On Characterizing Semantic Pollution of Proof Systems

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Proof theory and model-theoretic semantics provide different ways of proving results about logics, and soundness and completeness proofs reveal an intrinsic connection between these methods. However, Avron (1996) writes that a requirement of a 'good' proof system is that it should be "independent from any particular semantics". This has become known as syntactic purity of a proof system, as opposed to a semantically polluted one. The value of soundness and completeness proofs seems to come from a certain independence that the syntactic side has from the semantic side. If a proof system is semantically polluted, this may take away from its "proof-theoretical nature and the expected generality" (Avron, 1996). Labeled proof calculi are a standard example of semantically polluted systems, due to their internalization of Kripke semantics into the proof system (see e.g. Poggiolesi and Restall, 2012). They are the result of attempts to find a 'well-behaved' proof system for modal logic. Other examples for semantically polluted calculi can be found in the literature, including for instance semantic sequents and tableaux (Poggiolesi, 2010), or the inclusion of neighborhood semantics into the proof system (Negri, 2017).

The goal of this talk is to investigate whether there are ways to characterize what semantic pollution is, and to provide and compare ways of telling when a proof system is semantically polluted or not. This contributes to a better clarification of what a 'good' proof system can be, and encourages a more nuanced understanding of the distinction between syntax and semantics. The literature distinguishes between a strong and a weak definition of syntactic purity. *Strong* syntactic purity occurs when a proof system is "independent of any particular semantics" (Avron, 1996). This includes the idea that "one should not be able to guess, just from the form of the structures which are used, the intended semantic of a given proof system". *Weak* syntactic purity, on the other hand, says that a sequent calculus cannot make use of 'explicit semantic elements' (Poggiolesi, 2010). Poggiolesi argues that strong syntactic purity is too strong, since it implies that basic propositional sequent calculi already must be declared semantically polluted. Thus, she adopts weak syntactic purity, where she defines a 'semantic element' as an untranslatable ingredient of a sequent. This rules out, for example, expressions like *xRy* in labeled calculi, that explicitly incorporate the notion of possible worlds and the Kripke accessibility relation.

In this talk, we will discuss the different conceptions of semantic pollution, and connect them to possible formal measures. For weak syntactic purity, we discourage the idea that translatability is decisive in the formal description of a semantic element. Instead, we aim to spell out conditions on the (use of) formal language in a proof system in order to exclude semantic elements, which also helps us understand better why these elements are excluded. We will discuss the (dis)advantages of possible existing formalizations of 'recognizing' semantics from a proof system, and we sketch the beginning to a new approach, that focuses on semantic pollution of Kripke semantics in labeled calculi.

References

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