SEARCHING FOR SOLUTIONS

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> 26.09.2013 Tbil I C

SEARCHING FOR SOLUTIONS

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Puzzles

SEMANTICS FOR MODALS
ALL OR NOTHING

DR. PROCRASTINATE

ALTERNATIVES

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SEMANTICS FOR MODALS

STANDARD MODAL LOGIC

- ▶ Permission (\Diamond) is existential quantification (\exists).
- ▶ Obligation (\Box) is universal quantification (\forall).

Kratzer: contextual features for deontic modals

MODAL BASE A function f, such that f(w) represents the content of a body of laws in a world w.

ORDERING Worlds are ordered according to how close they are to the ideal world.

DEFINITION OF OBLIGATION

 $\Box p$ holds if the best worlds are all p worlds.

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KRATZER: CONTEXTUAL FEATURES FOR DEONTIC MODALS

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KRATZER CONDITIONALS

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CONDITIONAL

(1)If I agree with you, then we will both be wrong.

RESTRICTOR APPROACH

The antecedent of a conditional restricts the modal base against which the modal in the consequent is evaluated.

you may drive.

ALL OR NOTHING

(2)

- (3) $(p \land q) \rightarrow \Diamond r \models p \rightarrow \Diamond r$
 - 1. Restricted to $p \wedge q$, $\Diamond r$ holds. Requires a pgr world.

If the car passed its technical inspection and you

If the car passed its technical inspection, then

have your license, then you may drive.

- 2. Restricted to p, $\Diamond r$ holds. Requires a pr world.
- 3. A pgr world is a pr world.

SECOND PUZZLE: DR. PROCRASTINATE

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DR. PROCRASTINATI

WHAT SHOULD DR. PROCRASTINATE DO?

- (4) a. Dr. Procrastinate ought to accept and write the review.
 - b. Dr. Procrastinate ought not to accept.

REPRESENTATION

(5) a.
$$\Box(p \land q)$$
 b. $\Box \neg p$

(6)
$$\Box p$$



ALTERNATIVE-BASED SEMANTICS

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ALONI [2007]

- Disjunction adds alternatives into the semantics
- The alternatives are the denotations of the disjuncts

FREE CHOICE

- (7) a. A country may establish a research center or a laboratory.
 - b. $\Diamond(p \vee q)$

ANDERSON

EXAMPLES

- (8) a. Pawns must step at most 2 squares per move.
 - b. Pawns may step 2 squares on their first move.

ANDERSONIAN MODALS

(9) a.
$$\Box \varphi =_{df} \neg \varphi \rightarrow v$$

b.
$$\Diamond \varphi =_{df} \varphi \rightarrow \neg v$$



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SEVERAL VIOLATIONS NEEDED

SEVERAL VIOLATIONS (inconsistencies WITH LAW)

- (10) a. The jury finds the defendant in violation of article 1.
 - b. The jury finds the defendant **not** in **violation** of article 2.

EXAMPLE FROM DSU 344 OF THE WTO

- ► The [US] acted inconsistently with <u>Article 2.4.2 of the Anti-Dumping Agreement</u>
- ► The [US] did not act **inconsistently** with [...] <u>Articles</u> 2.1, 9.3 and 2.4 of the Anti-Dumping Agreement

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OLUTION:

ATOMS AND NEGATION

ATOMS

 $\sigma \models^+ p \text{ iff } \forall w \in \sigma \colon p \in w$ $\sigma \models^- p \text{ iff } \forall w \in \sigma \colon \overline{p} \in w$

NEGATION

$$\sigma \models^+ \neg \varphi \text{ iff } \sigma \models^- \varphi$$
 $\sigma \models^- \neg \varphi \text{ iff } \sigma \models^+ \varphi$

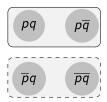


FIGURE 1: p

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OI LITIONS

DISJUNCTION

DEFINITION

$$\sigma \models^+ \varphi \lor \psi \text{ iff } \sigma \models^+ \varphi \text{ or } \sigma \models^+ \psi$$
$$\sigma \models^- \varphi \lor \psi \text{ iff } \sigma \models^- \varphi \text{ and } \sigma \models^- \psi$$

ILLUSTRATION

- (11) Sue sings or Mary dances.
 - a Yes, Sue sings.
 - b. Yes, Mary dances.
 - c. No, Sue won't sing and Mary won't dance.



FIGURE 2: $p \lor q$



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CONJUNCTION

CONJUNCTION

$$\sigma \models^+ \phi \land \psi \text{ iff } \sigma \models^+ \phi \text{ and } \sigma \models^+ \psi$$
$$\sigma \models^- \phi \land \psi \text{ iff } \sigma \models^- \phi \text{ or } \sigma \models^- \psi$$

ILLUSTRATION

- (12) Sue sings and Mary dances.
 - a. Yes, Sue sings and Mary dances.
 - b. No, Sue won't sing.
 - c. No, Mary won't dance.

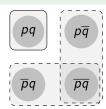


FIGURE 3: $p \land q$



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MAXIMAL SUPPORTING STATES

$$\mathsf{ALT}[\phi]^+ := \{\sigma \in [\phi]^+ | \neg \exists \tau \in [\phi]^+ : \sigma \subset \tau\}$$



FIGURE 4: $p \lor q$

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OLUTION

IMPLICATION

IMPLICATION

$$\sigma \models^+ \varphi \to \psi \quad \text{iff} \quad \forall \tau \in \mathsf{ALT}[\varphi]^+ : \tau \cap \sigma \models^+ \psi \\
\sigma \models^- \varphi \to \psi \quad \text{iff} \quad \exists \tau \in \mathsf{ALT}[\varphi]^+ : \tau \cap \sigma \models^- \psi$$

ILLUSTRATION

- (13) If Sue sings, then Mary will dance.
 - a. Yes, if Sue sings, then Mary will dance.
 - b. No, if Sue sings, then Mary won't dance.

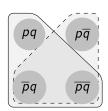


FIGURE 5: $p \rightarrow q$

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SOLUTION:

ALTERNATIVES AND NEGATING IMPLICATION

$$(p \lor q) \to r$$

- (14) If Sue sings or Mary dances, Pete will play the piano.
 - a. No, if Sue sings, Pete will not play.
 - b. No, if Mary dances, Pete will not play.

IMPLICATION

$$\sigma \models^- \varphi \rightarrow \psi$$
 iff $\exists \tau \in \mathsf{ALT}[\varphi]^+ : \tau \cap \sigma \models^- \psi$

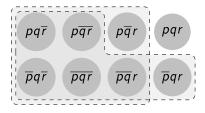


FIGURE 6: $[(p \lor q) \xrightarrow{} r]_{\Box}$

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$$\sigma \models^{+} \circ \varphi \quad \text{iff} \quad \forall \tau \in \mathsf{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{-} v \\
\sigma \models^{-} \circ \varphi \quad \text{iff} \quad \forall \tau \in \mathsf{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{+} v$$

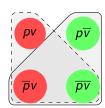


FIGURE 7: *p*

THREE-FOLD DEONTICS

- Permitted: only v worlds.
- Prohibited: only v worlds.
- Neutral: both v and \overline{v} worlds.

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SOLUTION:

DEONTIC MODALS DIFFER FROM IMPLICATION

 $\neg \Diamond (p \lor q)$

(15) A country may not establish a research center or a laboratory.

DEFINITION

 $\sigma \models^- \otimes \varphi$ iff $\forall \tau \in \mathsf{ALT}[\varphi]^+ : \tau \cap \sigma \models^+ v$

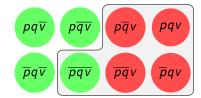


FIGURE 8: $[\neg \Diamond (p \lor q)]^+$

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ENTAILMENT

SUPPORT-ENTAILMENT:

$$\varphi \models_+ \psi$$
 iff $[\varphi]^+ \subseteq [\psi]^+$

$$[\varphi]^+ \subseteq [\psi]^-$$

REJECTION-ENTAILMENT:

$$oldsymbol{arphi}\models_-\psi$$
 iff $[\psi]^-\subseteq [oldsymbol{arphi}]^-$

ENTAILMENT:

$$\varphi \models \psi$$
 if

$$arphi \models \psi$$
 iff $arphi \models_+ \psi$ and $arphi \models_- \psi$

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FIRST PUZZLE: ALL OR NOTHING

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ALL OR NOTHING

- (16) a. If the car passed its technical inspection and you have your license, then you may drive.
 - b. If the car passed its technical inspection, then you may drive.

REPRESENTATION

$$(17) \qquad (p \land q) \rightarrow \Diamond r \not\models p \rightarrow \Diamond r$$

IMPLICATION AND PERMISSION

$$\begin{array}{ll} \sigma \models^+ \phi \rightarrow \psi & \text{iff} & \forall \tau \in \mathsf{ALT}[\phi]^+ : \tau \cap \sigma \models^+ \psi \\ \sigma \models^+ \otimes \phi & \text{iff} & \forall \tau \in \mathsf{ALT}[\phi]^+ : \tau \cap \sigma \models^- \nu \end{array}$$

CONSIDER THE STATE:

$$\sigma = \{ \{pqr\overline{v}\}, \{p\overline{q}r\overline{v}\}, \{p\overline{q}rv\} \}$$

φ SUPPORT-ENTAILS ψ IFF $[\varphi]^+ \subseteq [\psi]^+$

$$\sigma$$
 supports $(p \land q) \rightarrow \Diamond r$ $\{pqr\overline{v}\}$

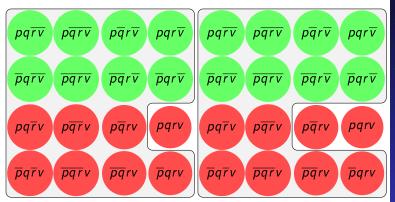
$$\sigma$$
 does not support $ho o \lozenge r$

$\{\{pqr\overline{v}\},\{p\overline{q}r\overline{v}\},\{p\overline{q}rv\}\}$

ALL OR NOTHING

(18)

- a. If the car passed its technical inspection and you have your driver's license, then you may drive. $(p \land q) \rightarrow \Diamond r$
- b. If the car passed its technical inspection, then you may drive. $p \rightarrow \Diamond r$



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FIGURE 10: $[\rho \rightarrow \Diamond r]^+$

ALL OR NOTHING

EXAMPLE

- (19) a. If the car passed its technical inspection and you have your driver's license, then you may drive. $(p \land q) \rightarrow \Diamond r$
 - b. If the car passed its technical inspection, then you may drive. $p \rightarrow \Diamond r$

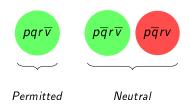


FIGURE 11: State σ

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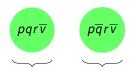
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EXAMPLE

- (20) a. If the car passed its technical inspection and you have your driver's license, then you may drive. $(p \land q) \rightarrow \Diamond r$
 - b. If the car passed its technical inspection, then you may drive. $p \rightarrow \Diamond r$



Permitted Permitted

FIGURE 12: State $\sigma \cap |p \rightarrow \Diamond r|$

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SECOND PUZZLE: DR. PROCRASTINATE

a. Dr. Procrastinate ought to accept and write the

b. Dr. Procrastinate ought not to accept.

WHAT SHOULD DR. PROCRASTINATE DO?

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REPRESENTATION

review

- $ightharpoonup \Box (p \land q)$
- **▶** □¬*p*

(21)

4 D > 4 P > 4 E > 4 E > 9 Q P

DESIDERATA: NON-MONOTONICITY

 $\Box(p \land q) \not\models \Box p$ (22)

OBLIGATION: $\Box \varphi \equiv \neg \Diamond \neg \varphi$

 $\sigma \models^+ \Box \varphi$ iff $\forall \tau \in ALT[\varphi]^- : \tau \cap \sigma \models^+ v$ $\sigma \models^- \Box \varphi$ iff $\forall \tau \in \mathsf{ALT}[\varphi]^- : \tau \cap \sigma \models^- \mathsf{v}$

REJECTION-ENTAILMENT: $\varphi \models_{-} \psi$ IFF $[\psi]^{-} \subseteq [\varphi]^{-}$

- \blacktriangleright {{ $p\overline{q}v$ },{ \overline{pqv} }}: Not writing $(\neg q)$ leads to a violation.
- $\triangleright \Box p$ is rejected.
- $ightharpoonup \Box(p \land q) \equiv \neg \Diamond(\neg p \lor \neg q)$
- $ightharpoonup \Box (p \land q)$ is not rejected.

As $[\Box p]^- \not\subseteq [\Box (p \land q)^-]$, then $\Box (p \land q) \not\models \Box p$

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DR. PROCRASTINATE

WHAT SHOULD DR. PROCRASTINATE DO?

- (23) a. Dr. Procrastinate ought to accept and write the review.
 - b. Dr. Procrastinate ought not to accept.
 - c. Dr. Procrastinate will not write the review.

REPRESENTATION

- $ightharpoonup v_1(p \wedge q)$
- $ightharpoonup |v_2| \neg p$
- ▶ ¬q

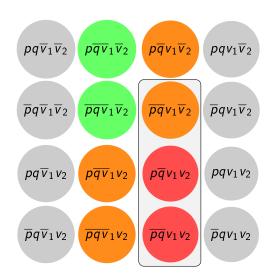


FIGURE 13: $[([v1](p \land q)) \land ([v2] \neg p) \land (\neg q)]^+$

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OLUTION



FIGURE 14: $[(v_1(p \land q)) \land (v_2 \neg p) \land (\neg q)]^+$

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FIGURE 15: $[(v_1(\rho \land q)) \land (v_2 \neg \rho) \land (\neg q) \land \rho]^+$

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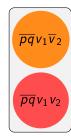


FIGURE 16: $[([v_1](p \land q)) \land ([v_2] \neg p) \land (\neg q) \land (\neg p)]^+$

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Thank you for listening Feedback: maher@uos.de

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