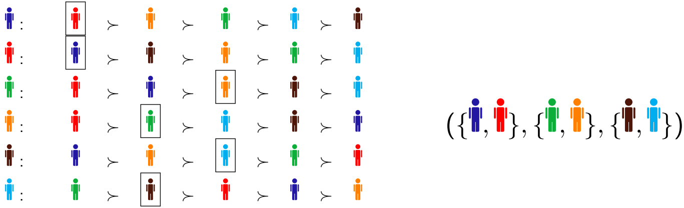

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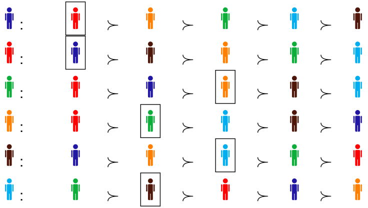

  - preferences with no odd ring

$$\begin{array}{lccc} 1: & \boxed{2} & \succ & 3 \\ 2: & \boxed{3} & \succ & \boxed{1} \\ 3: & 1 & \succ & \boxed{2} \end{array}$$

K.-S. Chung. On the existence of stable roommate matchings, *Games and Economic Behavior*, 2000

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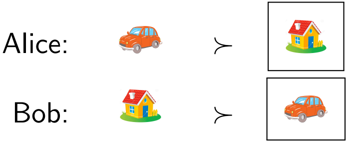
Swap stability

Swap: two agents prefer to exchange their current match



Swap-stable matching: a matching with no possible swap

J. Alcalde. Exchange-proofness or divorce-proofness? Stability in one-sided matching markets, *Economic design*, 1994

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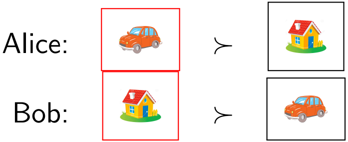

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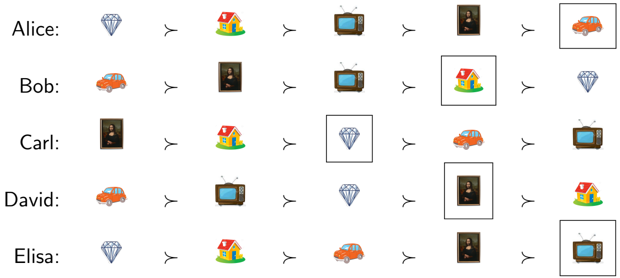
Swap-stable house allocation

Swap $\rightarrow$ Two agents are strictly better-off and no agent is worse-off

Convergence of the swap dynamics in $\mathcal{O}(n^2)$ steps



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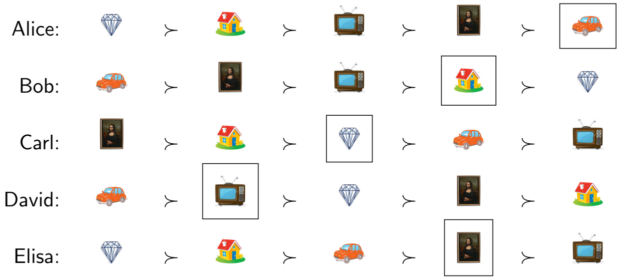

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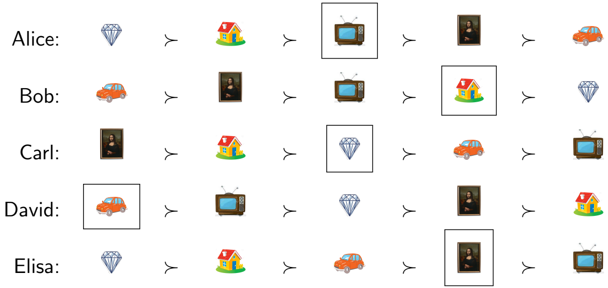

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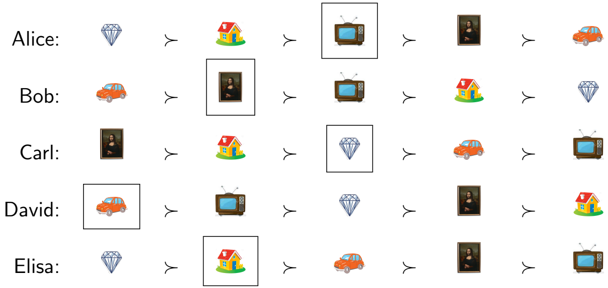

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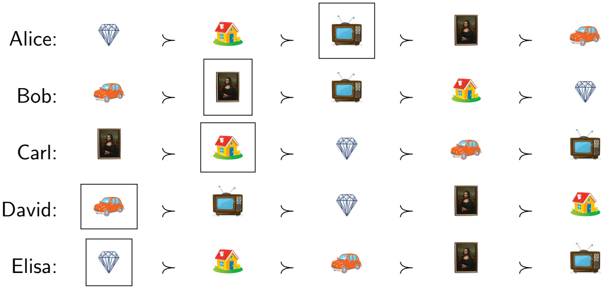
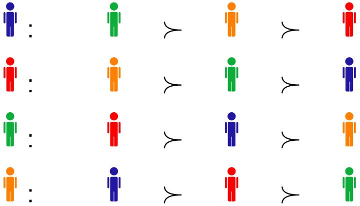
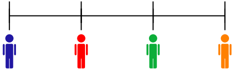


$\Rightarrow$ There always exists a swap-stable allocation

Swap-stable marriage / roommate matchings

- A swap-stable matching does not always exist even under single-peaked preferences





- Deciding whether a swap-stable matching exists is NP-complete

K. Cechlárová and D. F. Manlove. The exchange-stable marriage problem, *Discrete Applied Mathematics*, 2005

K. Cechlárová. On the complexity of exchange-stable roommates, *Discrete Applied Mathematics*, 2002

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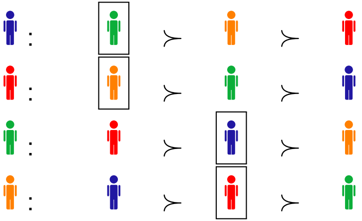

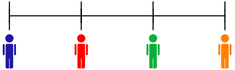

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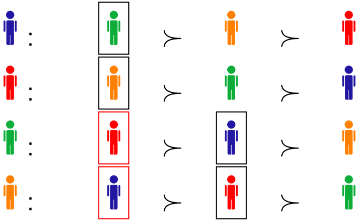

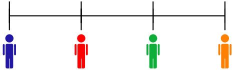

- Deciding whether a swap-stable matching exists is NP-complete

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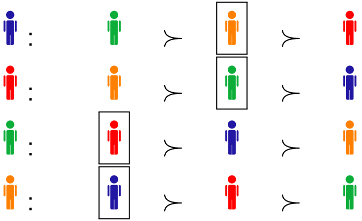

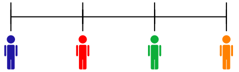

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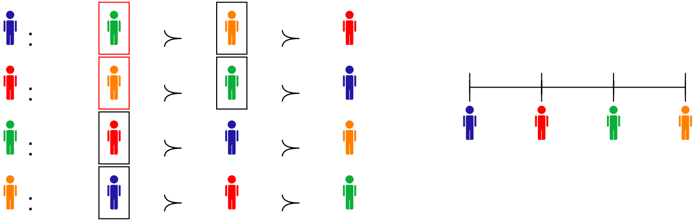

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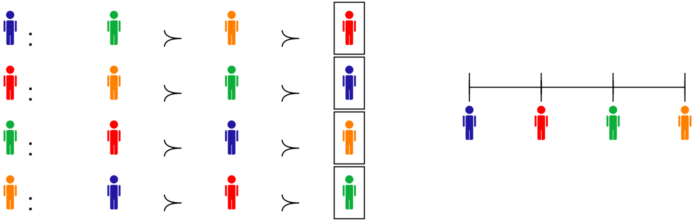

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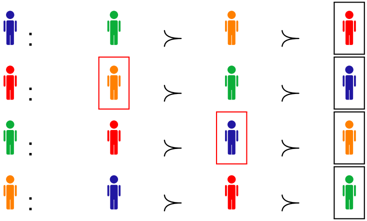

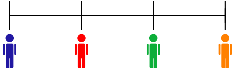

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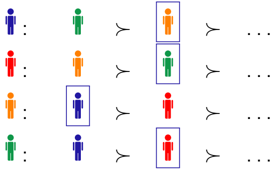
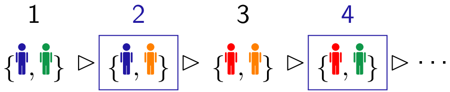
Restricted marriage and roommate settings

- A swap-stable matching always exists under IMB preferences
  - the iteratively mutual best pairs are matched
- The dynamics of swaps:
  - always converge under globally-ranked preferences





s = sum of ranks in the global order

$s = 6$

  - may cycle even under single-peaked and narcissistic preferences
- Deciding about convergence is co-NP-hard

A. Abizada. Exchange-stability in roommate problems, *Review of Economic Design*, 2019

F. Brandt, and A. Wilczynski. On the convergence of swap dynamics to Pareto-optimal matchings, *Proceedings of WINE-19*, 2019

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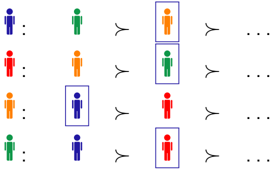

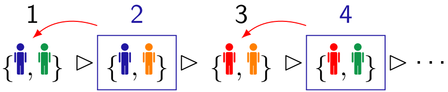

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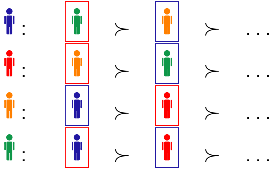

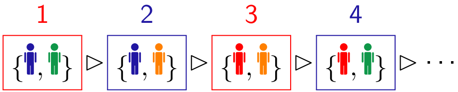

s = sum of ranks in the global order

$s = 6 \Rightarrow s = 4$

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A. Abizada. Exchange-stability in roommate problems, *Review of Economic Design*, 2019

F. Brandt, and A. Wilczynski. On the convergence of swap dynamics to Pareto-optimal matchings, *Proceedings of WINE-19*, 2019

Outline

Structured preferences

Stable matchings

Optimal matchings

Fair matchings

Outline

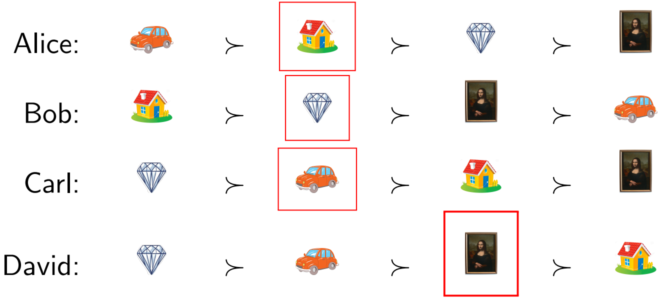
Structured preferences

Stable matchings
- Blocking-pair stable matchings
- Swap-stable matchings

Optimal matchings
- Pareto-optimal matchings
- Rank-maximal matchings
- Popular matchings

Fair matchings

Pareto-optimality (PO)



Pareto-optimal matching: a matching with no possible improving cycle

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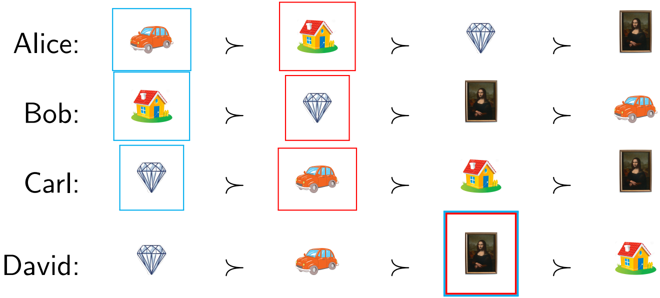
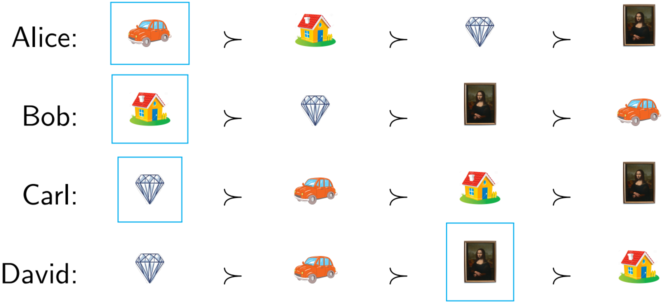


Pareto-optimal matching: a matching with no possible improving cycle

Characterization of all Pareto-optimal matchings

A matching is Pareto-optimal iff it can result from a serial dictatorship



$\langle$ Alice, Bob, Carl, David$\rangle$

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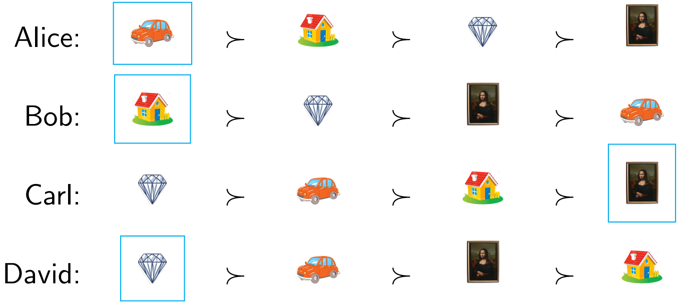

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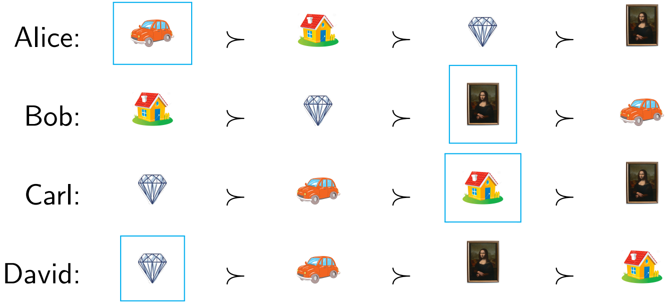

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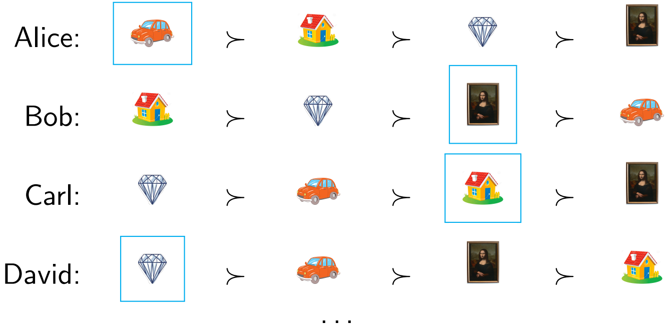
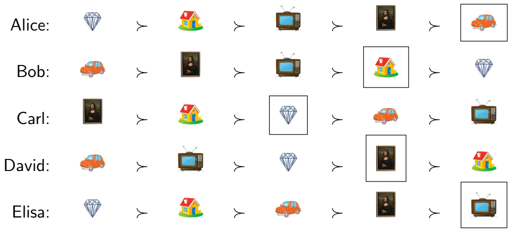


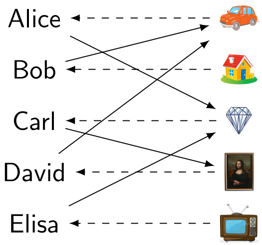
$\Rightarrow$ Worst case: n! Pareto-optimal house allocations

Pareto-optimality in housing market

initial allocation $\rightarrow$ Top Trading Cycle [attributed to Gale]

- Iterative implementation of the cycles in the graph where:
  - the agents point to their most preferred object
  - the objects point to their current owner





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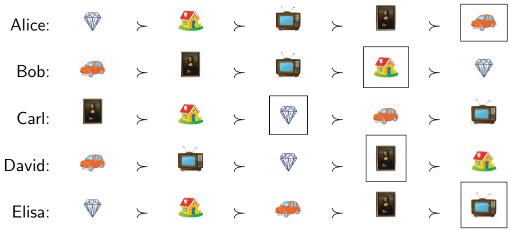

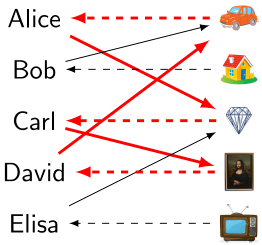

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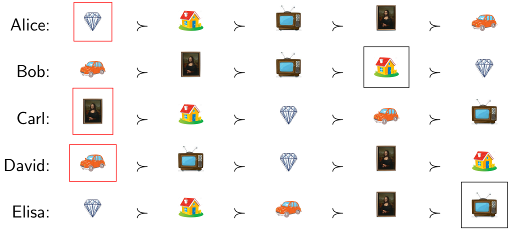

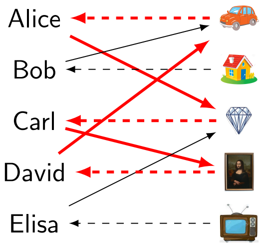

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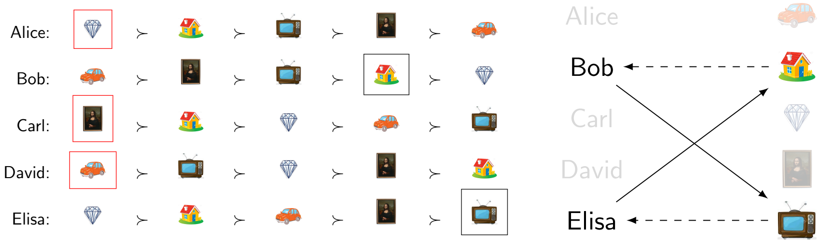

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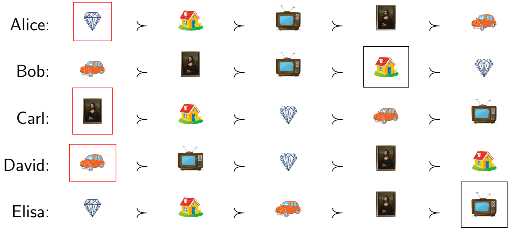

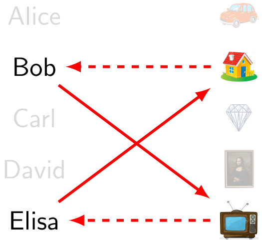

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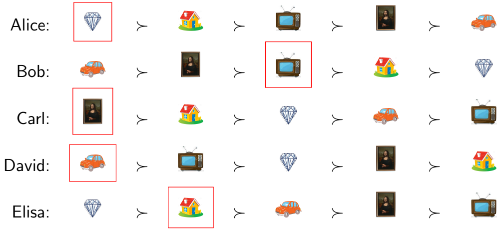
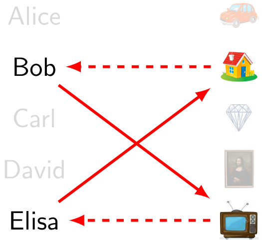


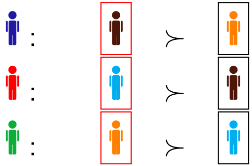


⇒ A mechanism is strategy-proof, Pareto-efficient and individually rational iff it is TTC [Ma, 1994]

L. Shapley, and H. Scarf. On cores and indivisibility, *Journal of mathematical economics*, 1974

Swap stability and Pareto-optimality

- Every Pareto-optimal house allocation is swap-stable
- Every swap-stable matching is PO under SP preferences



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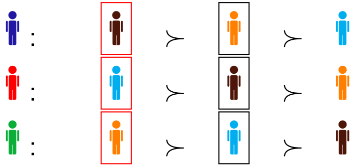

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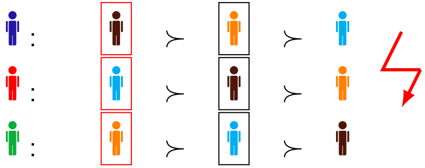

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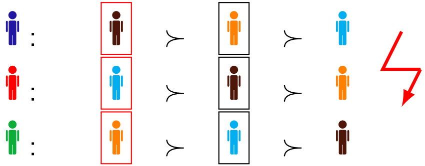

→ The swap dynamics always converge to a PO matching:
  - under single-peaked preferences for house allocation
  - under 1-Euclidean preferences for marriage and roommate settings

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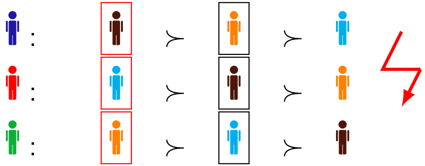

→ The swap dynamics always converge to a PO matching:
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- Deciding about convergence to a Pareto-optimal matching is hard

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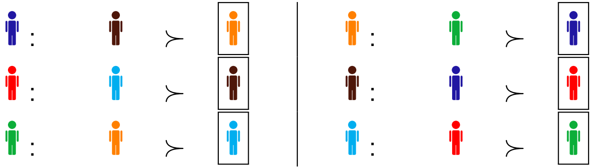
BP-stability and Pareto-optimality

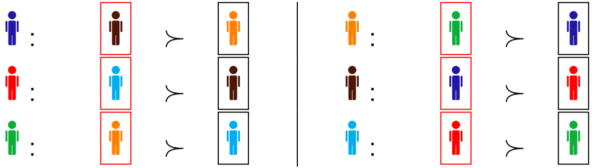
- Every BP-stable matching is Pareto-optimal



⇒ The outcome of Deferred-acceptance is BP-stable and Pareto-optimal in marriage settings

BP-stability and Pareto-optimality

- Every BP-stable matching is Pareto-optimal



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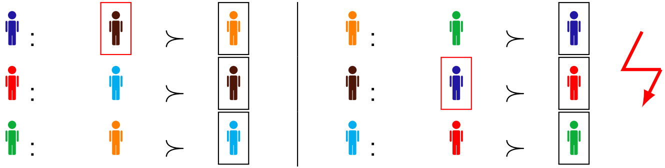

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BP-stability and Pareto-optimality

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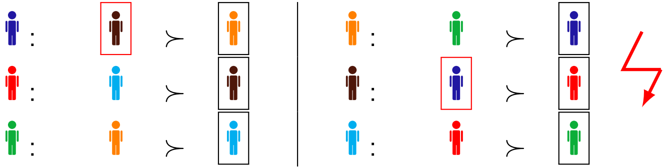


$\Rightarrow$ The outcome of Deferred-acceptance is BP-stable and Pareto-optimal in marriage settings

- A matching with the smallest number of blocking pairs is Pareto-optimal
  - Computing such a minimally unstable matching is NP-complete

D. J. Abraham, and D. F. Manlove. Pareto optimality in the roommates problem. Technical Report TR-2004-182, University of Glasgow, 2004

Outline

Structured preferences

Stable matchings
- Blocking-pair stable matchings
- Swap-stable matchings

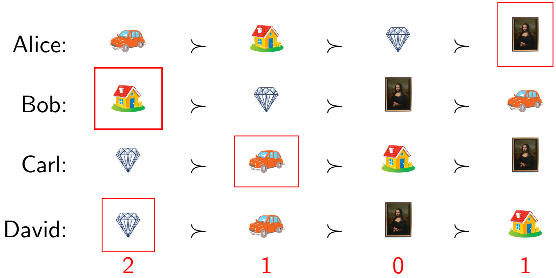
Optimal matchings
- Pareto-optimal matchings
- Rank-maximal matchings
- Popular matchings

Fair matchings

Rank-maximality

- Evaluation of matchings by their signature
- Lexicographic maximization

Rank-maximality ⇒ Pareto-optimality



Rank-maximal matching: a matching that lexicographically maximizes the signature

Rank-maximality

- Evaluation of matchings by their signature
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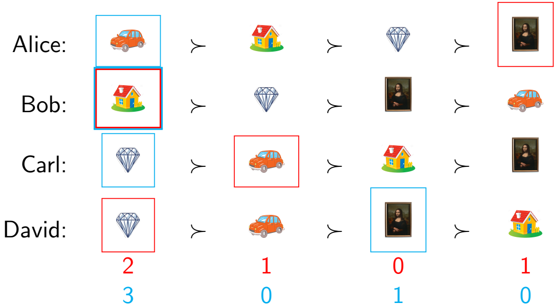
Rank-maximality ⇒ Pareto-optimality



Rank-maximal matching: a matching that lexicographically maximizes the signature

Computing a rank-maximal matching

- A rank-maximal matching always exists and can be computed in polynomial time
  - Maximum weight matching problem with exponential weights + scaling algorithm
  - Proper combinatorial algorithm based on augmenting paths
- Counting the number of rank-maximal matchings is #P-complete

R. W. Irving, T. Kavitha, K. Mehlhorn, D. Michail, and K. E. Paluch. Rank-maximal matchings, *ACM Transactions on Algorithms*, 2006

P. Ghosal, M. Nasre, and P. Nimbhorkar. Rank-maximal matchings–structure and algorithms. *Theoretical Computer Science*, 2019

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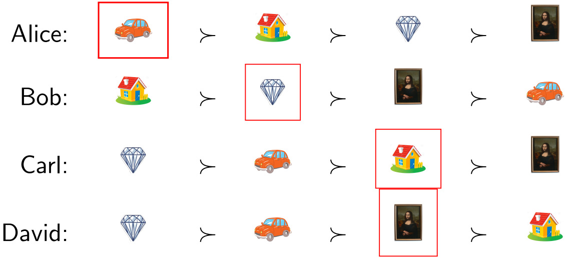
Popularity

- Pairwise comparisons of matchings

Popularity $\Rightarrow$ Pareto-optimality



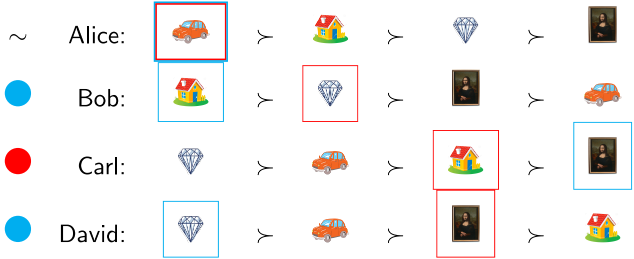
Popular matching: there is no other matching that is more popular

Á Cseh. Popular matchings, *Trends in Computational Social Choice*, 2017

Popularity

- Pairwise comparisons of matchings

Popularity ⇒ Pareto-optimality



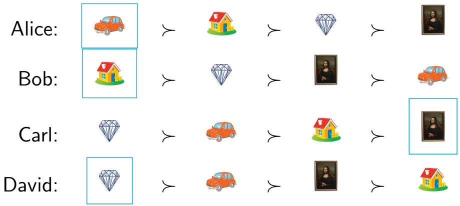
Popular matching: there is no other matching that is more popular

Á Cseh. Popular matchings, *Trends in Computational Social Choice*, 2017

Popular house allocation

An allocation is popular iff every agent is matched with either:

- her most preferred object, or
- her most preferred object that is not ranked first by someone.



⇒ Deciding whether a popular house allocation exists and finding one can be done in polynomial time

D. J. Abraham, R. W. Irving, T. Kavitha, and K. Mehlhorn. Popular matchings, *SIAM Journal on Computing*, 2007

BP-stability and popularity

Strong popularity ⇒ BP-stability ⇒ Popularity

- Deferred-acceptance algorithm ⇒ A popular marriage matching always exists and finding one can be done in polynomial time
- Checking the existence of a strongly popular matching can be done in polynomial time
  1. Check the existence of a BP-stable matching
  2. If yes, check whether the resulting BP-stable matching is strongly popular
- Testing whether a given matching is popular can be done in polynomial time

P. Biró, R. W. Irving, and D. F. Manlove. Popular Matchings in the Marriage and Roommates Problems, *Proceedings of CIAC-10*, 2010

Popularity in the roommate setting

- A popular roommate matching does not always exist
  → Complexity of the existence decision problem?
  Open problem for several years...
- Deciding whether a popular roommate matching exists is NP-hard [Faenza et al. 2019, Gupta et al. 2021]
- A popular matching always exists under IMB preferences
  - it is also BP-stable and swap-stable

Y. Faenza, T. Kavitha, V. Powers, and X. Zhang. Popular matchings and limits to tractability, *Proceedings of SODA-19*, 2019

S. Gupta, P. Misra, S. Saurabh, and M. Zehavi, Popular matching in roommates setting is NP-hard, *ACM Transactions on Computation Theory*, 2021

A. Wilczynski. Ordinal Hedonic Seat Arrangement under Restricted Preference Domains: Swap Stability and Popularity, *Proceedings of IJCAI-23*, 2023

Outline

Structured preferences

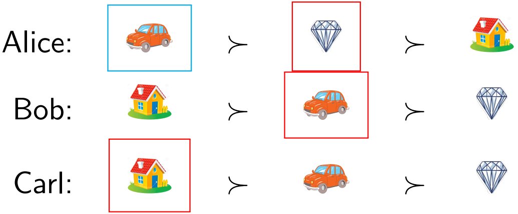
Stable matchings

Optimal matchings

Fair matchings

Rank-envy-freeness (r-EF)

Rank-envy: Agent i prefers the element that has been assigned to agent j over her own assigned element whereas she has ranked it better in her preferences than agent j



r-EF matching: matching with no rank-envy

F. Kojima and M. U. Ünver, The "Boston" school-choice mechanism: an axiomatic approach, *Economic Theory*, 2014

Rank-envy-freeness (r-EF)

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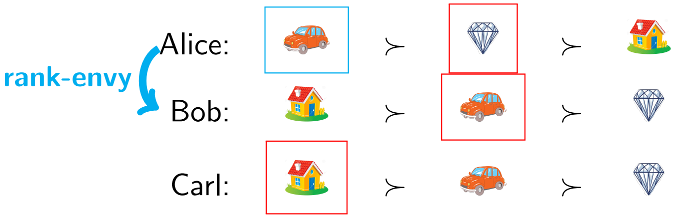

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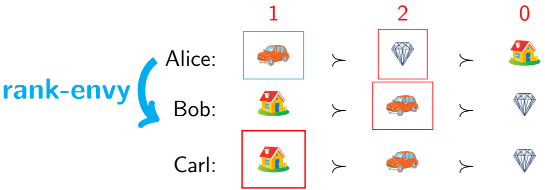
Rank-envy-freeness in house allocation

- Rank-maximality ⇒ r-EF



  - An r-EF matching always exists and can be computed in polynomial time

- Popularity ⇒ r-EF

K. Belahcène, V. Mousseau, and A. Wilczynski. Combining Fairness and Optimality when Selecting and Allocating Projects, *Proceedings of IJCAI-21*, 2021

Rank-envy-freeness in house allocation

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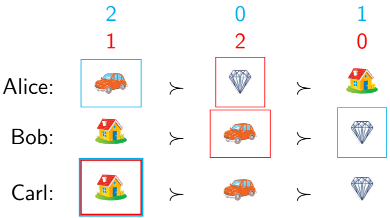

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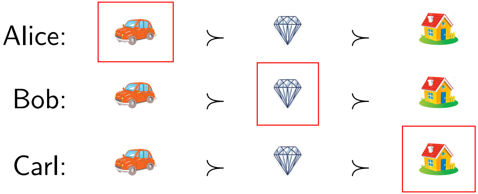
K. Belahcène, V. Mousseau, and A. Wilczynski. Combining Fairness and Optimality when Selecting and Allocating Projects, *Proceedings of IJCAI-21*, 2021

Rank$_k$-envy-freeness (r$_k$-EF)

Rank$_k$-envy: Agent i prefers the element that has been assigned to agent j over her own assigned element whereas:

- she has ranked it better in her preferences than agent j, or
- agent j does not rank it among her k first ranked elements



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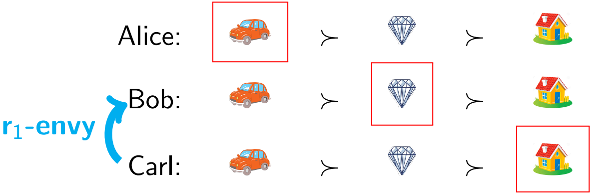
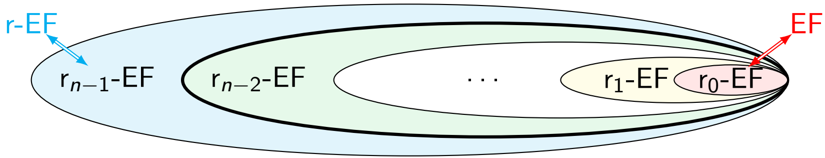


K. Belahcène, V. Mousseau, and A. Wilczynski. Combining Fairness and Optimality when Selecting and Allocating Projects, *Proceedings of IJCAI-21*, 2021

Rank_k-envy-freeness in house allocation



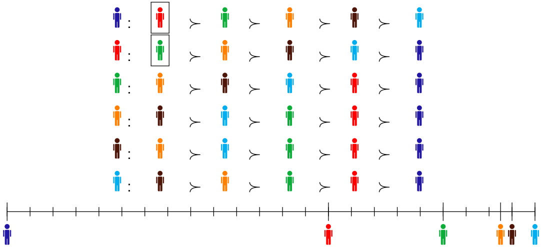
- An r_{n-1}-EF matching always exists
- An r_{n-2}-EF matching does not always exist

r_1-EF ⇔ Popularity

K. Belahcène, V. Mousseau, and A. Wilczynski. Combining Fairness and Optimality when Selecting and Allocating Projects, *Proceedings of IJCAI-21*, 2021

Rank-envy-freeness in marriage / roommate settings

- An r-EF marriage / roommate matching does not always exist even under 1-Euclidean preferences



- Deciding whether an r-EF marriage / roommate matching exists is NP-complete even under globally-ranked preferences
- Every r-EF matching is swap-stable

B. Coutance, P. Maddila, and A. Wilczynski. Rank-envy-freeness in roommate matchings, To appear in *Proceedings of ECAI-23*, 2023

Rank_k-envy-freeness in marriage / roommate settings

- A matching is r_1-EF iff every agent is matched with either:
  - her most preferred agent, or
  - her most preferred agent that is not ranked first by someone.

  → Constant characterization of r_1-EF
- Deciding whether an r_1-EF matching exists can be done in polynomial time
- Every r_1-EF matching is popular

→ These properties do not hold for r_2-EF...

B. Coutance, P. Maddila, and A. Wilczynski. Rank-envy-freeness in roommate matchings, To appear in *Proceedings of ECAI-23*, 2023

Outline

Structured preferences

Stable matchings

Optimal matchings

Fair matchings

Conclusion

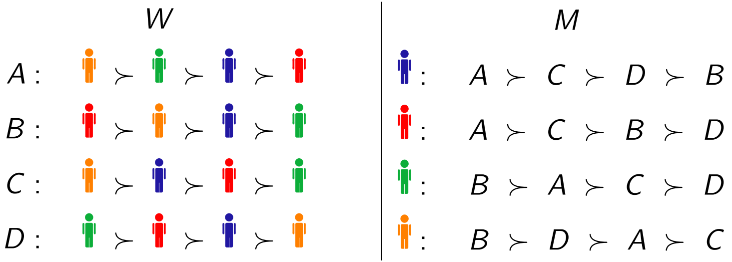
Summary

- Stability, optimality and fairness: different notions that can nevertheless be combined
- Importance of structure in the preferences
- Well-known algorithms:
  - Deferred-acceptance
  - Top-trading cycle

To go further

- More general preferences
  - Unacceptabilities: partial lists of preferences
  - Indifferences: ties in the preference lists
- Related models:
  - Many-to-one matchings
  - Hedonic games
- Omitted notions:
  - Strategy-proofness
- Other directions to reach more positive results:
  - Fractional matchings

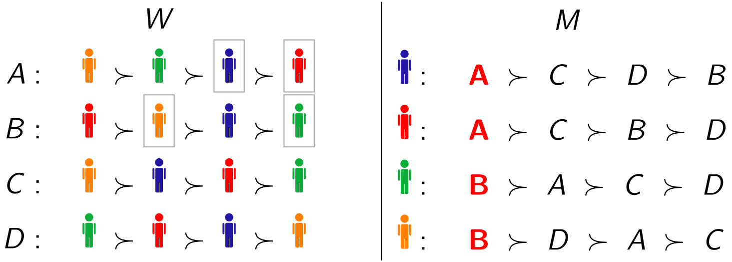
Execution of the Deferred-Acceptance algorithm



While there exist unengaged men:

1. Each single man proposes to the woman he prefers the most among the women who did not reject him yet
2. Each woman temporarily accepts the proposition of the man she prefers ("engagement") and rejects all the other propositions

Execution of the Deferred-Acceptance algorithm



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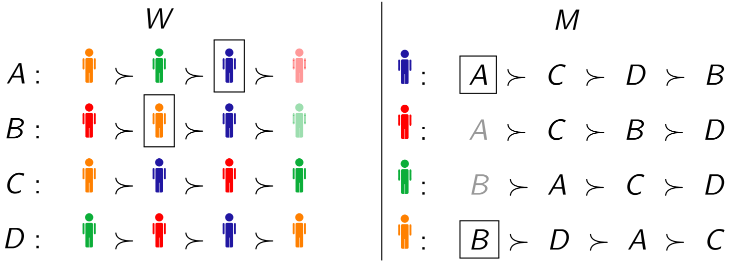

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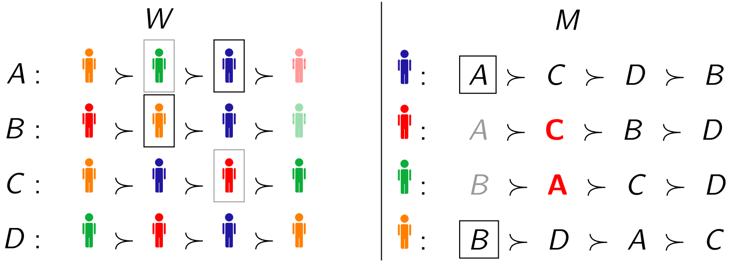

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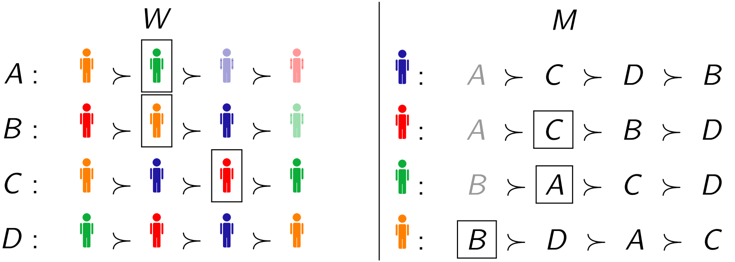

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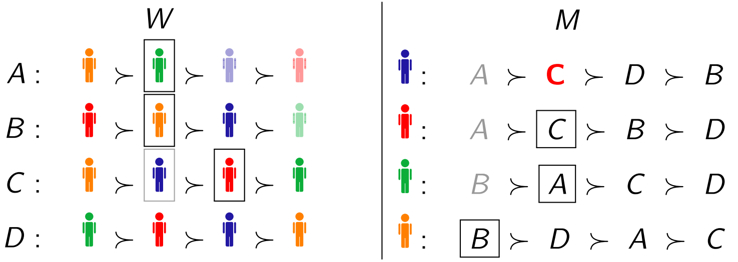

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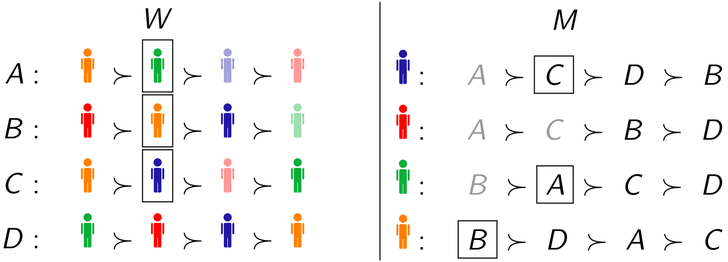

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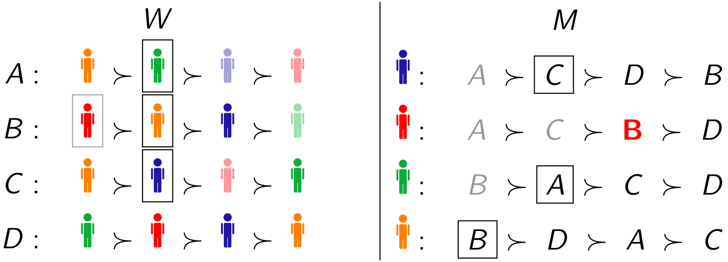

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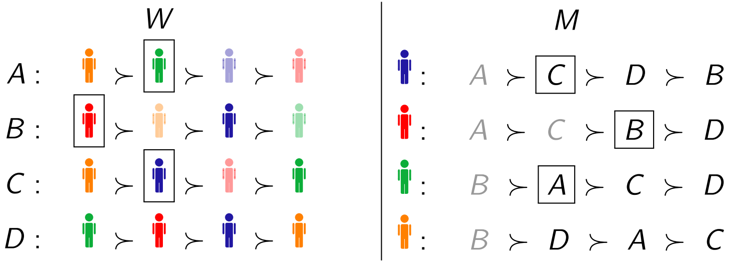

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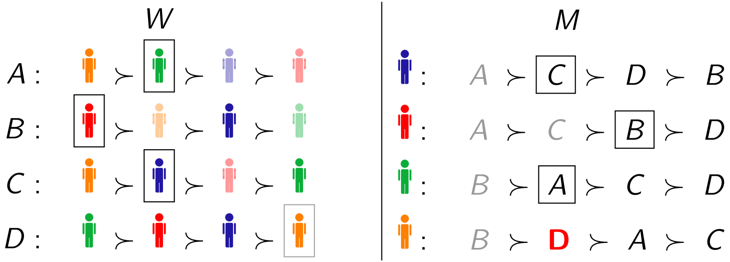

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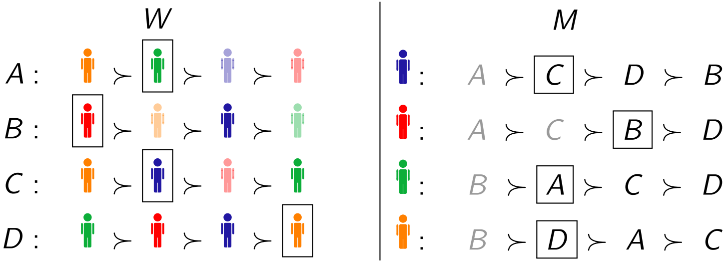

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Back