Computing Desirable Collective Decisions III
Approval-based Committee Elections

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Committee Elections

- A set C of candidates, k of which have to be elected
- Outcome: committee $W \subseteq C$, $|W| = k$.
- A set N of n voters
- Each voter $i \in N$ approves a subset $A_i \subseteq C$.
- We say that i's utility is $u_i(W) = |A_i \cap W|$
  (this is a dichotomous preference assumption).

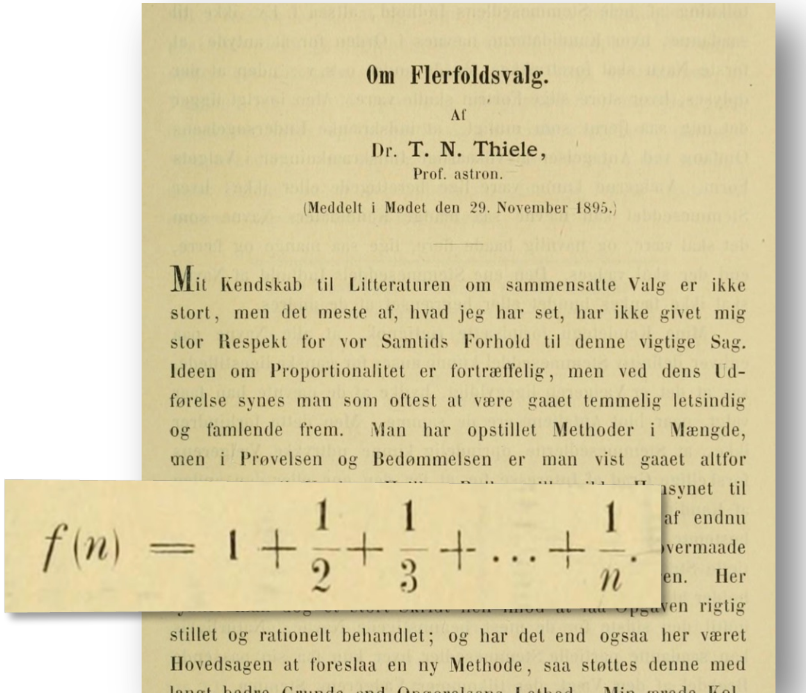
Thiele's methods

- Given a sequence $w_1, w_2, \ldots$, select a committee W that maximizes

$$\sum_{i \in N} w_1 + w_2 + \cdots + w_{u_{i(W)}}.$$

- Examples:
  - Approval Voting (AV):
    1, 1, 1, …
  - Chamberlin-Courant (CC):
    1, 0, 0, …
  - Proportional Approval Voting (PAV):
    $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

1895



Om Flerfoldsvalg.

Af

Dr. T. N. Thiele,

Prof. astron.

(Meddelt i Mødet den 29. November 1895.)

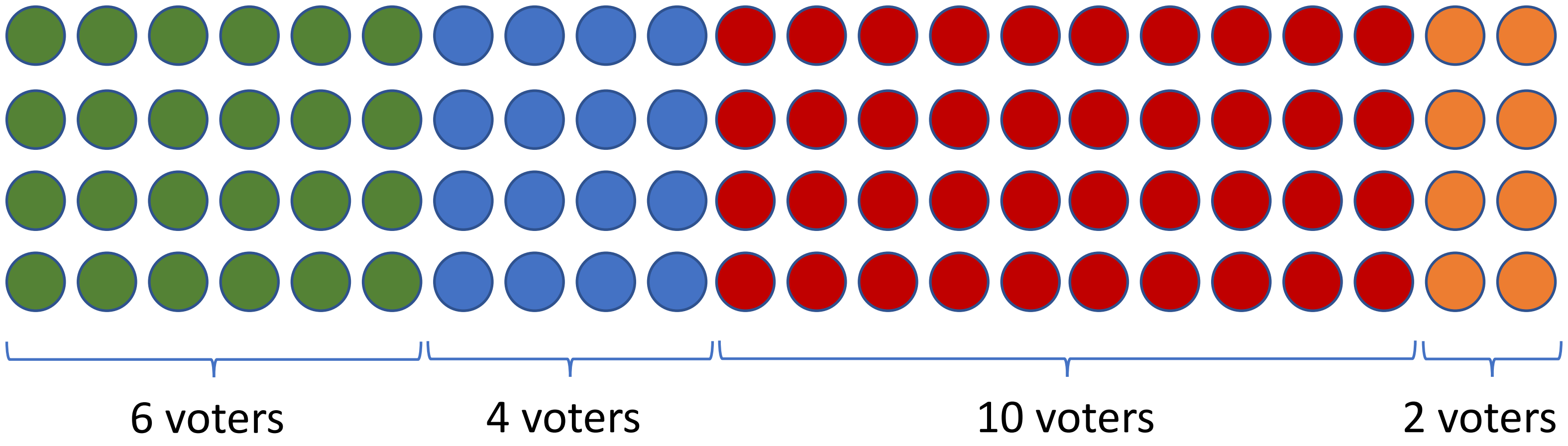
Mit Kendskab til Litteraturen om sammensatte Valg er ikke stort, men det meste af, hvad jeg har set, har ikke givet mig stor Respekt for vor Samtids Forhold til denne vigtige Sag. Ideen om Proportionalitet er fortræffelig, men ved dens Udførelse synes man som oftest at være gaaet temmelig letsindig og famlende frem. Man har opstillet Methoder i Mængde, men i Prøvelsen og Bedømmelsen er man vist gaaet altfor

$$f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}.$$

stillet og rationelt behandlet; og har det end ogsaa her været Hovedsagen at foreslaa en ny Methode, saa støttes denne med

Why harmonic numbers?

$k = 11$



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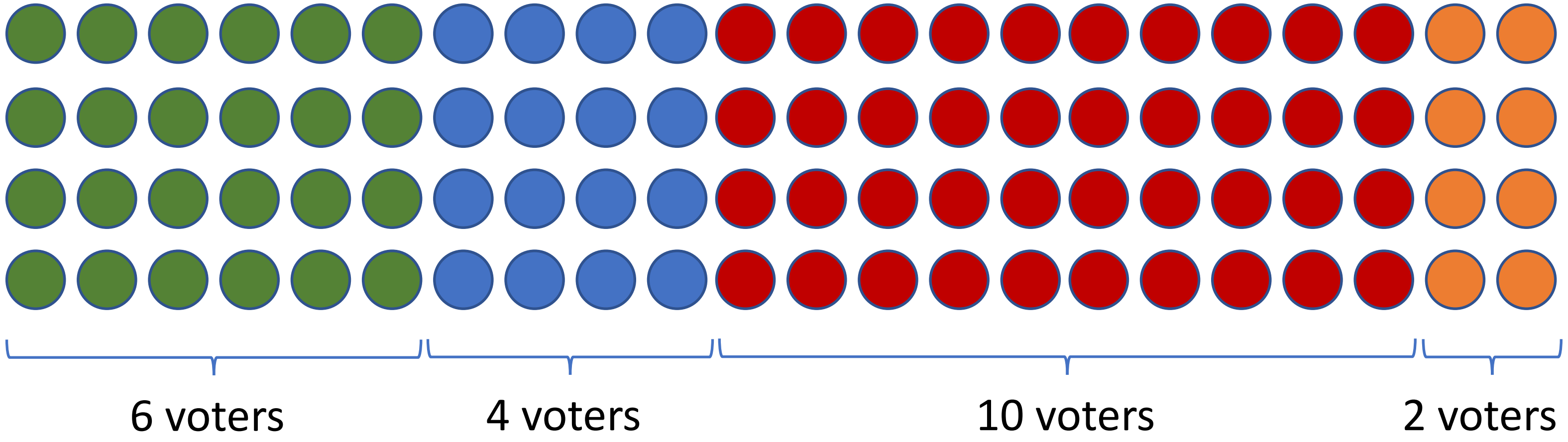

+6	+4	+10	+2
+3	+2	+5	+1
+2	+1.33	+3.33	+0.66
+1.5	+1	+2.5	+0.5
+1.2	+0.8	+2	+0.4
+1	+0.66	+1.66	+0.33

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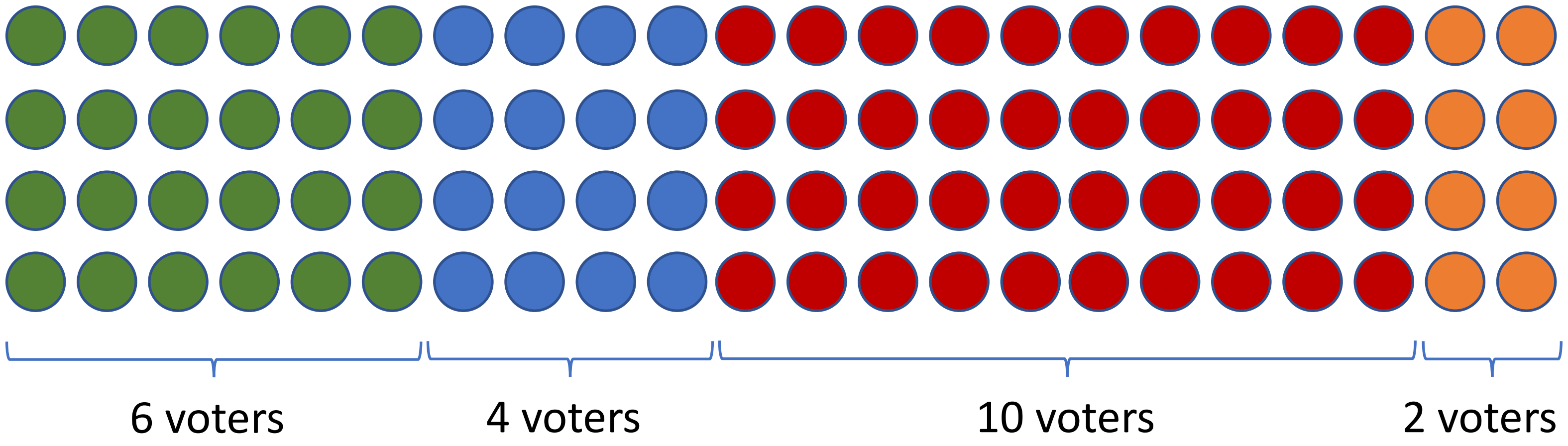

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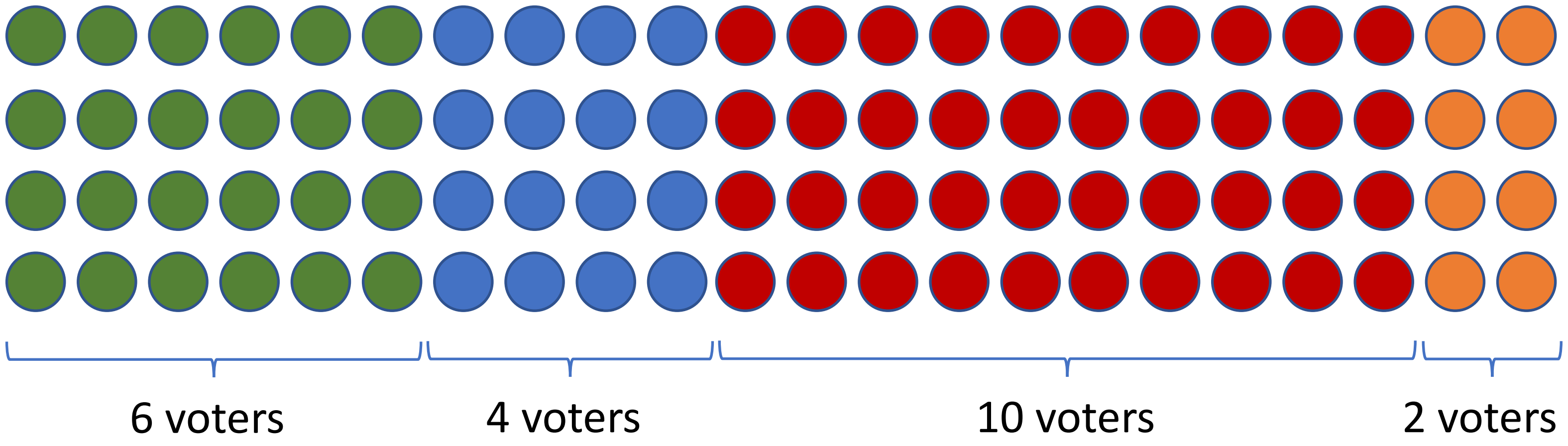

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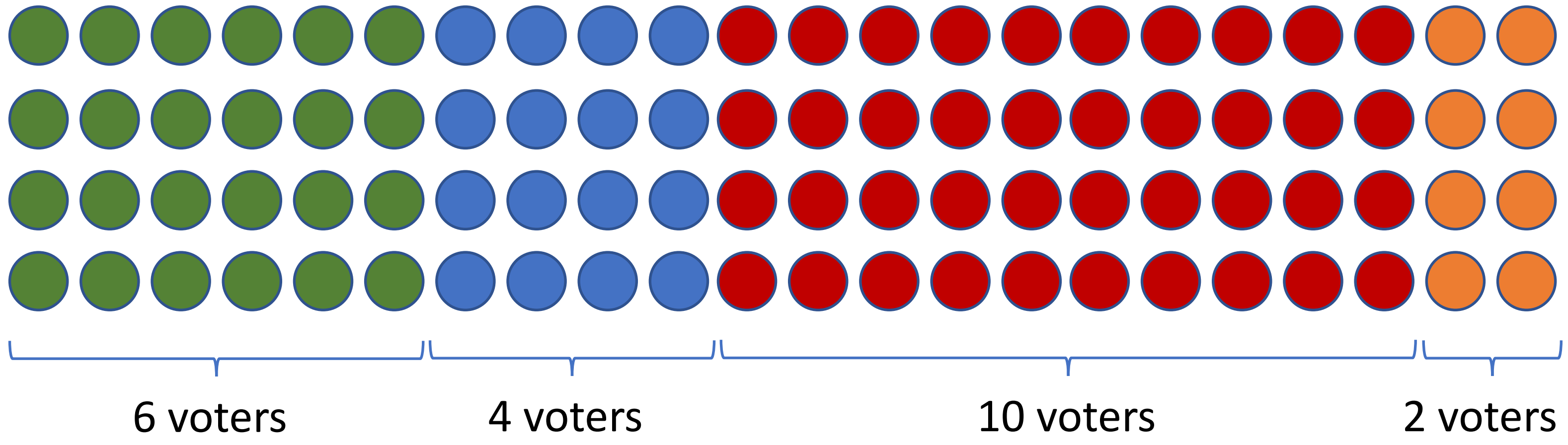

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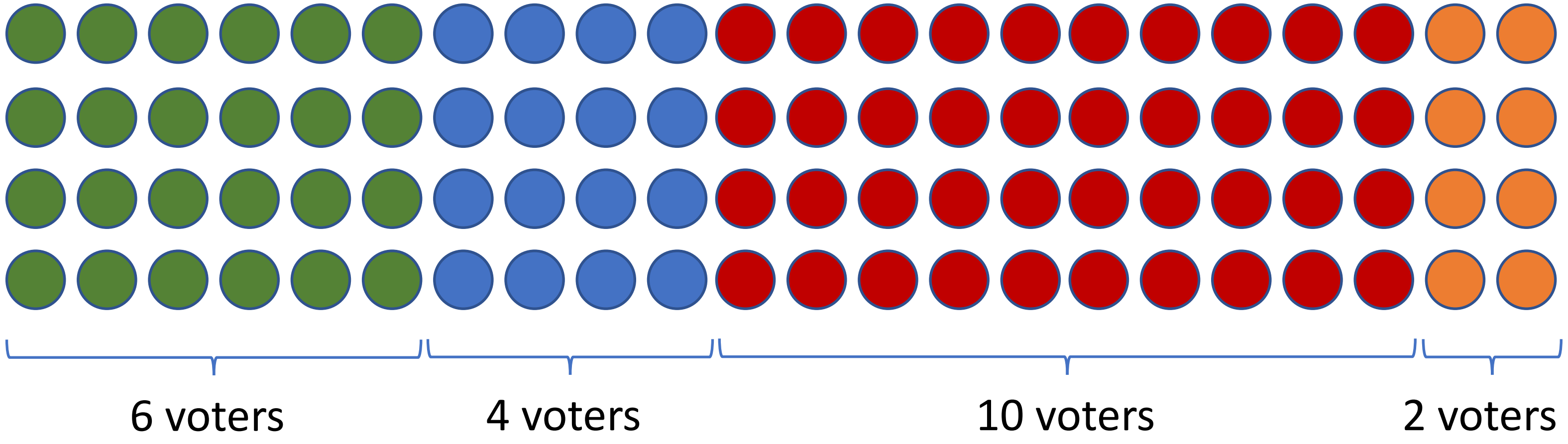

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Why harmonic numbers?

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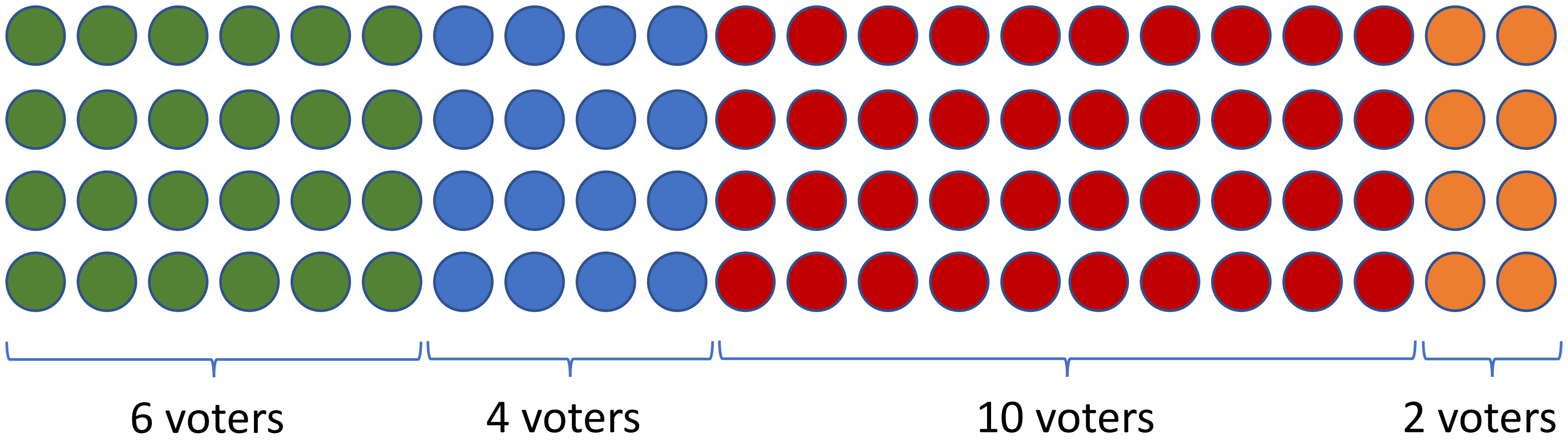

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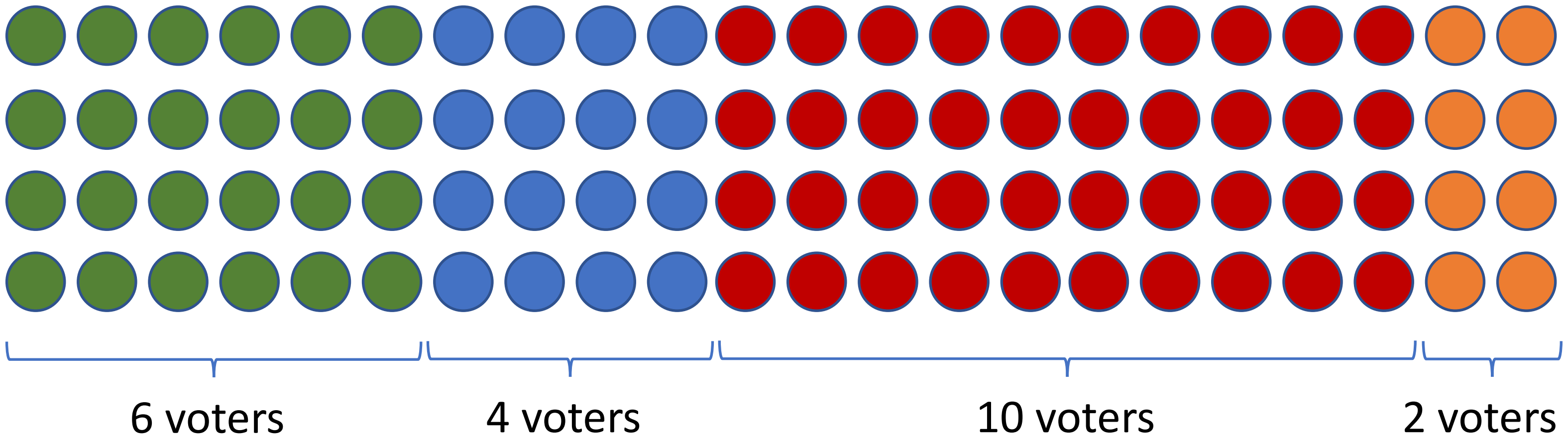


Suppose a party has x supporters, with $x \geqslant \ell \frac{n}{k}$. Then the party deserves at least ℓ seats. Note that

$$\frac{x}{1} > \frac{x}{2} > \frac{x}{3} > \cdots > \frac{x}{\ell} = \frac{n}{k}.$$

It follows that if we elect all seats with marginal increment $\geqslant \frac{n}{k}$ then all parties obtain at least what they deserve.

Why harmonic numbers?

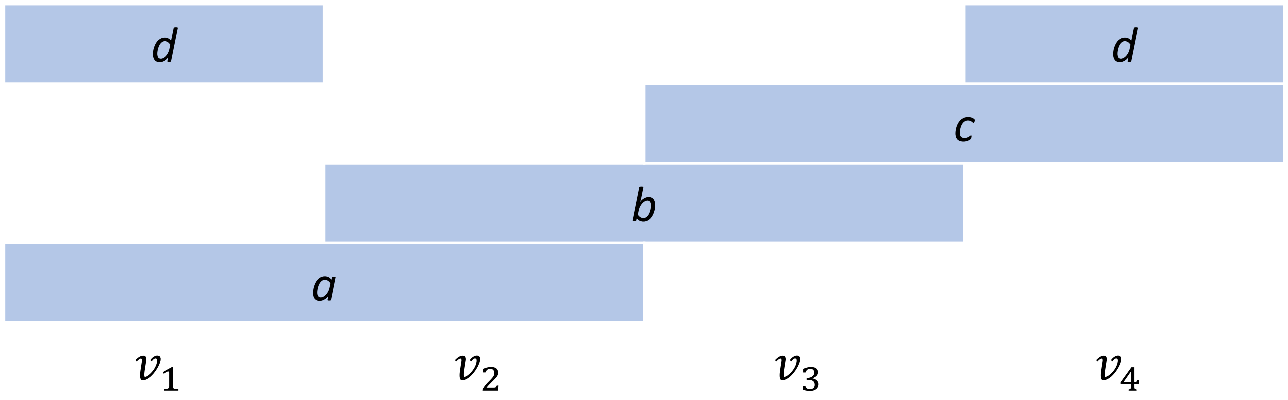
- $\boldsymbol{w} = \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right)$ is the unique sequence such that Thiele's method is proportional in the party list case. Paper
- PAV is the unique approval-based committee rule* that satisfies
  - symmetry
  - continuity
  - reinforcement
  - proportionality (D'Hondt) on party list profiles

  Paper
- *Next*: define proportionality when approval sets can intersect.

A representation axiom that is too strong

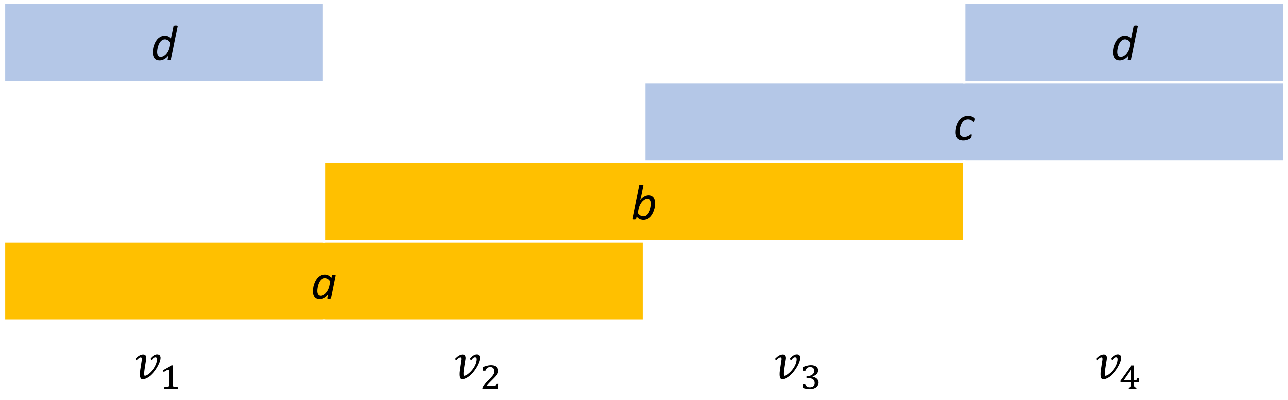
$k = 2$

"if $\frac{n}{k}$ voters have at least 1 candidate in common, then one of their common candidates should be elected"



Justified Representation

If $S \subseteq N$ with $|S| \geq \frac{n}{k}$ have a candidate in common, $|\bigcap_{i \in S} A_i| \geqslant 1$, then it cannot be that $u_i(W) = 0$ for all $i \in S$.



AV fails JR. CC and PAV satisfy JR.

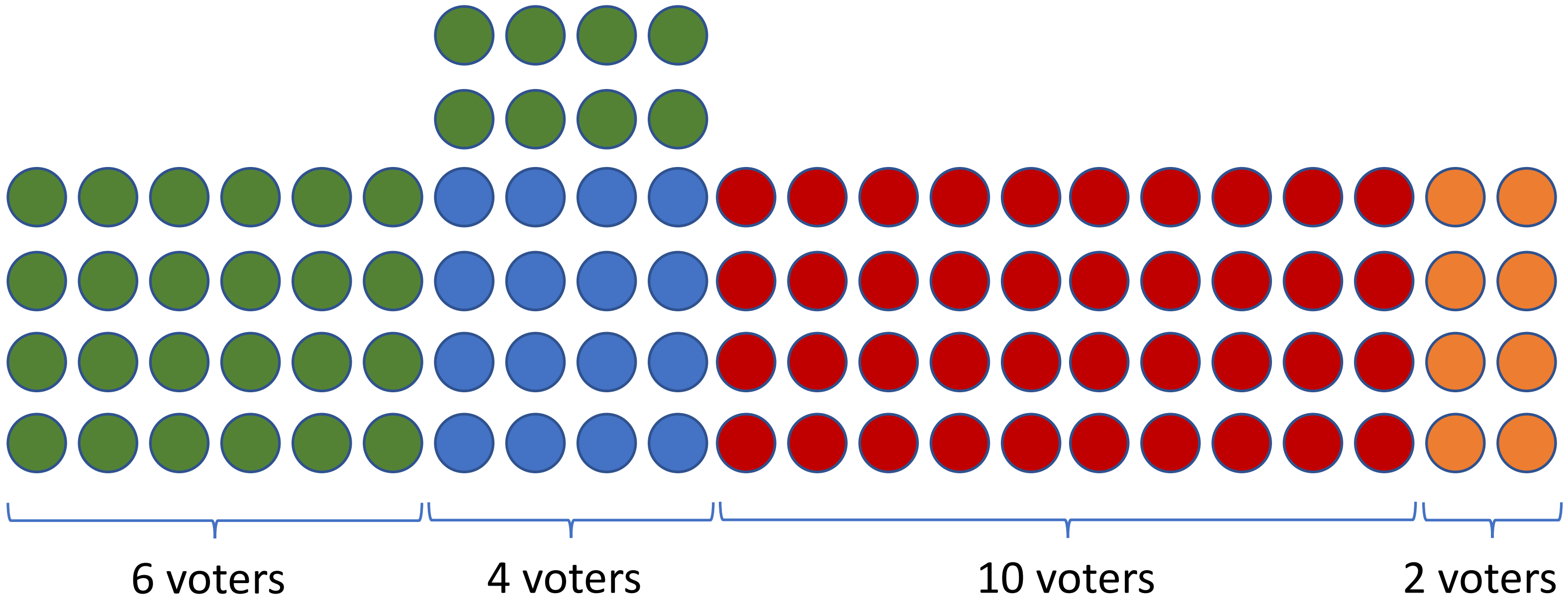
Paper

CC satisfies JR

- Let W be the CC committee, violating JR.
- Some number $n' < n$ of voters is covered by W.
- On average, each member of W covers $< \frac{n}{k}$ voters.
- Thus, some member $c^{\dagger} \in W$ covers $< \frac{n}{k}$ voters.
- Remove $c^{\dagger}$, and add the candidate approved by the JR group. This gives higher CC score.

Extended Justified Representation

If $S \subseteq N$ with $|S| \geq \ell \frac{n}{k}$ have ℓ candidates in common, $|\bigcap_{i \in S} A_i| \geqslant \ell$, then it cannot be that $u_i(W) < \ell$ for all $i \in S$.



AV and CC fail EJR. PAV satisfies EJR.

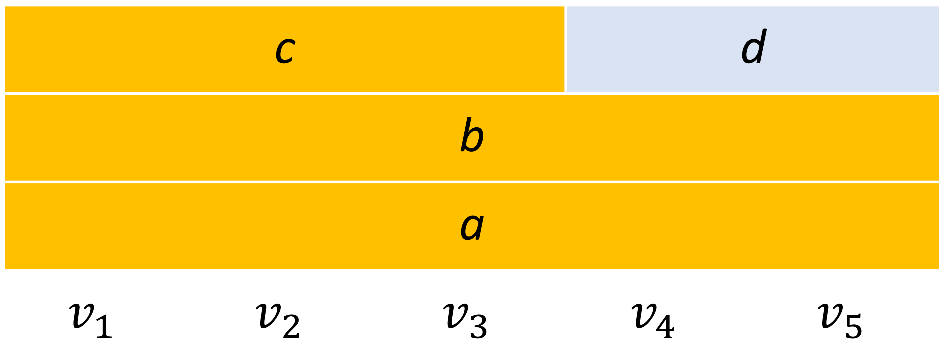
PAV satisfies EJR

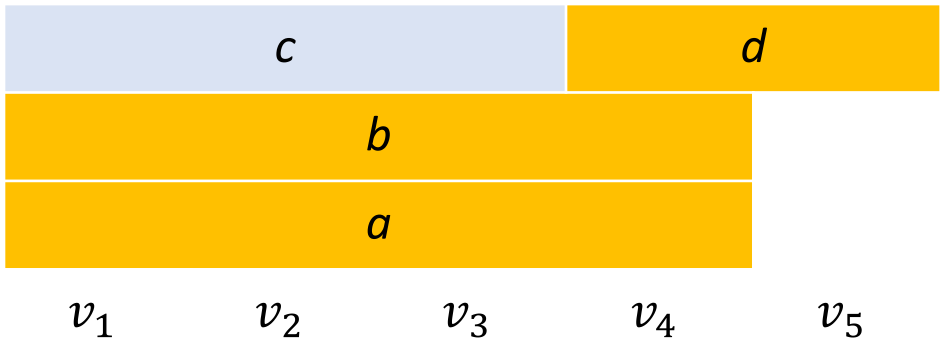
Paper

- Let W be the PAV committee. Suppose $S \subseteq N$ has size $\geqslant \ell \frac{n}{k}$, and $u_i(W) < \ell$ for all $i \in S$, but there is $c^* \in \bigcap_{i \in S} A_i \setminus W$.
- Let $\widetilde{W} = W \cup \{c^*\}$.
- Note $\text{PAV}-\text{score}(\widetilde{W}) \geqslant \text{PAV}-\text{score}(W) + |S| \frac{1}{\ell} \geqslant \text{PAV}-\text{score}(W) + \frac{n}{k}$.
- Claim: Can remove a member from $\widetilde{W}$ and lower PAV-score by $< \frac{n}{k}$.
- What is the average loss of PAV score from removal?
- $\frac{1}{k+1} \sum_{c \in \widetilde{W}} \sum_{i : c \in A_i} \frac{1}{u_i(\widetilde{W})} = \frac{1}{k+1} \sum_{i \in N} \sum_{c \in A_i \cap \widetilde{W}} \frac{1}{u_i(\widetilde{W})} \leq \frac{1}{k+1} \sum_{i \in N} 1 < \frac{n}{k}$.
- Hence there is some $c^\dagger \in \widetilde{W}$ with $\text{PAV}-\text{score}(\widetilde{W} \setminus \{c^\dagger\}) > \text{PAV}-\text{score}(W)$, contradiction.

PAV is not strategyproof

$k = 3$





Theorem. No committee rule is strategyproof and satisfies EJR.

Paper

ALGORITHM 1: Encode Problem for SAT Solving

```
Input: Set C of candidates, set N of voters, committee size k.
Question: Does a proportional and strategyproof committee rule exist?
for each profile P ∈ B^N do
    if P is a party-list profile then
        allowed[P] ← {C ∈ C_k : C satisfies EJR}
    else
        allowed[P] ← C_k
    for each committee C ∈ allowed[P] do
        introduce propositional variable x_{P,C}
for each profile P ∈ B^N do
    add clause ⋁_{C∈allowed[P]} x_{P,C}
    add clauses ⋀_{C≠C'∈allowed[P]} (¬x_{P,C} ∨ ¬x_{P,C'})
    for each voter i ∈ N do
        for each i-variant P' of P with P'(i) ⊆ P(i) do
            for each C ∈ allowed[P] and C' ∈ allowed[P'] do
                if C' ∩ P(i) ⊋ C ∩ P(i) then
                    add clause (¬x_{P,C} ∨ ¬x_{P',C'})
pass formula to SAT solver
return whether formula is satisfiable
```

Lemma 5.3. *There is no committee rule that satisfies proportionality and strategyproofness for $k = 3$, $n = 3$, and $m = 4$.*

Proof. Suppose for a contradiction that such a committee rule f existed. Consider the profile $P_1 = (ab, c, d)$. By proportionality, we have $c \in f(P_1)$ and $d \in f(P_1)$. Thus, we have $f(P_1) \in \{acd, bcd\}$. By relabelling the alternatives, we may assume without loss of generality that $f(P_1) = acd$.

Consider $P_{1.5} = (ab, ac, d)$. By Lemma 5.2, $d \in f(P_{1.5})$. Thus, $f(P_{1.5}) = acd$, or else voter 2 can manipulate towards P_1.

Consider $P_2 = (b, ac, d)$. By proportionality, $f(P_2) \in \{abd, bcd\}$. If we had $f(P_2) = abd$, then voter 1 in $P_{1.5}$ could manipulate towards P_2. Hence $f(P_2) = bcd$.

Consider $P_{2.5} = (b, ac, cd)$. By Lemma 5.2, $b \in f(P_{2.5})$. Thus, $f(P_{2.5}) = bcd$, or else voter 3 can manipulate towards P_2.

Consider $P_3 = (b, a, cd)$. By proportionality, $f(P_3) \in \{abc, abd\}$. If we had $f(P_3) = abc$, then voter 2 in $P_{2.5}$ could manipulate towards P_3. Hence $f(P_3) = abd$.

Consider $P_{3.5} = (b, ad, cd)$. By Lemma 5.2, $b \in f(P_{3.5})$. Thus, $f(P_{3.5}) = abd$, or else voter 2 can manipulate towards P_3.

Consider $P_4 = (b, ad, c)$. By proportionality, $f(P_4) \in \{abc, bcd\}$. If we had $f(P_4) = bcd$, then voter 3 in $P_{3.5}$ could manipulate towards P_4. Hence $f(P_4) = abc$.

Consider $P_{4.5} = (b, ad, ac)$. By Lemma 5.2, $b \in f(P_{4.5})$. Thus, $f(P_{4.5}) = abc$, or else voter 3 can manipulate towards P_4.

Consider $P_5 = (b, d, ac)$. By proportionality, $f(P_5) \in \{abd, bcd\}$. If we had $f(P_5) = abd$, then voter 2 in $P_{4.5}$ could manipulate towards P_5. Hence $f(P_5) = bcd$.

Consider $P_{5.5} = (b, cd, ac)$. By Lemma 5.2, $b \in f(P_{5.5})$. Thus, $f(P_{5.5}) = bcd$, or else voter 2 can manipulate towards P_5.

Consider $P_6 = (b, cd, a)$. By proportionality, $f(P_6) \in \{abc, abd\}$. If we had $f(P_6) = abc$, then voter 3 in $P_{5.5}$ could manipulate towards P_6. Hence $f(P_6) = abd$.

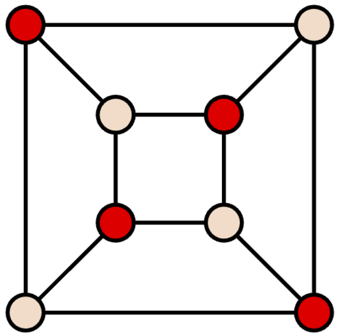
Consider $P_{6.5} = (b, cd, ad)$. By Lemma 5.2, $b \in f(P_{6.5})$. Thus, $f(P_{6.5}) = abd$, or else voter 3 can manipulate towards P_6.

Consider $P_7 = (b, c, ad)$. By proportionality, $f(P_7) \in \{abc, bcd\}$. If we had $f(P_7) = bcd$, then voter 2 in $P_{6.5}$ could manipulate towards P_7. Hence $f(P_7) = abc$.

Finally, consider $P_{7.5} = (ab, c, ad)$. By Lemma 5.2, $c \in f(P_{7.5})$. Thus, $f(P_{7.5}) = abc$, or else voter 1 can manipulate towards P_7. But then voter 3 can manipulate towards $P_1 = (ab, c, d)$, because by our initial assumption, we have $f(P_1) = acd$. Contradiction. ☐

PAV is NP-complete

- *Instance*: Profile P, size k, number $B \geqslant 0$.
- *Question*: Is there a committee W with $|W| = k$ such that $\text{PAV}-\text{score}(W) \geqslant B$?



- Clearly in NP. We'll show this is NP-hard by reducing from CUBIC INDEPENDENT SET:

- *Instance*: Graph $G = (V, E)$ with $d(v) = 3$ for all $v \in V$, size k.
- *Question*: Is there $V' \subseteq V$ with $|V'| = k$ such that for each $e = \{u, v\} \in E$, either $u \notin V'$ or $v \notin V'$?

Paper

PAV is NP-complete

- Let $G = (V, E)$ be a cubic graph and let $1 \leqslant k \leqslant |V|$.
- Introduce candidates $C = V$, and voters $N = E$. Each voter approves its endpoints. Set $B = 3k$.
- We prove: There is a k-committee with PAV-score B if and only if G has an independent set of size k.
- ⇐: Let V' be an independent set of size k. Then no voter approves 2 candidates in V'. Each candidate in V' is approved by the 3 incident edges. So the PAV-score of V' is $3k$.
- ⇒: Suppose W has PAV-score $3k$. Each candidate is approved by 3 voters, so can contribute at most 3 to the PAV score. Since the total score is $3k$, each member of W contributes 3. This can only happen if no voter approves more than 1 candidate in W, so it’s an independent set.

PAV can be computed by ILP

- In practice, using modern solvers like Gurobi, we can compute PAV as an integer linear program:

- Maximize $\sum_{i \in N} \sum_{\ell=1}^{k} \frac{1}{l} x_{i,\ell}$

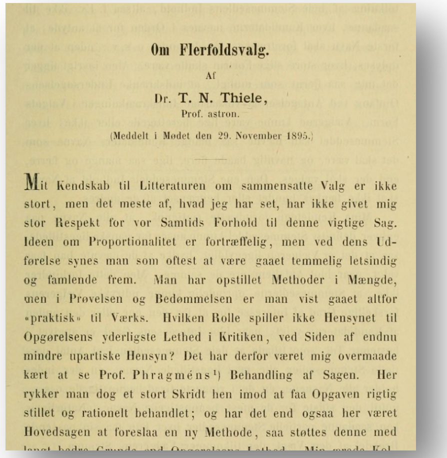
  subject to $\sum_{\ell=1}^{k} x_{i,\ell} = \sum_{c \in A_i} y_c$ for all $i \in N$

  $\sum_{c \in C} y_c = k$

  $y_c \in \{0,1\},\ x_{i,\ell} \in \{0,1\}$ for all i, ℓ, c.

- Fun fact: If profile is single-peaked (i.e. candidates ordered left-to-right, everyone approves an interval), the ILP can be solved in polynomial time.

Sequential PAV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- Greedy procedure for calculating PAV:
- $W \leftarrow \emptyset$
- **while** $|W| < k$ **do**
  - Find $c \in C$ that maximizes PAV-score($W \cup \{c\}$)
  - $W \leftarrow W \cup \{c\}$
- **return** W
- *Theorem:* Let W be the optimum PAV committee, and let W' be the committee identified by seqPAV. Then $\text{PAV}-\text{score}(W') \geqslant \left(1 - \frac{1}{e}\right) \text{PAV}-\text{score}(W)$.

63%

- Proof: PAV-score is submodular, and approximation is true in general for the greedy algorithm for maximizing a submodular function.

$f(W \cup \{c\}) - f(W) \geqslant f(W' \cup \{c\}) - f(W')$
if $W \subseteq W'$.

1	× 1	1		a	b	c	d	e	
1	× 1	1		a	b	c	d		f
9	× 1	9		a	b		d	e	
8	× 1	8		a	b		d		f
8	× 1	8		a		c		e	
10	× 1	10		a		c			f
1	× 1	1		a			d		f
4	× 1	4			b	c	d		
5	× 1	5			b	c			f
7	× 1	7			b			e	
2	× 1	2			b				f
4	× 1	4				c	d		
3	× 1	3				c		e	
1	× 1	1				c			f
9	× 1	9					d		
8	× 1	8						e	
9	× 1	9							f
18	× 1	18	z						
			18	38	37	37	37	36	37

1	× 1/2	1/2		*a*	*b*	*c*	*d*	*e*	
1	× 1/2	1/2		*a*	*b*	*c*	*d*		*f*
9	× 1/2	9/2		*a*	*b*		*d*	*e*	
8	× 1/2	4		*a*	*b*		*d*		*f*
8	× 1/2	4		*a*		*c*		*e*	
10	× 1/2	5		*a*		*c*			*f*
1	× 1/2	1/2		*a*			*d*		*f*
4	× 1	4			*b*	*c*	*d*		
5	× 1	5			*b*	*c*			*f*
7	× 1	7			*b*			*e*	
2	× 1	2			*b*				*f*
4	× 1	4				*c*	*d*		
3	× 1	3				*c*		*e*	
1	× 1	1				*c*			*f*
9	× 1	9					*d*		
8	× 1	8						*e*	
9	× 1	9							*f*
18	× 1	18	*z*						
			18	✓	55/2	27	27	27	27

1	× 1/3	1/3		*a*	*b*	*c*	*d*	*e*	
1	× 1/3	1/3		*a*	*b*	*c*	*d*		*f*
9	× 1/3	3		*a*	*b*		*d*	*e*	
8	× 1/3	8/3		*a*	*b*		*d*		*f*
8	× 1/2	4		*a*		*c*		*e*	
10	× 1/2	5		*a*		*c*			*f*
1	× 1/2	1/2		*a*			*d*		*f*
4	× 1/2	2			*b*	*c*	*d*		
5	× 1/2	5/2			*b*	*c*			*f*
7	× 1/2	7/2			*b*			*e*	
2	× 1/2	1			*b*				*f*
4	× 1	4				*c*	*d*		
3	× 1	3				*c*		*e*	
1	× 1	1				*c*			*f*
9	× 1	9					*d*		
8	× 1	8						*e*	
9	× 1	9							*f*
18	× 1	18	*z*						
			18	✓	✓	133/6	131/6	131/6	22

1	× 1/4	1/4		*a*	*b*	*c*	*d*	*e*	
1	× 1/4	1/4		*a*	*b*	*c*	*d*		*f*
9	× 1/3	3		*a*	*b*		*d*	*e*	
8	× 1/3	8/3		*a*	*b*		*d*		*f*
8	× 1/3	8/3		*a*		*c*		*e*	
10	× 1/3	10/3		*a*		*c*			*f*
1	× 1/2	1/2		*a*			*d*		*f*
4	× 1/3	4/3			*b*	*c*	*d*		
5	× 1/3	5/3			*b*	*c*			*f*
7	× 1/2	7/2			*b*			*e*	
2	× 1/2	1			*b*				*f*
4	× 1/2	2				*c*	*d*		
3	× 1/2	3/2				*c*		*e*	
1	× 1/2	1/2				*c*			*f*
9	× 1	9					*d*		
8	× 1	8						*e*	
9	× 1	9							*f*
18	× 1	18	*z*						
			18	✓	✓	✓	227/12	227/12	227/12

1	× 1/5	1/5		*a*	*b*	*c*	*d*	*e*	
1	× 1/5	1/5		*a*	*b*	*c*	*d*		*f*
9	× 1/4	9/4		*a*	*b*		*d*	*e*	
8	× 1/4	2		*a*	*b*		*d*		*f*
8	× 1/3	8/3		*a*		*c*		*e*	
10	× 1/3	10/3		*a*		*c*			*f*
1	× 1/3	1/3		*a*			*d*		*f*
4	× 1/4	1			*b*	*c*	*d*		
5	× 1/3	5/3			*b*	*c*			*f*
7	× 1/2	7/2			*b*			*e*	
2	× 1/2	1			*b*				*f*
4	× 1/3	4/3				*c*	*d*		
3	× 1/2	3/2				*c*		*e*	
1	× 1/2	1/2				*c*			*f*
9	× 1/2	9/2					*d*		
8	× 1	8						*e*	
9	× 1	9							*f*
18	× 1	18	*z*						
			18	✓	✓	✓	✓	1087/60	541/30

1	× 1/6	1/6		*a*	*b*	*c*	*d*	*e*	
1	× 1/5	1/5		*a*	*b*	*c*	*d*		*f*
9	× 1/5	9/5		*a*	*b*		*d*	*e*	
8	× 1/4	2		*a*	*b*		*d*		*f*
8	× 1/4	2		*a*		*c*		*e*	
10	× 1/3	10/3		*a*		*c*			*f*
1	× 1/3	1/3		*a*			*d*		*f*
4	× 1/4	1			*b*	*c*	*d*		
5	× 1/3	5/3			*b*	*c*			*f*
7	× 1/3	7/3			*b*			*e*	
2	× 1/2	1			*b*				*f*
4	× 1/3	4/3				*c*	*d*		
3	× 1/3	1				*c*		*e*	
1	× 1/2	1/2				*c*			*f*
9	× 1/2	9/2					*d*		
8	× 1/2	4						*e*	
9	× 1	9							*f*
18	× 1	18	*z*						
			18	✓	✓	✓	✓	✓	541/30

1	× 1/6	1/6		a	b	c	d	e	
1	× 1/6	1/6		a	b	c	d		f
9	× 1/5	9/5		a	b		d	e	
8	× 1/5	8/5		a	b		d		f
8	× 1/4	2		a		c		e	
10	× 1/4	5/2		a		c			f
1	× 1/4	1/4		a			d		f
4	× 1/4	1			b	c	d		
5	× 1/4	5/4			b	c			f
7	× 1/3	7/3			b			e	
2	× 1/3	2/3			b				f
4	× 1/3	4/3				c	d		
3	× 1/3	1				c		e	
1	× 1/3	1/3				c			f
9	× 1/2	9/2					d		
8	× 1/2	4						e	
9	× 1/2	9/2							f
18	× 1	18	z						
			18	✓	✓	✓	✓	✓	✓

$$n = 108, k = 6, \frac{n}{k} = 18$$

So EJR requires $z \in W$.

Sequential PAV fails EJR

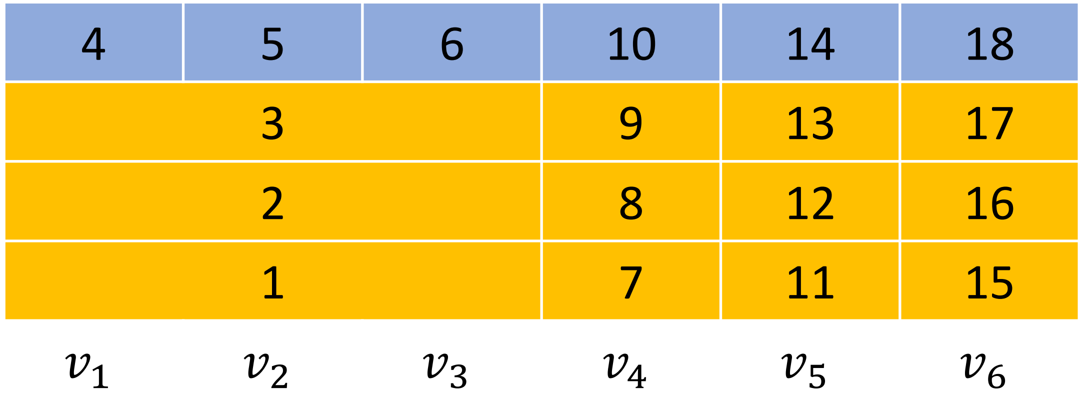
Paper

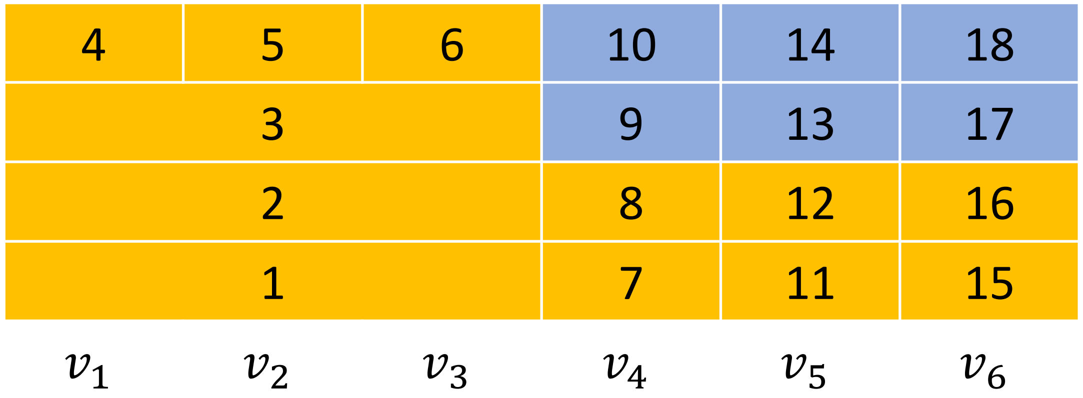
- This example is the smallest counterexample! (Though for $k = 7/8/9, n = 35/24/17$ is enough.)
- How to find such counterexamples? ILP!
- Fix k. In any given counterexample, we can relabel alternatives such that SeqPAV selects them in the order $c_1, c_2, \dots, c_k$, and does not select c_{k+1}. Since unselected candidates have no influence, we can take $C = k + 1$.
- For each $S \subseteq C$, add variable $z_S \in \mathbb{Z}$.
- Add constraints that for $j > i$, PAV-score$(\{c_1, \dots, c_i\}) >$ PAV$-$score$(\{c_1, \dots, c_{i-1}, c_j\})$
- Add constraint that $z_{\{c_{k+1}\}} \geqslant \frac{1}{k} \sum_S z_S$.
- Minimize $\sum_S z_S$.

Is PAV always right?

[Paper]

$k = 12$





EJR not strong enough to capture this!

Phragmén’s Sequential Rule (1894)

- It costs $1 to elect a candidate to the committee.
- Each voter has a virtual bank account, initially empty.
- We slowly fill up the bank accounts until some candidate has supporters who have $1 in total.
- We elect such a candidate and take the supporters’ money away.
- Finish when k candidates have been elected.

Phragmén's Sequential Rule (1894)

EDVARD PHRAGMÉN

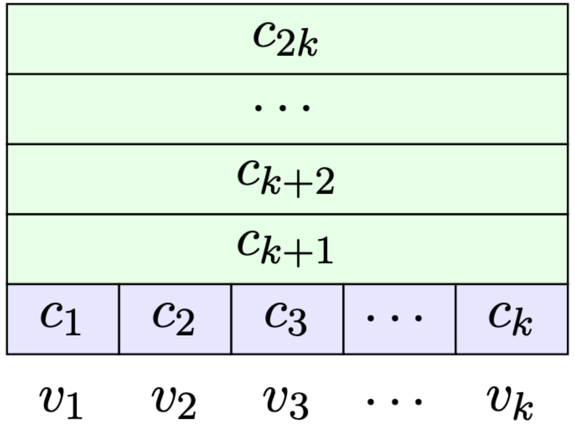
- The rule fails EJR.
- But it satisfies PJR ("Proportional Justified Representation"):

  Paper

  If $S \subseteq N$ with $|S| \geq \ell \frac{n}{k}$ have ℓ candidates in common, $|\bigcap_{i \in S} A_i| \geqslant \ell$,
  then it cannot be that fewer than ℓ candidates from $\bigcup_{i \in S} A_i$ are selected

- Why?
  - Phragmén cannot stop before at least \$$k$ have been given out.
  - By that point, S has received \$$\ell$. Before Phragmén gives out more than \$$k$, S must have <\$1 left, so has spent > \$$\ell - 1$.

Paper



Method of Equal Shares (2020)

- It costs $\$\frac{n}{k}$ to elect a candidate. Every voter gets \$1.
- Repeatedly, we go through the candidates and see if they can be purchased with the money of their supporters. We compute the way it can be purchased that minimizes the maximum payment ρ of any supporter.
- We elect the candidate with minimum ρ.
- This rule satisfies EJR.

Paper

Is PAV always right?

$k = 12$

4	5	6	10	14	18
	3		9	13	17
	2		8	12	16
	1		7	11	15
v_1	v_2	v_3	v_4	v_5	v_6

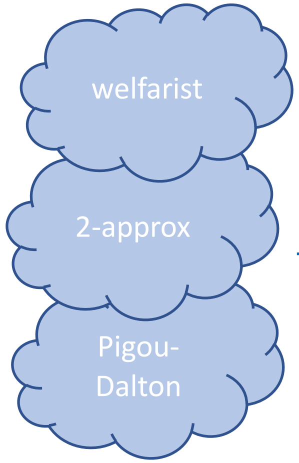
4	5	6	10	14	18
	3		9	13	17
	2		8	12	16
	1		7	11	15
v_1	v_2	v_3	v_4	v_5	v_6

What axiom can exclude this?

Core

- Let W be a committee.
- A group $S \subseteq N$ with $|S| \geq \ell \frac{n}{k}$ *blocks* W if there is $T \subseteq C$ with $|T| = \ell$ such that $u_i(T) > u_i(W)$ for all $i \in S$.
- W is in the *core* if it is not blocked.
- Core implies EJR: An EJR failure is a blocking coalition where $T \subseteq \bigcap_{i \in S} A_i$.
- *Open Problem*: does there always exist a committee in the core?

4	5	6	10	14	18
	3		9	13	17
	2		8	12	16
	1		7	11	15
v_1	v_2	v_3	v_4	v_5	v_6



Paper

Full Justified Representation (FJR)

If $S \subseteq N$ with $|S| \geq \ell \frac{n}{k}$ can propose a set T of ℓ candidates such that $u_i(T) \geq \beta$ for all $i \in S$, then it cannot be that $u_i(W) < \beta$ for all $i \in S$.

- Always satisfiable.
- Satisfiable for all monotone utility functions!
- No known natural rule or polytime algorithm that satisfies it.

Paper