Suppositional inquisitive semantics

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Support

- Inquisitive semantics takes sentences to express a proposal to the participants in the conversation to update the common ground of the conversation (CG) in one or more ways.

- The question in (1a) proposes two alternative ways to update the CG, which correspond to the two responses (1b-c).

(1)
a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
b. If Alf goes, then Bea will go as well. $p \rightarrow q$
c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$

- Basic inquisitive semantics (InqB) accounts for the intuition that (1b-c) are responses that, *if accepted by the other conversational participants*, yield a CG that supports the question in (1a), settling the proposal that it expresses.

Support and reject

- InqB does not account for the intuition that (1c) rejects the proposal expressed by (1b), and vice versa.

(1) a. If Alf goes to the party, will Bea go too? $p \to ?q$
b. If Alf goes, then Bea will go as well. $p \to q$
c. If Alf goes, then Bea will not go. $p \to \neg q$

- Radical inquisitive semantics (InqR) does account for this.

- It achieves this by not only specifying support-conditions, as InqB does, but simultaneously also rejection-conditions.

Support, reject, dismiss

- InqB and InqR do not account for the intuition that (1d) dismisses a supposition that is shared by (1a)-(1c).

(1)
a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
b. If Alf goes, then Bea will go as well. $p \rightarrow q$
c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$
d. Alf will not go to the party. $\neg p$

- This is just as much a way of settling the proposals that these sentences express, on a par with support and rejection.
- Suppositional inq semantics (InqS) aims to characterize when a response suppositionally dismisses a given proposal.
- To achieve this, it does not only specify conditions for support and rejection, but also for supposition dismissal.

Some basic notions

- We consider a language $\mathcal{L}$ of propositional logic.
- We let $?\varphi$ be an abbreviation of $\varphi \vee \neg\varphi$
- Sentences are evaluated relative to information states.
- An information state s is set of possible worlds.
- A possible world w is a valuation function that assigns the value 1 or 0 to each atomic sentence in $\mathcal{L}$.
- We use ω to denote the set of all worlds, the ignorant state.
- We refer to the empty set as the absurd or inconsistent state.

Global structure of the semantics

- The semantics for $\mathcal{L}$ is stated by simultaneous recursion of three notions:

  1. $s \models^{+} \varphi$ state s supports φ InqB
  2. $s \models^{-} \varphi$ state s rejects φ InqR
  3. $s \models^{\circ} \varphi$ state s dismisses a supposition of φ InqS

- By $[\varphi]^{\dagger}$ we denote $\{s \subseteq \omega \mid s \models^{\dagger} \varphi\}$. $^{\dagger} \in \{^{+}, ^{-}, ^{\circ}\}$

- In InqS the proposition expressed by φ, $[\varphi]$, is determined by the triple $\langle [\varphi]^{+}, [\varphi]^{-}, [\varphi]^{\circ} \rangle$.

- In *presenting* the semantics, we will often quantify over the maximal elements of $[\varphi]^{\dagger}$, called $\dagger$-alternatives.

- For any set of states $\mathcal{S}$: $\textsc{alt}\,\mathcal{S} = \{s \in \mathcal{S} \mid \neg \exists t \in \mathcal{S} : s \subset t\}$

Notation convention for representing states

- Let $|\varphi|$ denote the set of worlds where φ is classically true
- This gives us a convenient notation for states. For instance:

$$\begin{array}{lll} |p| & \models^{+} & p \vee q \\ |\neg p| & \models^{-} & p \wedge q \\ |\neg p| & \models^{\circ} & p \rightarrow q \end{array}$$

Downward closure / persistence

- A distinctive feature of InqB is that $[\varphi]^+$ is downward closed
  - If $s \models^+ \varphi$, then for any $t \subseteq s$: $t \models^+ \varphi$

  That is, in InqB support is persistent

- In InqR, both $[\varphi]^+$ and $[\varphi]^-$ are downward closed
  - If $s \models^+ \varphi$, then for any $t \subseteq s$: $t \models^+ \varphi$
  - If $s \models^- \varphi$, then for any $t \subseteq s$: $t \models^- \varphi$

  That is, in InqR both support and rejection are persistent

- Underlying idea: if s supports/rejects a sentence φ, then any more informed state $t \subseteq s$ will support/reject φ as well

- Information growth cannot lead to retraction of support/reject

Persistence and suppositional dismissal

- As soon as we take suppositional dismissal into account this central idea from InqB and InqR is no longer valid

- For instance, we want that:

$$|p \rightarrow q| \models^{+} p \rightarrow q$$

But we also want that:

$$|\neg p| \models^{\circ} p \rightarrow q$$
$$|\neg p| \not\models^{+} p \rightarrow q$$

- So: information growth can lead to suppositional dismissal, and thereby to retraction of support (or retraction of rejection)

Persistence modulo suppositional dismissal

- Fortunately, there is a natural way to adapt the idea that support and rejection are persistent to the setting of InqS

- Namely, in InqS we postulate that support and rejection are persistent modulo dismissal of a supposition, and that dismissal itself is fully persistent:

  - If $s \models^{+} \varphi$ and $t \subseteq s$, then $t \models^{+} \varphi$ or $t \models^{\circ} \varphi$
  - If $s \models^{-} \varphi$ and $t \subseteq s$, then $t \models^{-} \varphi$ or $t \models^{\circ} \varphi$
  - If $s \models^{\circ} \varphi$ and $t \subseteq s$, then $t \models^{\circ} \varphi$

Two more postulates

Second postulate

- The inconsistent state suppositionally dismisses any sentence φ, and never supports or rejects it. That is, for any φ:

$$\emptyset \models^{\circ} \varphi$$
$$\emptyset \not\models^{+} \varphi$$
$$\emptyset \not\models^{-} \varphi$$

Third postulate

- Support and rejection are mutually exclusive : $[\varphi]^{+} \cap [\varphi]^{-} = \emptyset$
- The postulates do not exclude that for some φ and $s \neq \emptyset$:
    - $s \models^{+} \varphi$ and $s \models^{\circ} \varphi$
    - $s \models^{-} \varphi$ and $s \models^{\circ} \varphi$

Finally

- Final postulate: any completely informed consistent state $\{w\}$ supports, rejects, or suppositionally dismisses any sentence:

$$\forall \varphi \in \mathcal{L} : \forall w \in \omega : \{w\} \in ([\varphi]^+ \cup [\varphi]^- \cup [\varphi]^\circ)$$

Propositions as conversational issues

- The postulates imply that the three components of a proposition jointly form a non-empty downward closed set of states that cover the set of all worlds:

$$\bigcup([\varphi]^+ \cup [\varphi]^- \cup [\varphi]^\circ) = \omega$$

- In terms of InqB, our propositions are issues over ω.
- The issue embodied by $[\varphi]$ is a conversational issue, it specifies several appropriate ways of responding to φ.

Some responsehood relations

- We can define a range of logical responsehood relations according to the following scheme, filling in different semantic relations for †:
  - $\psi \models^{\dagger} \varphi$ iff $\forall u \in \textsc{alt}[\psi]^{+}: u \models^{\dagger} \varphi$
- Three obvious responsehood relations are:
  - ψ supports φ: $\psi \models^{+} \varphi$
  - ψ rejects φ: $\psi \models^{-} \varphi$
  - ψ dismisses a supposition of φ: $\psi \models^{\circ} \varphi$
- But if, for example, we define a semantic property $s \models^{\otimes} \varphi$ as below, we obtain a new responsehood relation, which may be dubbed ψ suppositionally dismisses φ.
  - $s \models^{\otimes} \varphi$ iff $s \models^{\circ} \varphi$ and $\forall t \subseteq s: t \not\models^{+} \varphi$ and $t \not\models^{-} \varphi$.

Inquisitive and suppositional sentences

- φ is support inquisitive iff there are at least two support-alternatives for it, i.e., $\textsc{alt}[\varphi]^+$ contains at least two elements
- Rejection inquisitiveness and suppositional inquisitiveness are defined similarly
- We call a sentence φ suppositional iff there is a non-absurd state s such that $s \models^\circ \varphi$

Atomic sentences

- $s \models^{+} p$ iff $s \neq \emptyset$ and $\forall w \in s\colon w(p) = 1$
  $s \models^{-} p$ iff $s \neq \emptyset$ and $\forall w \in s\colon w(p) = 0$
  $s \models^{\circ} p$ iff $s = \emptyset$

- Atomic sentences are not suppositional, since only the inconsistent state can dismiss a supposition of p.
- Atomic sentences are not inquisitive, since there is only a single support-alternative and a single rejection-alternative:

$$\begin{aligned} \textsc{alt}[p]^{+} &= \{|p|\} \\ \textsc{alt}[p]^{-} &= \{|\neg p|\} \end{aligned}$$

Negation

$$s \models^{+} \neg\varphi \quad \text{iff} \quad s \models^{-} \varphi$$
$$s \models^{-} \neg\varphi \quad \text{iff} \quad s \models^{+} \varphi$$
$$s \models^{\circ} \neg\varphi \quad \text{iff} \quad s \models^{\circ} \varphi$$

- The suppositional content of φ is inherited by its negation $\neg\varphi$
- Unlike in InqB: $\neg\neg\varphi \equiv \varphi$

Disjunction

- $s \models^+ \varphi \vee \psi$ iff $s \models^+ \varphi$ or $s \models^+ \psi$

  $s \models^- \varphi \vee \psi$ iff $s \models^- \varphi$ and $s \models^- \psi$

  $s \models^\circ \varphi \vee \psi$ iff $s \models^\circ \varphi$ or $s \models^\circ \psi$

- The suppositional content of φ and ψ is inherited by the disjunction $\varphi \vee \psi$

- The disjunction $p \vee q$ is support-inquisitive: there are two support-alternatives for $p \vee q$:

$$\text{ALT}[p \vee q]^+ = \{|p|, |q|\}$$

Conjunction

- $s \models^+ \varphi \wedge \psi$ iff $s \models^+ \varphi$ and $s \models^+ \psi$
  $s \models^- \varphi \wedge \psi$ iff $s \models^- \varphi$ or $s \models^- \psi$
  $s \models^\circ \varphi \wedge \psi$ iff $s \models^\circ \varphi$ or $s \models^\circ \psi$

- The suppositional content of φ and ψ is inherited by the conjunction $\varphi \wedge \psi$

- The conjunction $p \wedge q$ is reject-inquisitive: there are two rejection-alternatives for $p \wedge q$:

$$\textsc{alt}[p \wedge q]^- = \{|\neg p|, |\neg q|\}$$

Triggering and projection of suppositional content

- None of the clauses in the semantics we have met so far trigger suppositional content.
- Atomic sentences are not suppositional, and negation, disjunction and conjunction only project suppositional content of their subformulas in a cumulative way.
- For the language at hand, implication is the only trigger of suppositional content.
- Implication also projects the suppositional content of its consequent, but relativized to its antecedent.

Supposition triggered by implication

- The supposition that is triggered by an implication concerns the supposability of its antecedent.
- The supposability of a sentence is determined by:

  (a) the existence of support-alternatives for it.

  (b) the supposability of its support-alternatives.

- Suppositional dismissal of an implication occurs in *s*, when there is no support-alternative for its antecedent, or when there is some support-alternative that is not supposable in *s*.

Supporting an implication: InqB versus InqS

- The clause for implication in InqB is as follows:

  $s \models \varphi \rightarrow \psi$ iff $\forall t$: if $t \models \varphi$, then $t \cap s \models \psi$

- We can also formulate this in terms of the alternatives for φ:

  $s \models \varphi \rightarrow \psi$ iff $\forall u \in \textsc{alt}[\varphi]: u \cap s \models \psi$

- Since in InqB support is fully persistent, it makes no difference whether we consider just the support-alternatives for φ or all states that support it.
- In InqS, where support is only persistent modulo suppositional dismissal, it does potentially make a difference.
- We should only consider the support-alternatives for φ, because other states that support φ may contain additional information which causes suppositional dismissal of ψ.
- This should not be a reason for support of $\varphi \rightarrow \psi$ to fail.

Implication in InqS: the intuitive idea

- s supports $\varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and for every $u \in \textsc{alt}[\varphi]^+$:
  - (a) it is possible to suppose u in s, and
  - (b) $s \cap u$ supports ψ
- s rejects $\varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and for some $u \in \textsc{alt}[\varphi]^+$:
  - (a) it is possible to suppose u in s, and
  - (b) $s \cap u$ rejects ψ
- s dismisses $\varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$, or for some $u \in \textsc{alt}[\varphi]^+$:
  - (a) it is impossible to suppose u in s, or
  - (b) $s \cap u$ dismisses a supposition of ψ

Implication in InqS: supposability

When is it possible to suppose a support-alternative u for φ?

- Normally, to suppose a piece of information u in a state s is thought of as going from s to the more informed state $s \cap u$
- Thus, we could say that is possible to suppose u in s iff in going from s to $s \cap u$ our state remains consistent
- However, in the present setting, u is not just an arbitrary piece of information: it is a piece of information that supports φ
- This property should be maintained in going from u to $s \cap u$:

$$\forall t \text{ from } u \text{ to } u \cap s : \ t \models^{+} \varphi$$

In words: support should persist in restricting u to s

Persisting support and suppositional dismissal

- Recall our first general postulate:

  Support should be persistent modulo suppositional dismissal

- Given this postulate, the only reason why support of φ may fail to persist in restricting u to s is that somewhere along the way, suppositional dismissal occurs

Persisting support and consistency

- Our persisting support condition: $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$ entails the basic requirement that $s \cap u$ should be consistent.
- Just requiring consistency is not always sufficient.
- Example: $p \to q$ has a single support-alternative $u = |p \to q|$. Let $s = |\neg p|$, then $u \cap s \neq \emptyset$. But $p \to q$ is not supposable in s.

Persisting support versus support in $u \cap s$

- We require persisting support all the way from u to $u \cap s$:

$$\forall t \text{ from } u \text{ to } u \cap s: \; t \models^{+} \varphi$$

- Just requiring support at $u \cap s$ is not always sufficient.

- Example:

  - Let $\varphi = (p \rightarrow q) \vee r$
  - Then φ has two support-alternatives: $|p \rightarrow q|$ and $|r|$
  - Let $u = |p \rightarrow q|$ and let $s = |\neg p \wedge r|$
  - Then $u \cap s = s$, and $s \models^{+} (p \rightarrow q) \vee r$, because $s \models^{+} r$
  - However, $(p \rightarrow q) \vee r$ should not count as supposable in s

Implication in InqS fully spelled out

- $s \models^+ \varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+$:
  1. $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$, and
  2. $u \cap s \models^+ \psi$
- $s \models^- \varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^+$:
  1. $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$, and
  2. $u \cap s \models^- \psi$
- $s \models^\circ \varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^+$:
  1. $\exists t$ from u to $u \cap s$: $t \not\models^+ \varphi$, or
  2. $u \cap s \models^\circ \psi$

Non-suppositional reductions

Reduction: φ not suppositional

- $s \models^+ \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+ : u \cap s \models^+ \psi$
- $s \models^- \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^+ : u \cap s \models^- \psi$
- $s \models^\circ \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^+ : u \cap s \models^\circ \psi$

Reduction: φ and ψ not suppositional

- $s \models^+ \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+ : u \cap s \models^+ \psi$
- $s \models^- \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^+ : u \cap s \models^- \psi$
- $s \models^\circ \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^+ : u \cap s = \emptyset$

Non-suppositional reductions

Reduction: φ not suppositional

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{+} \psi$
- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{-} \psi$
- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{\circ} \psi$

Reduction: φ and ψ not suppositional

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{+} \psi$
- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{-} \psi$
- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+}: u \cap s = \emptyset$

Non-inquisitive reductions

- Now suppose that besides being non-suppositional, φ is not support-inquisitive either (though still supportable)
- In this case, $\textsc{alt}[\varphi]^+$ consists of a single alternative, call it α_φ
- The clauses for $\varphi \to \psi$ then simpy reduce to:

$$s \models^+ \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi \models^+ \psi$$

$$s \models^- \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi \models^- \psi$$

$$s \models^\circ \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi \models^\circ \psi$$

- If ψ is non-suppositional, dismissal further reduces to:

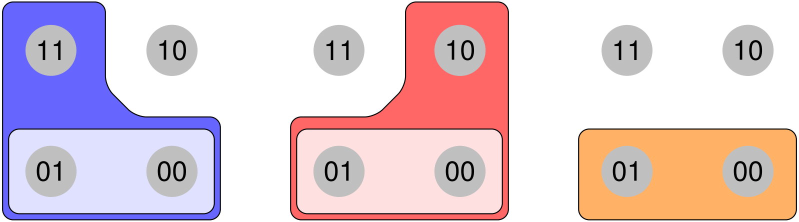
$$s \models^\circ \varphi \to \psi \quad \text{iff} \quad s \cap \alpha_\varphi = \emptyset$$

Our initial example: $p \to q$

$$s \models^{+} p \to q \quad \text{iff} \quad s \cap |p| \models^{+} q$$

$$s \models^{-} p \to q \quad \text{iff} \quad s \cap |p| \models^{-} q$$

$$s \models^{\circ} p \to q \quad \text{iff} \quad s \cap |p| = \emptyset$$



(a) support (b) reject (c) dismiss

How to read the pictures

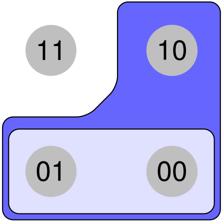
- Support is persistent modulo suppositional dismissal.
  - We depict maximal states that support φ, and if necessary also the maximal substates of these states that no longer support φ.
  - We think of these substates as support holes.
- Rejection is persistent modulo suppositional dismissal.
  - We depict maximal states that reject φ, and if necessary also the maximal substates of these states that no longer reject φ.
  - We think of these substates as rejection holes.
- Dismissal is fully persistent.
  - We depict only maximal states that dismiss a supposition of φ.
  - All substates thereof also dismiss a supposition of φ.

Our initial example: $p \to \neg q$

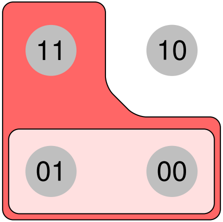
$$s \models^{+} p \to \neg q \quad \text{iff} \quad s \cap |p| \models^{+} \neg q$$

$$s \models^{-} p \to \neg q \quad \text{iff} \quad s \cap |p| \models^{-} \neg q$$

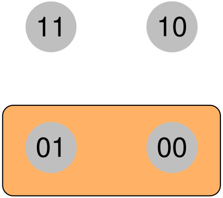
$$s \models^{\circ} p \to \neg q \quad \text{iff} \quad s \cap |p| = \emptyset$$



(a) support



(b) reject



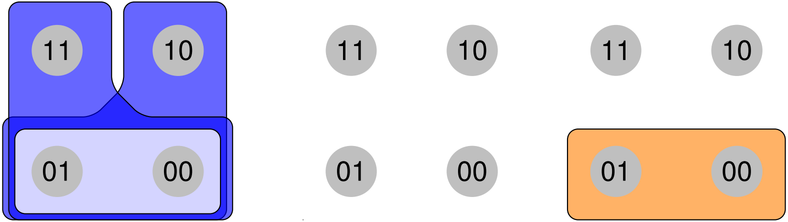
(c) dismiss

Our initial example: $p \to ?q$

$s \models^+ p \to ?q$ iff $s \cap |p| \models^+ q$ or $s \cap |p| \models^+ \neg q$

$s \models^- p \to ?q$ iff $s \cap |p| \models^- q$ and $s \cap |p| \models^- \neg q$ impossible

$s \models^\circ p \to ?q$ iff $s \cap |p| = \emptyset$



(a) support (b) reject (c) dismiss

Desired predictions

(1) a. If Alf goes to the party, will Bea go too? $p \to ?q$

b. If Alf goes, then Bea will go as well. $p \to q$

c. If Alf goes, then Bea will not go. $p \to \neg q$

d. Alf won't go. $\neg p$

- Both (1b) and (1c) support the conditional question in (1a):

$$p \to q \quad \models^{+} \quad p \to ?q$$

$$p \to \neg q \quad \models^{+} \quad p \to ?q$$

- (1b) and (1c) are contradictory, they reject each other:

$$p \to q \quad \models^{-} \quad p \to \neg q$$

$$p \to \neg q \quad \models^{-} \quad p \to q$$

Desired predictions

(1) a. If Alf goes to the party, will Bea go too? $p \rightarrow ?q$
b. If Alf goes, then Bea will go as well. $p \rightarrow q$
c. If Alf goes, then Bea will not go. $p \rightarrow \neg q$
d. Alf won't go. $\neg p$

- Finally, (1d) suppositionally dismisses (1a)-(1c) :

$$\neg p \models^{\otimes} p \rightarrow ?q$$

$$\neg p \models^{\otimes} p \rightarrow q$$

$$\neg p \models^{\otimes} p \rightarrow \neg q$$

- In particular:

$$\neg p \not\models^{+} p \rightarrow q$$

Three more complex examples

We will consider three more complex examples:

(1) Inquisitive antecedent: $(p \vee q) \rightarrow r$

(2) Suppositional consequent: $p \rightarrow (q \rightarrow r)$

(3) Suppositional antecedent: $(p \rightarrow q) \rightarrow r$

(1) Inquisitive antecedent: $(p \vee q) \to r$

- Both antecedent and consequent are non-suppositional
- There are two support-alternatives for the antecedent:

$$\textsc{alt}[p \vee q]^+ = \{|p|, |q|\}$$

- So we have:

$$s \models^+ (p \vee q) \to r \quad \text{iff} \quad \forall u \in \{|p|, |q|\}: u \cap s \models^+ r$$

$$s \models^- (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\}: u \cap s \models^- r$$

$$s \models^\circ (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\}: u \cap s = \emptyset$$

(1) Inquisitive antecedent: $(p \vee q) \rightarrow r$

$$s \models^{+} (p \vee q) \rightarrow r \quad \text{iff} \quad \forall u \in \{|p|, |q|\} : u \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s = \emptyset$$

- Some (non-)support examples:

$$(p \rightarrow r) \wedge (q \rightarrow r) \quad \models^{+} \quad (p \vee q) \rightarrow r$$

$$\neg p \wedge \neg q \quad \not\models^{+} \quad (p \vee q) \rightarrow r$$

(1) Inquisitive antecedent: $(p \vee q) \to r$

$$s \models^+ (p \vee q) \to r \quad \text{iff} \quad \forall u \in \{|p|, |q|\} : u \cap s \models^+ r$$

$$s \models^- (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s \models^- r$$

$$s \models^\circ (p \vee q) \to r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s = \emptyset$$

- Some rejection examples:

$$p \to \neg r \models^- (p \vee q) \to r$$

$$q \to \neg r \models^- (p \vee q) \to r$$

$$(p \to \neg r) \vee (q \to \neg r) \models^- (p \vee q) \to r$$

(1) Inquisitive antecedent: $(p \vee q) \rightarrow r$

$$s \models^{+} (p \vee q) \rightarrow r \quad \text{iff} \quad \forall u \in \{|p|, |q|\} : u \cap s \models^{+} r$$

$$s \models^{-} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s \models^{-} r$$

$$s \models^{\circ} (p \vee q) \rightarrow r \quad \text{iff} \quad \exists u \in \{|p|, |q|\} : u \cap s = \emptyset$$

- Some suppositional dismissal examples:

$$\neg p \models^{\otimes} (p \vee q) \rightarrow r$$

$$\neg q \models^{\otimes} (p \vee q) \rightarrow r$$

$$\neg p \vee \neg q \models^{\otimes} (p \vee q) \rightarrow r$$

(2) Suppositional consequent: $p \to (q \to r)$

- The antecedent is still non-suppositional, so the persistent support condition does not come into play

- Moreover, there is a single support-alternative for the antecedent:

$$\textsc{alt}[p]^{+} = \{|p|\}$$

- So we have:

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^{+} q \to r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^{-} q \to r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^{\circ} q \to r$$

(2) Suppositional consequent: $p \to (q \to r)$

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^{+} q \to r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^{-} q \to r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \models^{\circ} q \to r$$

- Since the consequent is a simple conditional, this can be further reduced to:

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

(2) Suppositional consequent: $p \to (q \to r)$

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some (non-)support examples:

$$(p \wedge q) \to r \quad \models^{+} \quad p \to (q \to r)$$

$$\neg p \quad \not\models^{+} \quad p \to (q \to r)$$

$$\neg q \quad \not\models^{+} \quad p \to (q \to r)$$

(2) Suppositional consequent: $p \to (q \to r)$

$$s \models^{+} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \to (q \to r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some rejection examples:

$$p \to \neg r \quad \models^{-} \quad p \to (q \to r)$$

$$q \to \neg r \quad \models^{-} \quad p \to (q \to r)$$

$$(p \wedge q) \to \neg r \quad \models^{-} \quad p \to (q \to r)$$

(2) Suppositional consequent: $p \rightarrow (q \rightarrow r)$

$$s \models^{+} p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{+} r$$

$$s \models^{-} p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \cap |q| \models^{-} r$$

$$s \models^{\circ} p \rightarrow (q \rightarrow r) \quad \text{iff} \quad s \cap |p| \cap |q| = \emptyset$$

- Some suppositional dismissal examples:

$$\neg p \models^{\otimes} p \rightarrow (q \rightarrow r)$$

$$\neg q \models^{\otimes} p \rightarrow (q \rightarrow r)$$

$$\neg p \vee \neg q \models^{\otimes} p \rightarrow (q \rightarrow r)$$

$$p \rightarrow \neg q \models^{\otimes} p \rightarrow (q \rightarrow r)$$

(3) Suppositional antecedent: $(p \to q) \to r$

- Now the antecedent is suppositional, so the persistent support condition finally comes into play
- There is a single support-alternative u for the antecedent:

$$u = |p \to q|$$

- So we have:

$$s \models^{+} (p \to q) \to r \;\text{ iff }\; \forall t \text{ from } u \text{ to } s \cap u\colon t \models^{+} p \to q \text{ and } s \cap u \models^{+} r$$

$$s \models^{-} (p \to q) \to r \;\text{ iff }\; \forall t \text{ from } u \text{ to } s \cap u\colon t \models^{+} p \to q \text{ and } s \cap u \models^{-} r$$

$$s \models^{\circ} (p \to q) \to r \;\text{ iff }\; \exists t \text{ from } u \text{ to } s \cap u\colon t \not\models^{+} p \to q \text{ or } s \cap u \models^{\circ} r$$

(3) Suppositional antecedent: $(p \rightarrow q) \rightarrow r$

$s \models^{+} (p \rightarrow q) \rightarrow r$ iff $\forall t$ from u to $s \cap u$: $t \models^{+} p \rightarrow q$
and $s \cap u \models^{+} r$

$s \models^{-} (p \rightarrow q) \rightarrow r$ iff $\forall t$ from u to $s \cap u$: $t \models^{+} p \rightarrow q$
and $s \cap u \models^{-} r$

$s \models^{\circ} (p \rightarrow q) \rightarrow r$ iff $\exists t$ from u to $s \cap u$: $t \not\models^{+} p \rightarrow q$
or $s \cap u \models^{\circ} r$

- Some (non-)support examples:

$$r \models^{+} (p \rightarrow q) \rightarrow r$$

$$\neg p \not\models^{+} (p \rightarrow q) \rightarrow r$$

$$p \wedge \neg q \not\models^{+} (p \rightarrow q) \rightarrow r$$

$$p \rightarrow \neg q \not\models^{+} (p \rightarrow q) \rightarrow r$$

(3) Suppositional antecedent: $(p \rightarrow q) \rightarrow r$

$s \models^+ (p \rightarrow q) \rightarrow r$ iff $\forall t$ from u to $s \cap u$: $t \models^+ p \rightarrow q$
and $s \cap u \models^+ r$

$s \models^- (p \rightarrow q) \rightarrow r$ iff $\forall t$ from u to $s \cap u$: $t \models^+ p \rightarrow q$
and $s \cap u \models^- r$

$s \models^\circ (p \rightarrow q) \rightarrow r$ iff $\exists t$ from u to $s \cap u$: $t \not\models^+ p \rightarrow q$
or $s \cap u \models^\circ r$

- Some rejection examples:

$$(p \rightarrow q) \rightarrow \neg r \quad \models^- \quad (p \rightarrow q) \rightarrow r$$

$$p \wedge (q \rightarrow \neg r) \quad \models^- \quad (p \rightarrow q) \rightarrow r$$

(3) Suppositional antecedent: $(p \to q) \to r$

$s \models^{+} (p \to q) \to r$ iff $\forall t$ from u to $s \cap u$: $t \models^{+} p \to q$
and $s \cap u \models^{+} r$

$s \models^{-} (p \to q) \to r$ iff $\forall t$ from u to $s \cap u$: $t \models^{+} p \to q$
and $s \cap u \models^{-} r$

$s \models^{\circ} (p \to q) \to r$ iff $\exists t$ from u to $s \cap u$: $t \not\models^{+} p \to q$
or $s \cap u \models^{\circ} r$

- Some suppositional dismissal examples:

$$\neg p \models^{\otimes} (p \to q) \to r$$

$$p \to \neg q \models^{\otimes} (p \to q) \to r$$

$$p \wedge \neg q \models^{\otimes} (p \to q) \to r$$

Conclusion

- The general perspective on meaning in inquisitive semantics is that sentences express proposals to update the CG in one or more ways
- There are several ways one may respond to such proposals, depending on one's information state
- InqB characterizes which states support a given proposal
- InqR also characterizes which states reject a given proposal
- InqS further distinguishes states that dismiss a supposition of a given proposal
- We thus arrive at a more and more fine-grained formal characterization of proposals, and thereby a more and more fine-grained characterization of meaning

Conclusion

- This in turn leads to a better account of the behavior of certain types of sentences in conversation
- InqS especially improves on InqB and InqR in its treatment of conditional statements and questions
- Paradigm example:

$$p \rightarrow q \quad \text{evaluated in the state} \quad |\neg p|$$

- InqB: support
- InqR: both support and reject
- InqS: suppositional dismissal

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