ŁUKASIEWICZ INFINITE-VALUED PROPOSITIONAL LOGIC: FROM THE PHILOSOPHY OF VAGUENESS TO THE GEOMETRY OF RATIONAL POLYHEDRA.

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It is the eve of the 2006 Football World Cup Final, when Italy is going to play France. Consider the sentence X= "Italy will score in the match against France". You do not know for sure whether X will turn out to be true or false after the match. That is why bookmakers take bets on (the event described by) the sentence X: because the information conveyed by X is *uncertain*. In the last few centuries, science has developed the theory of probabilities to deal with uncertainty. And although this is not always stressed in standard accounts, it is a fact that probability depends on logic. Consider the (compound) sentence $X \vee \neg X$= "Either Italy will score or Italy will not score in the match against France". The information conveyed by this new sentence is no longer uncertain: whatever happens on the day of the match, $X \vee \neg X$ will turn out to be true; betting on such *tautologies*, as these sentences are called in logic after Wittgenstein, is all but exciting. So why is X uncertain, whereas $X \vee \neg X$, regardless of the factual content of X, is not? The point is that *the logic of the original sentence X is classical* — and so, in particular, X satisfies the *tertium non datur law* that $X \vee \neg X$ is always true.

Consider next a slight variant of X, namely, the sentence Y= "Italy will score early in the match against France". The information conveyed by Y suffers from *two* distinct types of imperfection: (a) you do not know for sure which instants of time in the course of the match should count as "early"; and (b) even if you knew that, you would still not know whether Y will turn out to be true or false after the match. In other words, not only is the information conveyed by Y uncertain because of (b), as before; it is also *vague*, because of (a). Now it is no longer clear that there is such a thing as *the logic of Y*: may be it just does not make sense to reason about vague sentences. For example, would you say that $Y \vee \neg Y$ ought to be a tautology, or not?

The first part of this tutorial is an introduction to Łukasiewicz infinite-valued propositional logic as the logic of precise inferences among vague propositions. I discuss selected topics from the philosophy of vagueness, and show how they can be mathematized through Łukasiewicz logic, even though this was not the original purpose of Łukasiewicz himself.

The second part of the tutorial is a crash course on Chang's algebraization of Łukasiewicz logic using MV-algebras. I sketch the basic theory needed to understand the algebraic proof of the completeness theorem.

The third and last part of the tutorial offers some glimpses of more advanced topics in the study of Łukasiewicz logic and MV-algebras, including the connection with the piecewise linear geometry of rational polyhedra.

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