Searching for Solutions

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TbiLLC

Semantics for modals

Standard Modal Logic

- Permission ($\Diamond$) is existential quantification ($\exists$).
- Obligation ($\Box$) is universal quantification ($\forall$).

Kratzer: contextual features for deontic modals

Modal base A function f, such that $f(w)$ represents the content of a body of laws in a world w.

Ordering Worlds are ordered according to how close they are to the ideal world.

Definition of obligation

$\Box p$ holds if the best worlds are all p worlds.

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Kratzer Conditionals

Conditional

(1) If I agree with you, then we will both be wrong.

Restrictor approach

The antecedent of a conditional restricts the modal base against which the modal in the consequent is evaluated.

First puzzle: All or nothing

All or nothing

(2) a. If the car passed its technical inspection and you have your license, then you may drive.
b. If the car passed its technical inspection, then you may drive.

Representation

(3) $(p \wedge q) \rightarrow \Diamond r \models p \rightarrow \Diamond r$

1. Restricted to $p \wedge q$, $\Diamond r$ holds. Requires a pqr world.
2. Restricted to p, $\Diamond r$ holds. Requires a pr world.
3. A pqr world is a pr world.

Second puzzle: Dr. Procrastinate

What should Dr. Procrastinate do?

(4) a. Dr. Procrastinate ought to accept and write the review.
b. Dr. Procrastinate ought not to accept.

Representation

(5) a. $\Box(p \wedge q)$
b. $\Box\neg p$

(6) $\Box p$

Alternative-based semantics

Aloni [2007]

- Disjunction adds alternatives into the semantics
- The alternatives are the denotations of the disjuncts
- $\Diamond\varphi =_{df} \forall\alpha\Diamond\varphi$
- $\Box\varphi =_{df} \exists\alpha\Box\varphi$

Free choice

(7) a. A country may establish a research center or a laboratory.

b. $\Diamond(p \vee q)$

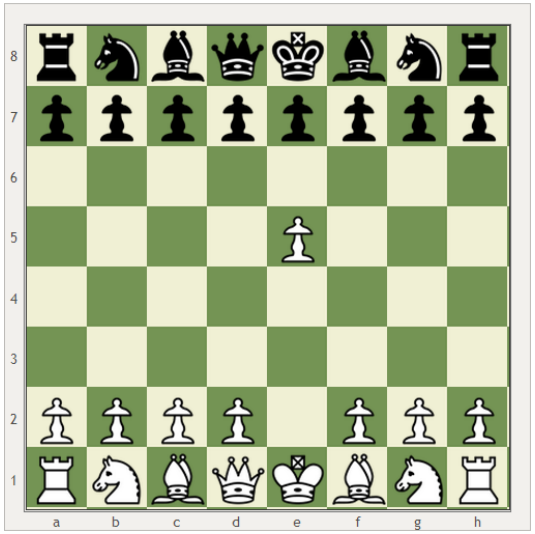
Anderson

Examples

(8) a. Pawns must step at most 2 squares per move.
b. Pawns may step 2 squares on their first move.

Andersonian modals

(9) a. $\Box\varphi =_{df} \neg\varphi \rightarrow v$
b. $\Diamond\varphi =_{df} \varphi \rightarrow \neg v$



Several violations needed

Several violations (*inconsistencies* with law)

(10) a. The jury finds the defendant **in violation** of article 1.

b. The jury finds the defendant **not in violation** of article 2.

Example from DSU 344 of the WTO

- The [US] acted **inconsistently** with Article 2.4.2 of the Anti-Dumping Agreement
- The [US] did not act **inconsistently** with [...] Articles 2.1, 9.3 and 2.4 of the Anti-Dumping Agreement

Atoms and negation

Atoms

$\sigma \models^+ p$ iff $\forall w \in \sigma : p \in w$
$\sigma \models^- p$ iff $\forall w \in \sigma : \overline{p} \in w$

Negation

$\sigma \models^+ \neg\varphi$ iff $\sigma \models^- \varphi$
$\sigma \models^- \neg\varphi$ iff $\sigma \models^+ \varphi$

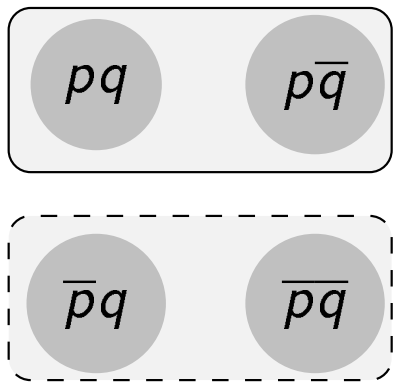


Figure 1: p

Disjunction

Definition

$\sigma \models^+ \varphi \vee \psi$ iff $\sigma \models^+ \varphi$ or $\sigma \models^+ \psi$
$\sigma \models^- \varphi \vee \psi$ iff $\sigma \models^- \varphi$ and $\sigma \models^- \psi$

Illustration

(11) Sue sings or Mary dances.

a. Yes, Sue sings.
b. Yes, Mary dances.
c. No, Sue won't sing and Mary won't dance.

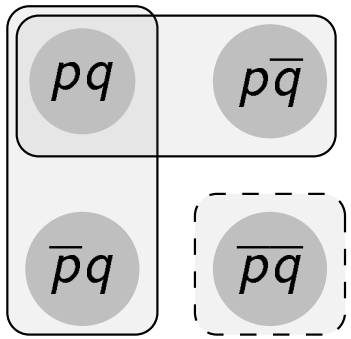


Figure 2: $p \vee q$

Conjunction

Conjunction

$\sigma \models^+ \varphi \wedge \psi$ iff $\sigma \models^+ \varphi$ and $\sigma \models^+ \psi$
$\sigma \models^- \varphi \wedge \psi$ iff $\sigma \models^- \varphi$ or $\sigma \models^- \psi$

Illustration

(12) Sue sings and Mary dances.

a. Yes, Sue sings and Mary dances.
b. No, Sue won't sing.
c. No, Mary won't dance.

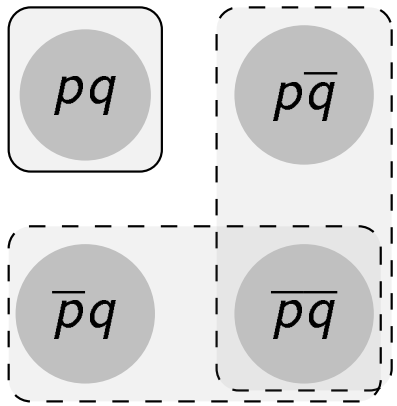


Figure 3: $p \wedge q$

Alternatives

Maximal supporting states

$$\mathsf{ALT}[\varphi]^+ := \{\sigma \in [\varphi]^+ | \neg\exists\tau \in [\varphi]^+ : \sigma \subset \tau\}$$

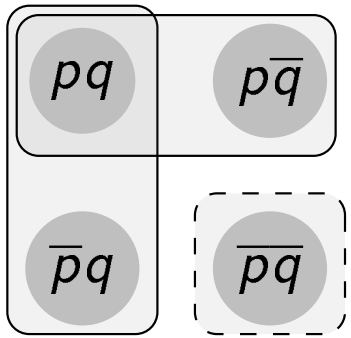


Figure 4: $p \vee q$

Implication

Implication

$\sigma \models^{+} \varphi \rightarrow \psi$ iff $\forall \tau \in \mathrm{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{+} \psi$

$\sigma \models^{-} \varphi \rightarrow \psi$ iff $\exists \tau \in \mathrm{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{-} \psi$

Illustration

(13) If Sue sings, then Mary will dance.

a. Yes, if Sue sings, then Mary will dance.

b. No, if Sue sings, then Mary won't dance.

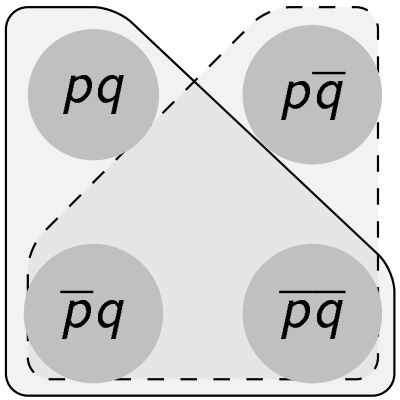


Figure 5: $p \rightarrow q$

Alternatives and negating implication

$(p \vee q) \rightarrow r$

(14) If Sue sings or Mary dances, Pete will play the piano.

a. No, if Sue sings, Pete will not play.

b. No, if Mary dances, Pete will not play.

Implication

$\sigma \models^{-} \varphi \rightarrow \psi$ iff $\exists \tau \in \mathrm{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{-} \psi$

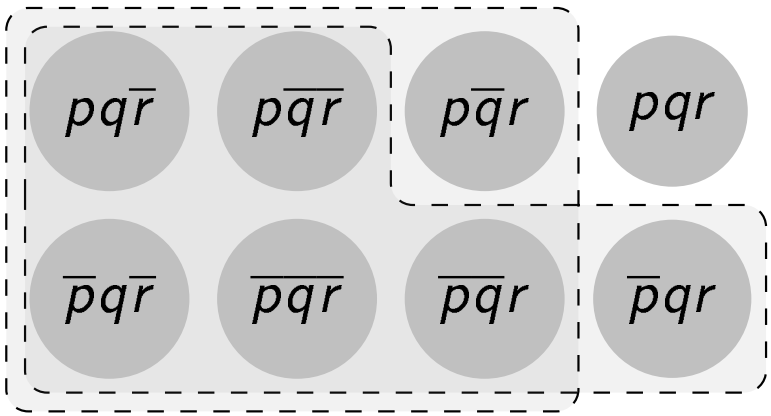


Figure 6: $[(p \vee q) \rightarrow r]^{-}$

Deontics

Deontic *may*

$$\sigma \models^{+} \Diamond_{v} \varphi \quad \text{iff} \quad \forall \tau \in \mathrm{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{-} v$$
$$\sigma \models^{-} \Diamond_{v} \varphi \quad \text{iff} \quad \forall \tau \in \mathrm{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{+} v$$

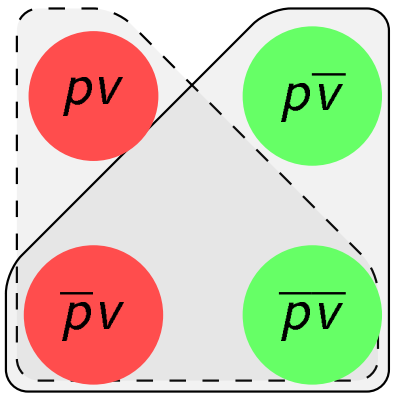


Figure 7: $\Diamond p$

Three-fold deontics

- Permitted: only $\overline{v}$ worlds.
- Prohibited: only v worlds.
- Neutral: both v and $\overline{v}$ worlds.

Deontic modals differ from implication

$\neg\Diamond(p \vee q)$

(15) A country may not establish a research center or a laboratory.

Definition

$\sigma \models^{-} \Diamond\!\!\!\!\!\!\bigcirc_{v}\, \varphi \quad \text{iff} \quad \forall \tau \in \mathrm{ALT}[\varphi]^{+} : \tau \cap \sigma \models^{+} v$

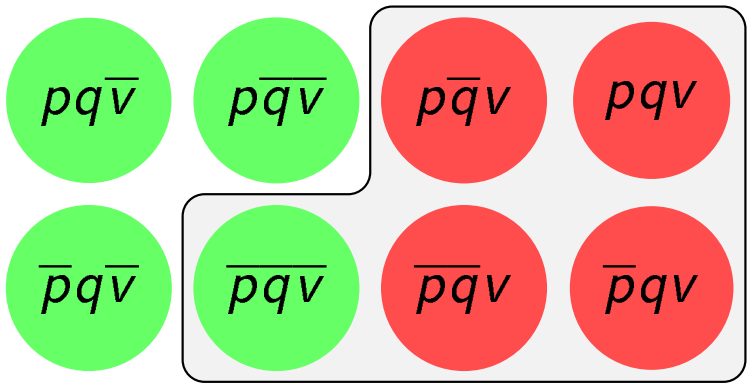


Figure 8: $[\neg\Diamond(p \vee q)]^{+}$

Entailment

Entailment

Support-entailment: $\varphi \models_+ \psi$ iff $[\varphi]^+ \subseteq [\psi]^+$

Rejection-entailment: $\varphi \models_- \psi$ iff $[\psi]^- \subseteq [\varphi]^-$

Entailment: $\varphi \models \psi$ iff $\varphi \models_+ \psi$ and $\varphi \models_- \psi$

First puzzle: All or nothing

All or nothing

(16) a. If the car passed its technical inspection and you have your license, then you may drive.
b. If the car passed its technical inspection, then you may drive.

Representation

(17) $(p \wedge q) \rightarrow \Diamond r \not\models p \rightarrow \Diamond r$

All or Nothing

Implication and Permission

$\sigma \models^+ \varphi \to \psi$ iff $\forall \tau \in \mathsf{ALT}[\varphi]^+ : \tau \cap \sigma \models^+ \psi$

$\sigma \models^+ \langle v \rangle \varphi$ iff $\forall \tau \in \mathsf{ALT}[\varphi]^+ : \tau \cap \sigma \models^- v$

Consider the state:

$\sigma = \{\{pqr\bar{v}\}, \{p\bar{q}r\bar{v}\}, \{p\bar{q}rv\}\}$

φ support-entails ψ iff $[\varphi]^+ \subseteq [\psi]^+$

σ supports $(p \wedge q) \to \Diamond r$

$\{pqr\bar{v}\}$

σ does not support $p \to \Diamond r$

$\{\{pqr\bar{v}\}, \{p\bar{q}r\bar{v}\}, \{p\bar{q}rv\}\}$

All or nothing

(18) a. If the car passed its technical inspection and you have your driver's license, then you may drive. $(p \wedge q) \rightarrow \Diamond r$

b. If the car passed its technical inspection, then you may drive. $p \rightarrow \Diamond r$

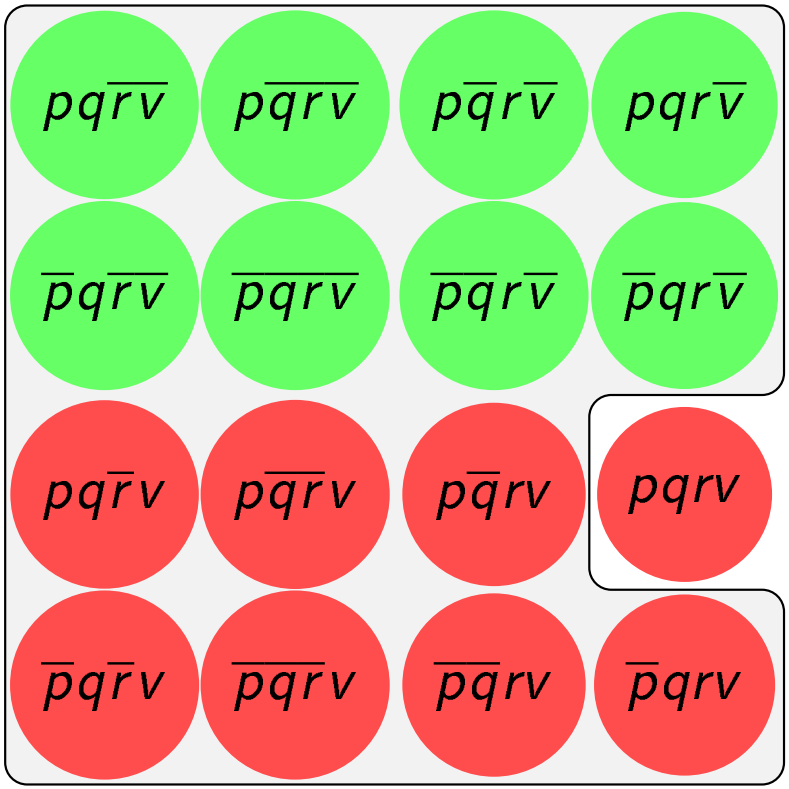


Figure 9: $[(p \wedge q) \rightarrow \Diamond r]^+$

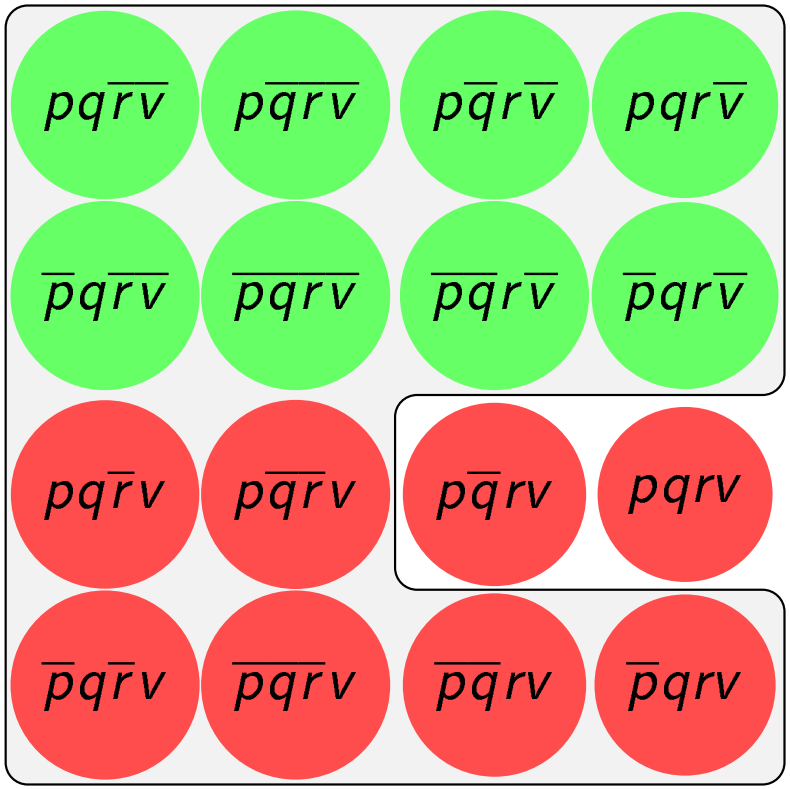


Figure 10: $[p \rightarrow \Diamond r]^+$

All or nothing

Example

(19) a. If the car passed its technical inspection and you have your driver's license, then you may drive. $(p \wedge q) \rightarrow \Diamond r$

b. If the car passed its technical inspection, then you may drive. $p \rightarrow \Diamond r$

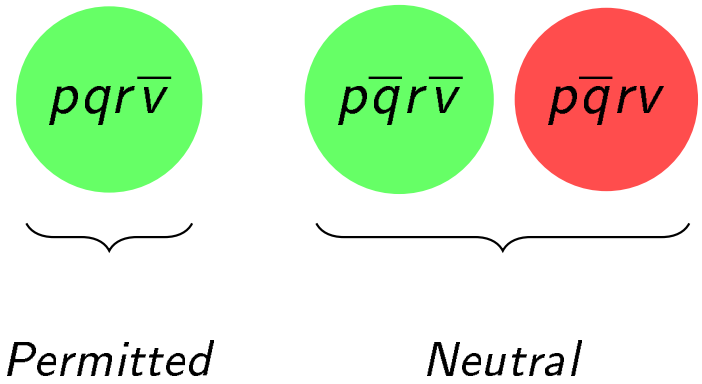


Figure 11: State σ

All or nothing

Example

(20)
a. If the car passed its technical inspection and you have your driver's license, then you may drive. $(p \wedge q) \rightarrow \Diamond r$

b. If the car passed its technical inspection, then you may drive. $p \rightarrow \Diamond r$

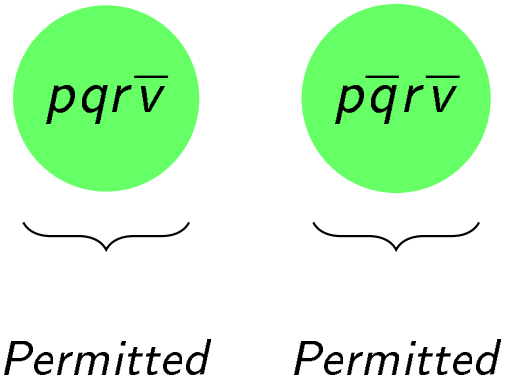


Figure 12: State $\sigma \cap |p \rightarrow \Diamond r|$

Second Puzzle: Dr. Procrastinate

What should Dr. Procrastinate do?

(21) a. Dr. Procrastinate ought to accept and write the review.
b. Dr. Procrastinate ought not to accept.

Representation

- $\Box(p \wedge q)$
- $\Box\neg p$

Dr. Procrastinate

Desiderata: non-monotonicity

(22) $\Box(p \wedge q) \not\models \Box p$

Obligation: $\Box\varphi \equiv \neg\Diamond\neg\varphi$

$\sigma \models^+ \Box\varphi$ iff $\forall\tau \in \mathrm{ALT}[\varphi]^- : \tau \cap \sigma \models^+ v$
$\sigma \models^- \Box\varphi$ iff $\forall\tau \in \mathrm{ALT}[\varphi]^- : \tau \cap \sigma \models^- v$

Rejection-entailment: $\varphi \models_- \psi$ iff $[\psi]^- \subseteq [\varphi]^-$

- $\{\{p\overline{q}v\}, \{\overline{pqv}\}\}$: Not writing $(\neg q)$ leads to a violation.
- $\Box p$ is rejected.
- $\Box(p \wedge q) \equiv \neg\Diamond(\neg p \vee \neg q)$
- $\Box(p \wedge q)$ is not rejected.

As $[\Box p]^- \not\subseteq [\Box(p \wedge q)^-]$, then $\Box(p \wedge q) \not\models \Box p$

Dr. Procrastinate

What should Dr. Procrastinate do?

(23)
a. Dr. Procrastinate ought to accept and write the review.
b. Dr. Procrastinate ought not to accept.
c. Dr. Procrastinate will not write the review.

Representation

- $\boxed{v_1}(p \wedge q)$
- $\boxed{v_2} \neg p$
- $\neg q$

Dr. Procrastinate

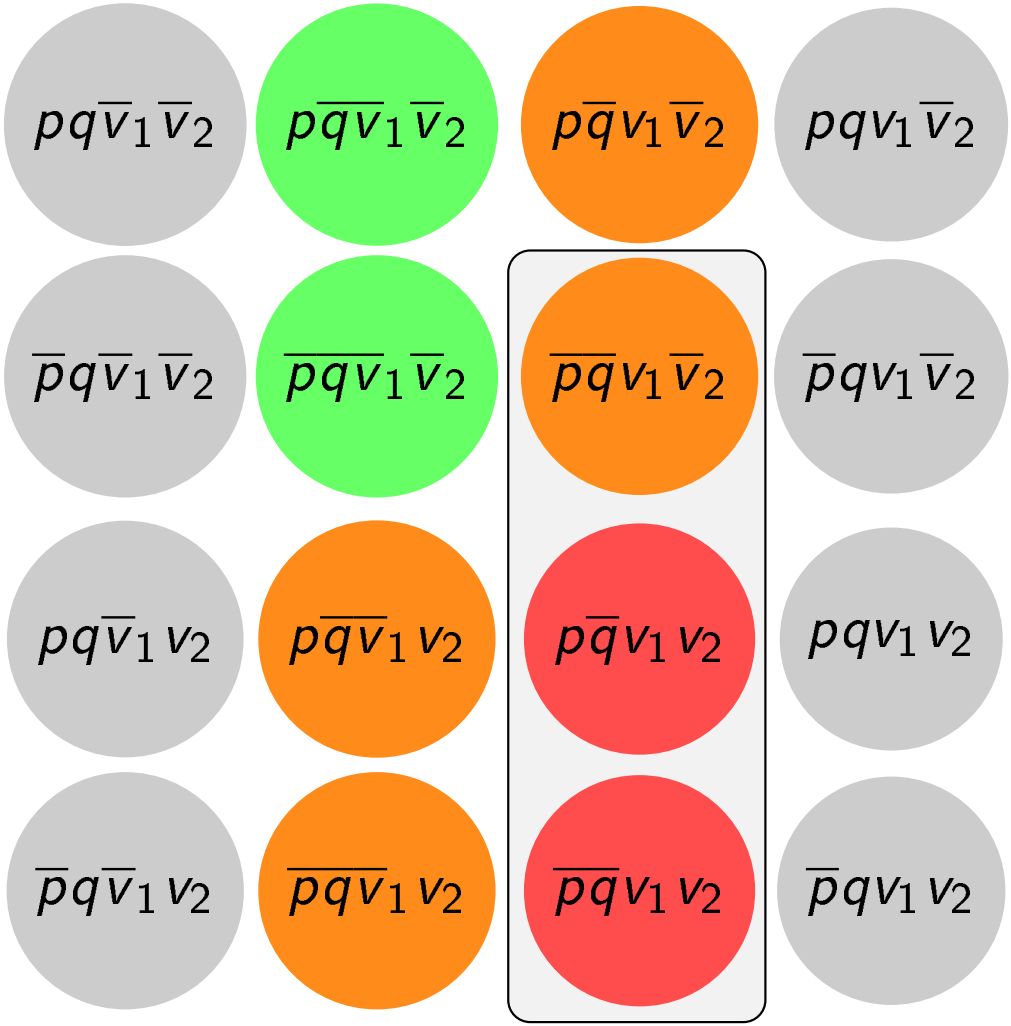


Figure 13: $[(\boxed{v1}(p \wedge q)) \wedge (\boxed{v2}\neg p) \wedge (\neg q)]^{+}$

Dr. Procrastinate

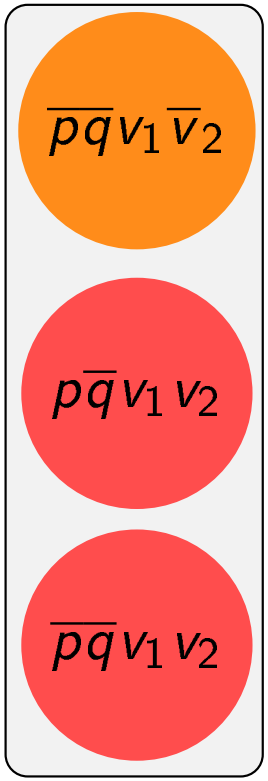


Figure 14: $[(\boxed{v1}(p \wedge q)) \wedge (\boxed{v2}\neg p) \wedge (\neg q)]^{+}$

Dr. Procrastinate

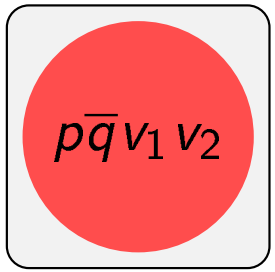


Figure 15: $[(\boxed{v1}(p \wedge q)) \wedge (\boxed{v2}\neg p) \wedge (\neg q) \wedge p]^{+}$

Dr. Procrastinate

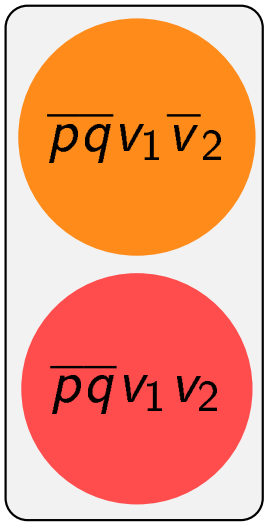


Figure 16: $[(\boxed{v1}(p \wedge q)) \wedge (\boxed{v2}\neg p) \wedge (\neg q) \wedge (\neg p)]^{+}$

The end

Thank you for listening
Feedback: maher@uos.de