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Table of Contents

The 2001 Borjomi Conference Proceedings

Dorit Ben Shalom.
Completeness and Decidability of the Logic of Relativized End-extension Set Algebras.

Henk Zeevat and Gerhard Jäger.
A Reinterpretation of Syntactic Alignment.

Introduction

This volume, jointly published by the CLLS at the University of Tblisi and the ILLC at the University of Amsterdam, collects the proceedings of the 3rd Amsterdam-Tblisi International Symposium on Logic, Language and Computation held in September 1999 in Batumi, Georgia and the 4th Amsterdam-Tblisi International Symposium on Logic, Language and Computation held in Borjomi, Georgia in September 2001.

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The volume collects papers in the areas of empirical linguistics, general linguistics, computational linguistics, linguistic semantics and pragmatics, logic and language, logic, logic and computation and computation.

Linguistics

Several of the presented papers contribute to the theoretical description of Georgian grammar - Asatiani on uniformities in reflexive and middle constructions, Sackokia about analytical perfects, Kikvidze and Tchantouria on honorifics and Ivanishvili and Soselia about the passive construction. Chikoidze, Dokvadze and Godabrelidze address the semantics of the verbal prefixes *v(o)-* and *vy-* in Russian. More abstract linguistic contributions are those of Shengelaia on the semantics of 'empty words', Mel'čuk about 'empty morphemes' and Zeevat and Jaeger on a statistical explanation of harmonic alignment.

Computational Linguistics

In the category of computational linguistic studies, Rezaei proposes an implementation of syntax and semantics by communicating processes, Nilsenová discusses pragmatic processing in Bayesian networks and Margvelani and Samsonadze focus on the morphological processing in an architecture for a Georgian text-to-speech processor. Margvelani's report concerns morphological processing for spell-checking and Alberti, Balogh and Kleiber investigate a possible implementation of a radical lexicalist grmmar framework.

Computation

The contribution of Nedjah and Mourelle concerns the design of a functional programming language.

Logic and Language

For the language and logic interface, there are contributions by Jaeger on a relational semantics for multimodal categorial logics, Aloni and Van Rooy concerning domain restrictions by topics in dynamic logic, Lecomte and Retoré, who reconstruct Chomsky's minimalism in a minimal logic and two contributions by Latrouite, Naumann and Osswald applying dynamic arrow logic to aspect systems and the description of Tagalog.

Logic

Fundamental studies in logic are the contributions of Litak on intermediate logics, Seuren with discussion of existential import, and Ben-Shalom who investigates a new semantics for predicate logic. Two studies on the relation of logic and games are Van Benthem and Japaridze. Hodkinson contributes results in relation algebras and Gregory develops relevance semantics as an approach to intensionality.

In the hope that the volume reflects the joyful spirit of both the conferences,

Dick de Jongh
Marie Nilsenová
Henk Zeevat
(volume editors)

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Markedness and the Dominant Category

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Abstract

Based on a careful analysis of Georgian nominal and verbal markers, we propose a hierarchy governing their assignment. We introduce the 'principle of communicative act' to account for the dichotomy in I/II and III series. We argue that languages cannot simply be characterized as 'mixed ergative systems' but it is necessary to state of which hierarchical relations they make use.

1 Introduction

The concept of markedness has played a significant role in the linguistic theory. It developed from the work of Trubetzkoy through [11], [8], [7], [5] and others and nowadays is mostly used as a tool for cognitive explanations of morpho-syntactic markedness patterns. According to [7], grammar is the set of instructions on processing of discourse and structurally marked items are usually cognitively marked. The marked and unmarked values can be conceived of as paradigmatic alternatives, and the concept of markedness may consequently be regarded as the main concept in the strategy of mapping a conceptual representation into a linguistic representation. Linguistic and extralinguistic factors determine the markedness category differently in different languages, based on criteria such as the following:

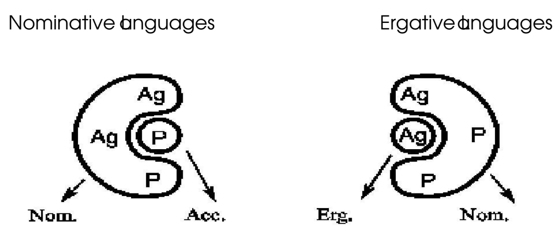
- socio-cultural - e.g., some languages with the grammatical category of gender distinguish feminine gender as the marked category, while masculine gender is frequently the unmarked one;
- cultural traditions - e.g., markedness of polite forms in Japanese;
- religious - e.g., in Batsbi, the word God never appears with ergative case marker;
- universal semantic oppositions - e.g., the formal opposition of singular (unmarked) vs. plural (marked) in numerous languages.[1]

[1]Note that the fact that the conceptually marked and unmarked categories are both for-

2 Markedness and cases

The markedness feature also determines case assignment: the unmarked noun is qualified as the functionally main one and represented by the unmarked, direct case (absolutive, nominative).[2] How can we explain the morphological equality and integrity of S-P (Subject-Patient) in the ergative languages?[3]

Figure 1.: Case distribution in nominative and ergative languages.



We can conceive the existence of a particular semantic category, which unites the S-P in one morphological class: the semantic orientation on patient for transitive verbs and on subjects for intransitive verbs. In intransitive semantics, the concrete lexical meaning of the verb is defined according to the semantics of the subject: e.g., a person *walks*, a bird *flies*, a snake *slithers*, a boat *sails*, etc. On the other hand, in case of general transitive semantics, the verb meaning is defined by the semantics of the object: *a letter → to write*, *a picture → to draw, a house → to build*, etc. Such semantic orientation of the verb may be called Semantic Agreement (SA). It can be further demonstrated by Georgian verb couples that differ only by animacy of the nouns: for transitive semantics, the decisive role is the Patient (more exactly its animacy). e. g. *caiqvana* 'took

mally marked (e.g., in Latvian, both sing. and pl. have non-zero markers, while in Mandarin Chinese sing,. and pl. have zero markers), or formally unmarked does not mean that the cognitively marked category is not universally marked (see [6] for a discussion about the markedness of the value plural). The only counterevidence would be posed by a language that has the opposite configuration of zero and non zero markers for number (or any other category).

[2]Nominative (or absolutive) is the case, that is employed in citation (viz [14]'s definition, p.55). Although nominative in Georgian has no zero marker, it is functionally 'main' (it is used as the citation form and governs verb agreement) and unmarked, especially in opposition with other cases, which indicate marked functions and roles of nouns.

[3]There are now several critical works on ergativity in the Kartvelian languages: [3], [15], [17].

somebody' (P is [+anim]) but *caiyo* 'took something' (P is [-anim]); for the Subject *cevs* 'somebody is lying' but *devs* 'something is lying'.

We can hypothesize that the passive and ergative constructions exclude each other because they both express grammaticalization of one category - Semantic Agreement, but on different levels.[4] The ergative construction implies the realization of the SA at the first stage of sentence generation, whereas the passive construction is, in the first place, associated with the expression of the S-O category and only in the second place with the SA realization. As a result, in both cases the same construction is derived: Ag (in indirect case) and P (in direct case).

These synchronic and diachronic contacts are traced in typological linguistics, the main aim of which is to define the deep, universal structures. However, which is the underlying structure, the nominative or the ergative one?

In our opinion, it would be better not to try to reduce these two constructions to one, but rather to describe them by those categories that are dominant in determining their semantics: for the nominative construction, it is the category S-O and for the ergative one - SA. Thus, in the system of grammatical categories of the nominative languages the dominant semantic category is S-O. All other categories are on a lower hierarchical level, thus S-O > SA. However, in the system of grammatical categories of the ergative languages, the most important feature is the marking of SA; thus SA > S-O. Therefore, we can say that the nominative and ergative constructions have the same underlying structure, but different dominant semantic category and hierarchies of grammatical categories.

Ergative constructions can emerge in nominative languages as a result of P actualization and usually are restricted to past, perfect and perfective aspect forms. In such cases the importance of P is increased: the result of the complete action is available and P actualization is more necessary than in the case of an incomplete action.

3 The notion of a Dominant Semantic Category

The categories S/O, SA and Activity seem to be cognitively universal concepts. Nevertheless, some languages prefer to grammaticalize S/O, while other languages choose either for SA or for Activity.

In general we can say that while the grammars of all human languages share a set of constraints in markedness, these constraints are so simple and general that they conflict in many contexts; they cannot all be satisfied simultaneously. The grammar of an individual language resolves these conflicts by ranking the universal constraints into a constraint hierarchy, conflicts being resolved in favor of higher-ranked constraints, with each constraint having absolute priority over all lower-ranked constraints.

[4] This idea originates from P. Uslar and was developed by other scholars. It was the favorite explanation of ergative constructions in XIX. century linguistics.

The dominant category further implicates the grammaticalisation of other semantic or functional categories. E.g., in languages with the dominant S/O category, we expect markedness in gender, definitness, present tense, imperfect, passive voice, dynamic verbs and so on, while in languages with the SA grammatical category class, it would be indefinitness, past tense, perfect, static verbs and so on. Obviously, the notation of the dominant category is very important for an adequate analysis of a given language. However, in Georgian (and other Kartvelian languages) neither O-S[5], nor SA, nor Activeness[6] is decisive. In the next section, we will have a look at the Georgian data in detail.

4 Georgian data

Formal analysis of the Georgian case patterns gives us the following information:

1. The I/II personal pronouns *me* (I), *en* (you-sing.), *even* (we), *tkven* (you-plural) never add case markers, they are always unmarked.

2. All other nouns show three main case patterns:

   (a) **NOM - (DAT) -** I series forms: present, imperfect, present subjunctive, future, conditional, future subjunctive

K'ac-i	*cxovrobs.*	('A man lives.')
K'ac-i	*surat-s xat'avs.*	('A man paints a picture.')
K'ac-i	*c'evs.*	('A man lies.')

   (b) **ERG-(NOM) -** II series forms: aorist, aorist subjunctive

K'ac-ma	*icxovra.*	('A man lived.')
K'ac-ma	*surat-i daxata.*	('A man painted a picture.')
K'ac-i	*dac'va.*	('A man lay.')

   (c) **DAT-(NOM) -** III series forms: perfect, pluperfect, III-subjunctive.

K'ac-s	*ucxovria.*	('A man has lived.')
K'ac-s	*surat-i dauxat'avs.*	('A man has painted a picture.')
K'ac-i	*dac'olila.*	('A man has lain.')

These systems prove the existence of the I/II - III formal dichotomy: the I/II person subsystem does not distinguish the roles of the nouns, since Ag and P (and Ad(dressee)) can all be unmarked. In the III person subsystem in the I series, S is always in nominative: *-i* (after consonants) // - Ø (after vowels),

[5] The reason is that no formal marking expresses the S-O category (See [16], [4] and others). Nevertheless S-O is relevant on the level of syntax, especially for the present tense verb forms (see [9]).

[6] It seems natural to define the Kartvelian languages as active languages ([13]), although some problems still remain ([9], [3]). In particular, there is a group of verbs like *igineba* - 'to curse' - which have active semantics, but passive morphology. Also, some active verbs do not trigger ergative: *kaci midis* ('A man-nom is walking'), *kaci cavida* ('A man-nom went').

and O in dative (-*s*).[7] It is thus a nominative system, while II and III are ergative systems: S of the transitive verb in II series appears with a special case represented by the suffix **ma** (after consonants) // **-m** (after vowels); in III series with the dative ending **-s**. All other nouns (S_{intr} and O_{tr}) are in nominative and trigger verb concord.

There are two types of verbal affixes, the V-type and the M-type:

	V-type		M-type	
	sing.	pl.	sing.	pl.
I.	**v-**	**v- -t**	**m-**	**gv-**
II.	**-t**	**g-**	**g-**	**-t**
III.	**-s, a, o**	**-n, en, an, nen, es** Ø	**h, s, Ø-**	**h, s, Ø- -t** Ø

Traditionally, the V-type endings are considered to be subject markers, while the M-type are object markers. Note, however, that this is not always the case: in the III series and also with affective verbs the subject appears with the M-type and object with the V-type: *mia* ('I am hungry'), *meinia* ('I am afraid'), *mciva* ('I am cold'), *mezineba* ('I want to sleep'), *minda* ('I want'). For that reason, most Georgian scholars traditionally qualify these forms as inversive ones.

In general, it seems better to analyze these markers without any functional qualification, simply by their relation to cases ([4]):

1. Noun in dative always triggers M-type affixes;
2. Noun in ergative always triggers V-type affixes;
3. Noun in nominative triggers either
   (a) V-type (if there is no ergative linked with the verb), or
   (b) M-type (in case there is an ergative linked with the verb), or
   (c) zero (if both ergative and dative appear in the construction).[8]

Based on the rules above, we can make the following generalizations:

1. The V-type affixes should be represented in the verb form even in cases when V-triggering noun is empty. [9]
2. From the point of view of the triggering V-type affixes, there are the following hierarchical relations: ergative (always) > nominative (sometimes)> dative (never). With respect to verb concord, I/II personal pronouns behave similarly as ergative, dative or nominative nouns. Consequently, we

[7]There is no special case for Patient; Addressee and Patient both have the same (dative) ending: *kaci ucers cign-s* (P) *deda-s* (Ad) ('A man- nom writes a book-dat for mother-dat').

[8]The zero allomorph is used when the nouns stands in dative with the I series forms, but in nominative with the II-III series.

[9]Empty" denotes such head-nouns which never occur in surface structures (e.g. *c'vims* 'It rains').

may speak about I/II ergative (always triggers V-), I/II dative (always triggers M-) and nominative (triggers either M-, or V-, or Ø).

3. I/II pronouns always trigger prefixes, while other nouns mostly suffixes.[10]

4. In case of several prefixes, a competitive situation arises, as there is no possibility of prefix clustering.[11]

When there is a competition among prefixes (both I/II persons are linked with the verb), the hierarchically weaker case wins? :[12]

1. I/II nominative and I/II dative → I/II dative

2. I/II ergative and I/II dative → I/II dative

3. I/II ergative and I/II nominative → I/II nominative

In case there is no competitive situation (this happens mostly when I/II meets III), both markers will be present on polypersonal verb form:[13]

4.1 Semantic and functional analysis of these forms

I/II personal pronouns are always unmarked by cases and they thus qualify as functionally 'main'. They are always represented on the verb form by V- or M-type person prefixes and the plural suffix *-t*. The difference in the opposition of V- and M-types may be explained by different semantics of the nouns: (a) if the volitionality of the agent (expressed by the noun) is implied by the action (e.g. 'I play', 'I work', 'I write'), it will be represented by the V-type markers; (b) if the volitionality of the agent is not implied by the verbal semantics (e.g., 'he writes to me', 'he paints me', 'I am afraid', 'I am hungry', 'It seems I have been here', and so on), it will trigger an M-type marker. The rule reflecting the competition between prefixes reveals that non-volitional I/II agents which are in dative and trigger M-type markers, always take precedence over other nouns. From this fact, we can conclude that they are marked. All other nouns (non-participants of the communicative act) behave differently. Depending on various tense forms (I, II or III series), they follow either the nominative or the ergative (extended to the activity category) case patterns. Differentiation by the roles of Ag, P or Ad is more relevant for them. Constraints which govern the choice of either ergative

[10]The III person dative is marked by the allomorphs: **h-** (before the sounds /b/, /p/, /g/, /k/, /q/, etc.); **s-**(before the sounds /d/, /t/, /t'/, /z/, /č/, /č'/, or Ø elsewhere).

[11]The only obvious exception is the combination of I person (nominative - ergative) plus III person (dative) marker in literary Modern Georgian: *v-s-cer* (cmp. spoken Georgian: **v-***er*) ('I write it (to) him').

[12]With affective verbs and with the III series verb forms, however, both nouns are represented: *me sen m-iqu 'var-x-ar* 'I love you' (I (dat) you (nom) I-love-you-is).

[13]If there are three nouns linked with the verb, more complex constructions with the reflexive *tav-* can arise: *man en emi tavi da-g-I-xat-a* (he (erg) you (dat) mine (nom) refl (nom) prev-you-version-wrote-he).

or nominative systems are simple: nominative (with S/O dominant category) for the present tense forms and extended ergative (with SA (Act) as the dominant category) for the past-perfect forms.[14] There is a direct correlation between case patterns and verb concord for I and III series:

- I series exhibit the nominative system:
  - S → V-type markers, which correspond to nominative noun;
  - Dir O → M-type markers, which correspond to dative noun;
  - Indir O → M-type markers, which correspond to dative noun.
- III series show ergative system (extended to the category of activity):
  - Ag → M-type markers, which correspond to dative noun;
  - P → V-type markers, which correspond to nominative noun.

A complex situation arises in the II-series, where noun cases exhibit the ergative system, while the verb concord is nominative.

This peculiar division of marked and unmarked nouns indicates that in the II series, there are two constraints at conflict. The I/II-III dichotomy defines the III person as marked, while the role opposition (Ag-P) most naturally for past tenses (such as the II series) selects for the SA category as dominant and consequently, P (which is prototypically III person) is unmarked. Thus, III person is conceptually unmarked (according to the roles) and marked (according to the I/II-III dichotomy) at the same time. (And vice versa: I/II person is cognitively unmarked according to the dichotomy). As we have seen, Georgian resolves this conflict peacefully, by division of the case patters and verb concord and by destroying the direct correlation between them: case marking reflects the unmarkedness of the Patient (which appears in nominative), while verb concord defines its markedness by M-type affixes. The zero allomorph for the III person Patient in M-type affixes shows that the III person (especially inanimate one, as a prototypical Patient) is cognitively unmarked and consequently formally unmarked.[15]

A functional analysis of the polypersonal verb forms can clarify the main function of polypersonality: the obligatory denotation of I/II persons (unmarked) in the verb form. That is, the I/II persons are always represented in verb concord

[14] If we don't distinguish nouns by the S/O functions and analyze them by the semantic feature 'volitionality' instead, so-called inversive verbs (which are semantically affective ones) do not differ from others in triggering verb concord : *m-ia* ('I am hungry'), *me-inia* ('I am afraid'), *m-civa* ('I am cold') and so on. The subject of these verbs appears in the dative and has to be expressed by the M-type affixes. E.g., the subject of the verb *miqvars* (to love) in Georgian stands in dative and triggers M-type markers. We could say that according to the Georgian language, love is an involuntary action - the subject loves independently of his/her will and the object causes the affection.

[15] Although there are some exceptions: *ga-s-ca* ('he gave something'), *ga-s-cia* ('he went).

without any constraints, while the III person only under certain circumstamces.[16]

4.2 Definition of the Dominant category in Georgian

The I/II-III dichotomy that has become obvious after our analysis of the marked-unmarked distinction among nouns (as indicated by case marking and verb concord and proved also by the main function of polypersonal verb forms) is strengthened by many other grammatical categories of Georgian:

- **the category of version**
  If the Patient belongs to the I/II persons, the verb has the prefix **i-**, e.g., *m-i-cer* ('you write to me'). If the Patient belongs to the III person, the verb has either the prefix **i-** (in case the III person is the subject), or **u-** (in case the III person is the indirect object): *i-éer-s* ('he writes for himself') *u-éer-s* ('he writes to him').

- **the category of direction**
  If the action is directed towards the III person, the verb forms are denoted by simple prefixes expressing direction: *a-dis* ('he goes up'), *ča-dis* ('he goes down'), *ga-dis* ('he goes out'), *še-dis* ('he goes into') and so on.

If the action is directed towards the I/II persons, the prefix **mo-** and its compounds are used: *amo-dis* ('he goes up to me/you'), *čamo-dis* ('he goes down to me/you'), *gamo-dis* ('he goes out to me/you'), *šemo-dis* ('he goes into to me/you') and so on.

These facts support the hypothesis that in Georgian the dominant category is the one which causes the I/II and III formal dichotomy. We propose that the relevant category is **participation in the communicative act** (CA).

According to this category, the participants in the communicative act (I/II persons, locutors) are unmarked and functionally main, while non-participants (III person, non-locutor) are marked and functionally minor. In other words, Georgian reflects the reality via the prism of a communicative act. This very category stands on the highest level in the hierarchy of grammatical categories and governs case patterns, as well as verb concord. From this particular perspective, the I/II persons are obligatory and consequently they are regarded as conceptually unmarked. The III person nouns, on the other hand, are regarded as marked.

The perspective can be made broader via a differentiation of III person roles. This differentiation is more restricted and complex: markedness is defined according to various tenses and moods and different hierarchies among the categories S/O, SA, Activity and Communicative Act can be observed:

[16]The III person is contained in portmanteau affixes: **-s** expresses subject of active verbs in present and subject of verbs in subjunctive mood; **-a** subject of passive verbs in present and subject of verbs in past; **-o** some verbs in aorist etc.

I series: CA > S/O > SA(Act)
II series: CA > S/O > SA(Act)
III series: CA > SA(Act)> S/O

In that manner, some units become more usual and frequent via turning marked units into unmarked ones.

5 Typological Perspectives of the Dominant Category

Through a careful analysis of the Georgian data, a new dominant category has been established. There could exist other types of languages with different dominant categories - e.g., in Japanese, Topic seems to be the dominant category, given that it governs verb agreement:

Topic is marked	by	**-wa**
S (nom)	by	**-ga**
Dir O(acc)	by	**-o**
Indir O (dat)	by	**-ni**

Competition rules: **-ga-wa** > **-wa**, **-o-wa** > **-wa**

From these facts we can conclude that topic is hierarchically stronger, i.e., Topic > S/O. [12] in his comprehensive relational typology proposes that the three main dimensions of relational structuring are those of semantic roles, information flow, and deistic anchoring. Languages can be divided into three major types, depending on the extent to which these dimensions are grammaticalised: "pivotless" languages, with no or little grammaticalization of any of these dimensions, "pure" languages, strongly grammaticalizing only one of them (usually that of semantic roles), and "mixed" languages, strongly grammaticalizing more than one dimension. According to this typology, Georgian is a mixed language, more precisely a role-deixis-oriented language, but the same qualification would appply to Batsbi (Tsova-Tush), where ergative construction is used provided that the noun is (i) Ag, (ii) I/II person, and (iii) acts according to his/her will and controls the action.[17] However, verb concord (class markers) is not restricted to I/II person and the Patient doesn't show the I/II versus III dichotomy either (no person markers for it). Therefore, it seems that in Tsova-Tush, SA is stronger and stands over the communicative act category: SA > Communicative Act > Activity. In Georgian, on the other hand, a different hierarchy should be used to arrive at adequate linguistic structures: CA > SA//S/O. To

[17] As [10] mentions: "When I constructed the first person form for the verb 'get poor' in Tsora-Tush using [Sa] (ergative noun) marking, my consultant did not say categorically that it wasn't possible. She said it isn't possible because you would never want to be poor" (p. 115).

summarize, although Batsbi is similar to Georgian in one respect, it also differs from it in a significant way. The same could be said about Basque, where there is a nominative system in the past tense verb concord, and an ergative system in the present. This seems unnatural and typologically unexpected, since most of the time ergative system is characteristic for the past, and nominative for the present tense forms. It is relevant to note that nominative constructions arise only in case the patient is the III person. This fact suggests that in Basque, the I/II versus III dichotomy plays a significant role, although the hierarchy among relevant categories seems to be different from Georgian: SA > CA > S/O.

We can conclude from the examples above that for purposes of a comprehensive description of languages, it is not enough to define languages as merely mixed systems, but also state of which hierarchies they make use.

The idea of defining the dominant category and hierarchies among grammatical categories should be further explored in further typological studies. The dominant category can be used as the qualifying criterion for identification of groups of typologically close languages and their classification, along with hierarchical oppositions between grammatical categories. The hierarchies are defined according to the priority given to marked categories during surface realizations.

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The Semantics of Russian verbal prefixes: v(o)-/vy- (‘in’/’out’)

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Our research is based on an understanding of natural language meaning that attaches crucial importance to the share of spatial relations. According to this approach the content of a sentence is conceived as a projection in the Language Space (LS), a somewhat transformed reflection of the real space. Projections are thus one component of LS, the other being metaphorical reflections of quality degrees, functioning of the perception organs, social relations, etc. The characteristic features of LS are the position of the observer, the focus of her/his attention, direction of their movements (back, forward, up, down, etc.). The most important role is attached to the border, its position and orientation relative to the observer and the object of his/her attention and their movements.

Besides the prepositions and verb roots a paramount contribution to the organization of LS projections is due to verbal prefixes, at least for languages with highly developed verb prefixation (Russian, German, Georgian, etc.) In our present work we deal with schematizing the LS of the two most common Russian verbal prefixes: *v–* (‘in’) and *vy–* (‘out’). A comparative perspective on the problem is supported by parallel examples from German and Georgian.

In the conclusion, the interpretation of the meaning of verbal prefixes is brought into correlation with metaphorical expressions.

1. Introduction

In the course of research carried out several years ago, we have been trying to formulate the features of Linguistic Space (LS) sufficiently to describe differential characteristics of the system of basic or prototypical meanings of Russian verbal prefixes. As a matter of fact, these features must define LS as such with the further perspective to represent as projections in LS all the rest of the verbal prefixation meanings, including those reflecting real space relations. The latter case is not quite trivial, since LS is conceived not as a pure, but rather as somewhat transformed reflection of real space.

The pivot of LS is a ‘border’ - a passive object. It is sometimes, especially in the case of *v(o)-*, *vy-* (‘in’/’out’) verb prefixes, conceived as closed, constituting a body with its internal and external areas . Some prefixes refer solely to the surface of the border (*na-*’on’ etc.), while others imply a relation between the areas on each side of it (besides *v(o)-*, *vy-*, also *za-*, *pere-*‘over’ etc.).

The other, active, part of a LS-structure is formed by trajectories of an active or moving object (MO), the moves of which relative to the border (BRD) constitute the basic, prototypical (spatial) meaning of verbal prefixes. These trajectories can be categorized by two basic directions in LS: radial and tangential. The former symbolizes trajectories by which MO is moving towards or from the BRD, its additional feature being crossing/non-crossing of the border: obviously, both the verbal prefix in question are radial and crossing, yet *v(o)-* has the orientation towards the BRD (a positive one), whereas *vy-* is characterized by the opposite orientation: from the BRD (a negative one). To make the picture more complete, we can add to the ‘+’ and ‘-‘ orientations another orientation which can be most naturally marked as zero and which characterizes the cases when a verbal prefix means just crossing the BRD as such (*pro*-‘through’, *pere*-‘over’). As to the pair which will be considered here, their crossing is definitely oriented: perhaps the corresponding movements could be described more thoroughly as a leaving (*vy-*) and entering (*v(o)-*) of the inner space of BRD, which is always conceived as closed one, though in the most, if not all, cases it is never really closed (a house, a yard, or a glass are never closed in any strict geometrical sense).

There are two directions which are not relevant to the meaning of the prefix *vy-/v(o)*: (i) the tangential direction, a part of spatial meaning of a verbal prefix designating the movement ‘on’ (*na-*) or ‘around’ (*o-/ob-/obo-*) the BRD, which in the case of the latter is also preferably conceived as a closed one; and (ii) the vertical direction characterized by 'up' and 'down' values, which are indispensable components of the spatial meaning of the verbal prefixes *v(o)z-*, *s(o)*, *niz(o)-*, *pod(o)*.

The above mentioned characteristics suffice for schematizing the main features of LS and correspondingly of the structure of the spatial meaning of Russian verbal prefixes. In other words, their general spatial scheme can be represented by some combination of these characteristics, though a more detailed analysis may require distinguishing specific areas around BRD, such as 'upper' (*pere-*), 'under' (*pod(o)-*) or 'in front of' (*pred(o)-*).

2. General Description of *vy/v(o)*

The pair we devote our primary attention to (*v(o)-* and *vy-* verbal prefixes) can be characterized in the above introduced terms as follows:

1. BRD – closed,
2. Trajectory – radial, crossing (the BRD);
3. Orientation: positive for *v(o)-*, negative for *vy-*.

Fig. 1 depicts the schemes which are supposed to mirror the basic features 1-3 of our pair. The dash line components of the trajectories are intended to underline the indeterminable character of the corresponding parts of the MO movement, in particular of its initial and final positions, specified only by the terms 'in'/'out'.

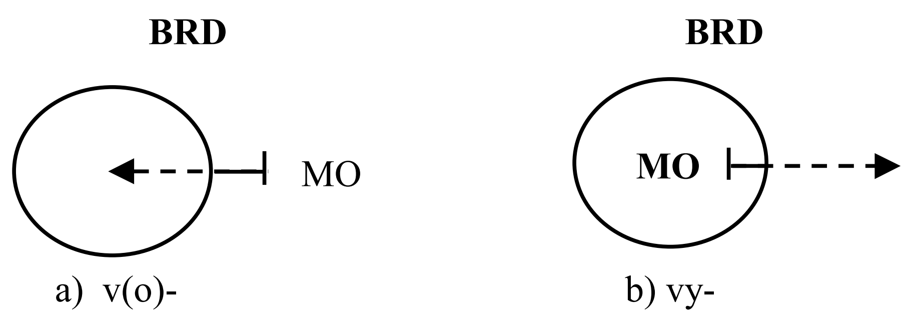

Fig. 1. General schemes representing the main spatial features of the v(o)- and vy- Russian verb prefixes; BRD- Border, MO-Moving Object

3. Realizations of the General Scheme

In subsections of this section we shall consider some particular realizations of these general schemes in the contexts of different verb roots compatible with the pair of Verbal prefixes under consideration. We shall primarily classify these contexts as appertaining to the real spatial relations, the five senses , social life, etc., and then try to define for each of these classes the character of the projections in MO and BRD, including the details of the changes of trajectories and some deviations from the initial primitive structures approximately depicted in Fig. 1.

3.1 Purely Spatial Meanings

The natural starting-point of our analysis is a purely spatial kind of Russian verbal prefixes meanings, for which the schemes in Fig. 1 are the most thorough approximation. Characteristically often, they create pairs of mutually opposed combinations with a single verb root. They have almost regular correspondences not only in English, as is obvious from the list below, but also in German and Georgian, the corresponding verbal prefixes being '*ein-/aus-*' and '*she-/ga*' –, respectively. Curiously enough, both the above mentioned languages, in spite of their striking differences, have in common an important feature of LS lacking in Russian or English: their purely spatial verbal prefixes may depend on the position of an observer (OBS). Georgian marks the trajectory of MO oriented towards OBS by the additional prefix '-*mo*-':

she-dis (OBS outside of the BRD)	he is going 'in'	*ga-dis* (OBS in the BRD)	he is going 'out'
she-mo-dis (OBS in the BRD)		*ga-mo-dis* (OBS outside the BRD)	

It appears that the -*mo*-/'zero' opposition functions in Georgian similarly to '*her-/hin*'- in German.

Below, we listed the most usual pairs of Russian verbs with *v(o)-/vy*- verbal prefixes which have (among other) purely spatial meanings:

v-jexat'//*vy-jexat'* – 'drive into'//'come/go out'
vo-jti//*vy-jti* – 'go/come/come in'//go out'
v-bezhat'//*vy-bezhat'* – 'run/come into'//'run out'
v-nesti//*vy-nesti* – 'bring/carry in'//'carry/take out'
v-tashchit'//*vy-tashchit'* – 'drag in/into'//'take/drag out'
v-letet'//*vy-letet'* – 'fly in'//'fly out'
v-porxnut'//*vy-porxnut'* – 'flit in'//'flit out'
v-brosit'//*vy-brosit'* – 'throw in'//'throw out'
v-katit'//*vy-katit'* – 'roll in'//'roll out'
v-vesti//*vy-vesti* – 'introduce/bring into'//'take/lead out'

All these verbs have a meaning which implies the real moving object as MO and BRD as a real (three-dimensional) body or some restricted two-dimensional area (a yard, a plot, a square, etc.). The moving object (MO) may be represented by a mass object: a liquid, a gaseous or dry substance or even a number of persons/things:

v-lit'//*vy-lit'* – 'pour in'//'pour out' (liquid substance)
v-meshchat's'a – 'contain/go in'
v-kachat'//*vy-kachat'* – 'pump into' //'pump out'
v-doxnut'//*vy-doxnut'* – 'breathe in'//'breath out'
v-sypat'//*vy-sypat'* – 'pour in'//'pour out' (dry substance)
v-valit'//*vy-valit'* – 'throw in/into'//throw out'

German and English demonstrate quite regular correspondences ('*ein*- // *aus*–' and 'in // out') in this case also; Georgian retains for the 'in' orientation the *she* – verbal prefix for things/persons but otherwise prefers *cha*- verbal prefix with the prototypical meaning 'down'. As for opposite orientation, Georgian uses a variety of verbal prefixes (e.g., *ga*- ('out'), *gada*- ('over')).

A more peculiar example of the same spatial type meaning of our pair of verbal prefixes is the case when the BRD is conceived as an obstacle for the MO. It can be something exercising resistance to its movement or a whole, compact (not hole) body into which the MO intrudes, perhaps in order to become its integral component.

Examples for 'in'– movement:
v-lomit's'a - 'break into'
v-tisnut' - 'squeeze/cram in/into'
v-gryzt's'a - 'gnaw in /into'
v-rezat's'a - 'cut/fit in'

vo-tknut' - 'run/stick/drive in/into'
v-meshat' - 'mix in'
v-stavit' - 'put/fix into/in'
v-shit' - 'saw in'
v-sunut' - 'put/stick in/into'

As a rule, verbs of this type are lacking antonyms derived from the same verb root. The rare exceptions may be exemplified as below, but in general some oppositions are doubtful.

v-davit'//vy-davit' - 'press in'//'squeeze out'
v-pixnut'//vy-pixnut' - 'shave/push in'//'shave/push out'
v-rezat'//vy-rezat' - 'fit into'//'cut out' (e.g.lock)

German correspondences are uniformly prefixed by '*(hin)ein*', but Georgian in the most cases prefers the *cha*– verbal prefix with the prototypical meaning – 'down', which perhaps mirrors the association with the force exerted on the BRD by gravity. Moreover, the verbs expressing an 'out' – movement are often prefixed by the prefix *amo*-, prototypically corresponding to the 'up' – orientation.

Examples of 'out' – movement:
vy-dernut' - 'pull out'
vy-rvat' - 'pull/tear out'
vy-svobodit' - 'free/let out'
vy-gryzt' - 'gnaw out'
vy-kopat' - 'dig up/scrape out', etc.

Lastly, it is worthwhile to mention the cases when not the BRD but the MO itself hinders its movement which is caused by some external agent:

vy-gnat' - 'drive/turn out'
vy-stavit' - 'turn/chuck out'
vy-dvorit' - 'turn out'
vy-shvyrnut' - 'fling/hurl out'
vy-manit' - 'entice from/(lure from/out)'

Sometimes, these verbs form meaning with both orientations:

v-tolknut'//vy-tolknut' - 'push/shave into'//'chuck/throw out'
v-tashchit'//vy-tashchit' - 'drag in/into'//'drag/pull out'

3.2 Non-spatial Meanings

The other, non-spatial, meanings of *v(o)-, vy-* verbal prefixes may be considered superpositions on the basic scheme in Fig.1. Perhaps the simplest among these non-spatials are those which we shall refer to as social, that is expressing some social relations and actions between persons or groups of persons. In what follows we shall refer to organized groups (a firm, an institution, a government/legislative body, an army, a peculiar stratum of society, etc.) as Definite Circle of People (DCP), and to the rest as Person (PRS) implying group of persons also. Besides DCP and PRS, the social space in some cases refers to a multitude of people which may be considered as an Indeterminate Mass of Persons (IMP). DCP and IMP are projected on the BRD (Fig. 1) in many examples of social space, though in some instances the BRD requires a different interpretation, e.g., in a difficult situation, be it a dangerous one (an illness, a crime or other troubles) or useful and auspicious at the end (work, study, etc.). We shall refer to it as TRS (Trial Situation).

The most frequent interpretation of MO would be PRS, yet at least in one very important type of our Verbal prefixes, the social meaning will be represented by an object having a social and primarily material value (e.g., money) SV – Social Value.

Just in the latter case of MO=SV we may not avoid indispensable addition to the scheme in Fig.1: an external (for the BRD) agent, which represents a source (SRC) or target (TRG) of MO's trajectory (see Fig. 2).

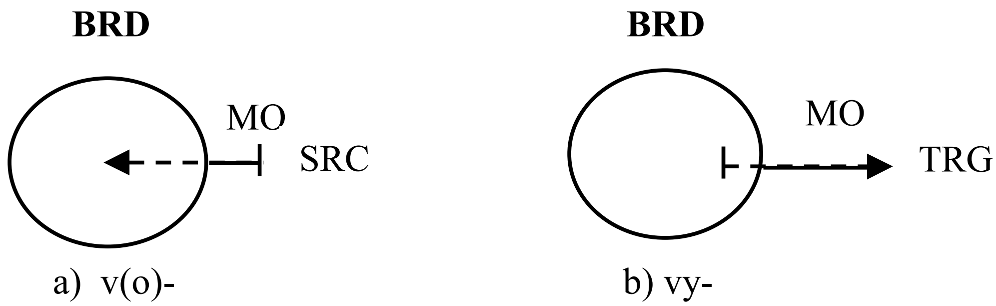

Fig. 2 LS schemes for v(o)- and vy- Verbal prefixes including image of an external agent: a) SRC (SouRCe) and b) TRG (TaRGet).

Below we list some examples of verbs prefixed by our pair of Verbal prefixes with social meaning (we refer to the scheme in Fig.1 by default):

a. BRD= DCP, MO=PRS

v(o)-

v-xodit' - 'participate in'/'be a member of'
v-vesti - 'make a member of'
v-kl'uchit' - 'include'
v-krast's'a - 'insinuate/worm into'
v-teret's'a - 'insinuate/worm into'

Examples:
– *vxodit'/vvesti/vkl'uchit' kogo-libo v sostav comiteta* - 'be a member of/make a member of/include in the Committee'
– *vkrast's'a/vteret's'a v kompaniju* - 'insinuate/worm into a company'
– *vteret's'a v chje-libo doverije* - 'insinuate/worm into somebody's confidence'

In the last case *doverije* - 'confidence' marks the sphere (BRD) of a person (PRS) or of a definite circle of people (DCP), into which an essentially alien, though outwardly friendly person 'insinuates/worms' himself .

vy-

vy-byt' – 'leave/quit'
vy-cherknut' - 'strike out/of'
vy-gnat' - 'fire/sack/expel from'
vy-dat'(zamuzh) - 'marry to'
vy-svatat' - 'ask to marry'

Examples:
– *vybyt' iz stroja (armiji, partiji i t. p.)* -'quit the ranks (of army, party, etc.)'
– *vycherknut' kogo-libo iz spiska (sotrudnikov, uchastnikov nekotorogo meroprijatij)* - 'strike somebody off the list (of the staff, of the participants of some undertaking, etc.)'
– *vygnat' iz shkoly* – 'expel from school'
– *vydat' zamuzh/vysvatat' chju-libo doch* – ' marry off/ask to marry somebody's daughter'

In the last case, the DCP implies a family. As for *vysvatat'* ('ask to marry'), it obviously requires the scheme of Fig. 2 comprising the TRG/SRC (target/source) element.

b. BRD= IMP, MO= PRS
vy-brat' – 'elect'
vy-jti – 'be by origin'
vy-dvinut's'a/vy-sluzhit's'a – 'rize/work way up'

Examples:
– *vybrat' v kachestve nashego predstavitel'a* – 'elect/nominate as our representative'
– *vyjti iz krest'jan* – 'be a peasant by origin'
- *vysluzhit's'a iz r'adovyx v oficery* – 'work one's way from a private to an officer'

In the first case, a class of people may be defined only pragmatically ("we"), while the other two imply the vast and, as a result, quite indeterminate mass of persons (peasants, private soldiers).

c. BRD= TRS, MO= PRS
v(o)-
v - vergat' – 'fling/plunge into'
v - v'azyvat' – 'mix in/put up'
v - meshat' – 'implicate/mix up'
vo - vlekat' – 'involve in/inveigle into'
v - travlivat'/v - t'agivat' – 'draw into/involve'
v - letet'/v - lipnut'/v - t'apat's'a – 'get into (a mess)'
v - meshat's'a – 'interfere/intervene in'
v - pr'agat's'a – 'harness to'
v - kl'uchat's'a – 'join in'

All the above listed verbs imply that a person/body of persons (PRS) is subjected to a trial situation (TRS). With the exception of the last two cases, all these TRS must be evaluated negatively ; the initial five lines comprise cases in which some active external agent is implied (active TRG), who involves/inveigles/draws into his/her victim – MO.

With the exception of the obvious distinction in the direction of the MO –trajectory, all that was said above is also valid for the following examples of verbs with the *vy-* prefix:

vy –
vy - gorazhivat' – 'screen/shield'
vy - zvolit' – 'help/get out'
vy - ruchit' – 'rescue/help out'
vy - svobodit' – 'free/let out'
vy - vernut's'a – 'slip/get out'
vy - karabkat's'a – 'scramble/get out'
vy - putat's'a – 'pull through'.

d. BRD= DCP, MO= SV, SRC/TRG = PRS (see Fig. 2).

Although the SV (Social Value) is mainly represented by some material value, in some situations it may be different, e.g. an action of DCP (=BRD) favorable for target PRS (person). The latter meaning, characteristic for some *vy-* prefixed verbs, ascribes corresponding authority to the Definite Circle of Persons representing a BRD.

v(o)-
v - buxat' – 'over-invest'
v - verit' – '(en) – trast in'
v - kladyvat' – 'invest'
v - nesti -' pay in'
v – ruchit' – 'entrust to'.

Examples:
– *vbuxat' uymu deneg v postrojku doma* – 'over-invest in housebuilding'
– *vnesti kvartirnuju platu* - 'pay in a rent'

vy -
vy - gadat' – 'gain'
vy - kl'anchit' – 'cadge/plague for'
vy - manit' – 'coax/wheedle out'
vy - kupit' – 'redeem'
vy - menivat' – 'barter/swap for'
vy - platit' – 'pay off'

The first triple of verbs assigns to the external person (PRS) a TRG (target) value, the next two imply it (both of the SRC and TRG meanings), and the last one ascribes to it the role of a SRC only.

Examples:
– *vykl'anchit' u nachal'stva otpusk* – ' cadge a leave of absence of the chief'
– *vykupit' zalozhennyje veshchi po dvojnoj cene* – 'redeem pledged goods for a double price'
– *vyplatit' dolg* – 'pay off a debt'.

None of the four directions above represent examples of verbs creating antonymous pairs. As for the interlingual correspondences,it appears that English translations quite often demonstrate coincidences *in/into* with *v(o)-*, yet'*vy-* translations are far more random. German in these cases also prefers the *ein-aus* opposition, but deviations from it are far more frequent than for the spatial meanings of *v(o)-*, *vy-*. Finally, Georgian most frequently uses *cha-* instead of *she-* to express the in-meaning, and a small variety of verbal prefixes, apart from the regular *ga-*, for the opposite one: *f-*(up – orientation), *cha-* ('down'), *da-* ('down/on').

3.3 Vy/V(o)- in Combination with Special Verb Roots

The scheme in Fig.2 used in the last point of the previous subsection is unavoidable for many representatives of one more type of meaning, expressed by our pair of Verbal prefixes in combination with some verb roots. These are verbs expressing mental, emotional and sensory states or activities. The abbreviations below have the following meaning: FLA – Field of Attention; MES – Mental/Emotional/Sensory organs/capacities (the small letter indexes **a** and **p** designate organs/capacities of the agent and patient, respectively); IFA – Information about Focus of Attention; MSA – Mental/Sensory/emotional Activity point, ASI – Active Source of Information.

Since the verbs with the pair of verbal prefixes under consideration do not form antonymous oppositions in the MES area, we shall consider them separately.

v(o)-:

a. BRD= FLA, SRC= MES, MO= MSA:

v - gl'adet's'a – 'peer at/into'
v - perit'(vzor) – 'fix on (gaze)'
v - niknut' – 'go deep into'
v - dumat's'a – 'think/ponder over'
v - chitat's'a –'read carfully'
v - chuvstvovat's'a – 'feel deeply'

All the above listed verbs imply that a person's mental/emotional/sensory (MES) activity (MSA) is directed to a field of her/his interests (FLA).

Examples:

– *on vgl'adels'a v eje lico* – 'he peered into her face'
– *vchuvstvovat's'a v rol' Gamleta* – '(begin to) feel deeply the part of Hamlet'

b. BRD=MESp, SRC=MESa, MO=MSA

v - dalblivat' – 'ram into'
v - tolkovat' – 'ram/din into'
v - tem'ashit' – 'take into (head)'
v - pechatlit' – 'impress'
v - l'ubit' - 'make fall in love'
vo-odushevvit' - 'inspire'

Agent's mental/emotional (MESa) activities (MSA) influence the MES of the patient:

– *vtolkovat' pravilo ucheniku* - 'ram the rule into pupil'
– *jego vystuplenije voodushevilo publiku* – 'his performance inspired the audience'

vy-

a. BRD=FLA, TRG=MES, MO=IFA

vy-gl'adet' - 'find/spy out'
vy-smotret' - 'look/spy out'
vy-chitat' - 'find (in a book, etc)'
vy-schitat'/vy-chislit' - 'calculate/figure out'
vy-javit' - 'reveal/bring out'
vy-jasnit' - 'clear up/find out'

The mental /sensory organs (MES) of the target person (TRG) receive information about the focus (IFA) of the field of attention (FLA):

– *on vysmotrel eje v tolpe* – 'he spied her out in the crowd'
– *nakonec on vyjasnil, kto ukral jego chasy* – 'at last he found out who had stolen his watch'

b. BRD=MESa, TRG=MESp, MO=IFA
vy-skazat' - 'state/say/express'
vy-boltat' - 'let/blab out'
vy-razit' - 'express'
vy-dumat' - 'make up/fabricate'

The only difference from the previous point is that BRD is represented by the MES of some agent who sends the message (IFA) to another person (TRG), playing the role of the patient (which may be represented by Indeterminate Mass of Persons (IMP) as well):

– *vyrazhaju vam moju iskrenn'uju blagodarnost'* – 'I express to you my sincere gratitude '
- *on vydumal etu istoriju s nachala do konca* – 'he has fabricated the story from beginning to end'

In English, the prepositions *in/into* corresponds to *v(o)-* rather often, but *vy-* verbal prefixes correspondences are mixed; German shows the inverted picture: *vy-* and *aus-* express the same meaning much more often than *v(o)-* and *ein-*, since *be*, *an-*, *ver-*, etc. are used instead. Georgian definitely prefers *cha-*('down') for *v(o)-*, though it expresses rather frequently *vy-* by the prototypical *ga-*.

3.4 Different Functions of BRD
The next and the last type which we shall consider is, in some sense, close to that discussed in the previous section.We begin with the case of verbs for which the *vy-* component may be interpreted as following the scheme of Fig. 1 without the TRG element. Nevertheless, in order to keep a certain homogeneity with the closely related previous type, we will rather consider it as a particular case of the scheme in Fig. 2, with TRG representing the Potential Observer (PTO).

a. In case of this subtype the border divides the LS into an observable (OBS) and an unobservable (UNO) part, thus functioning as a screen (SCR). The verbs of this subtype designate an act of emergence of some portion of Information which may become a Focus of Attention (IFA) of some PTO.

BRD=SCR, TRG=PTO, MO-IFA
vy-javit's'a – 'come to light/manifest itself'
vy-jasnits'a – 'turn out'
vy-stavit' – 'exhibit/display/set out'
vy-vesti – 'conclude/infer'

However, a more exact definition of the last example requires its blending with the scheme of the previous type, where TRG was instantiated by some MES.

Examples:
– *togda-to i vyjavilis' vse jego nedostatki* – 'just then all his shortcomings came to light'
– *nakonec-to on vystavil svoji kartiny* – 'at last he exhibited his paintigs'
– *iz etix dannyx ja vyvel sledjusheje zakl'uchenije...* – 'from these data I concluded that...'

b. The next subtype is characterized by a more definite content of the BRD: this time it is considered a Process (PRC), bringing in an observable Result (RES) for the Object (OBJ) of the PRC. Omitting in this case the PTO (nonetheless implied) , and considering the BRD itself as a Limit of the Process (LPR) we have:

BRD= LPR, MO= OBJ, TRG= RES:
vy - zdorovet' – 'recover'

vy - kormit' – 'bring up /rear'
vy - krestit' – 'baptize / christen'
vy - nudit' – 'force / compel / oblige'
vy - porot' – 'lash / birch'
vy - rabotat'/vy- delat' – 'produce / manufacture'
vy - rasti – ' grow up '
vy - strojit' – 'build'

Examples:
– *vyzdorovet' posle grippa* – 'recover from a flu'
– *jego vynudili podat' v otstavku* – 'he was compelled to resign'
– *on tak vyros, chto vygl'adit sovsem kak vzroslyj* – 'he is so grown up that looks quite like adult'

c. Some of the verbs of this type require, in some sense, the opposite interpretation: the BRD represents the Limits of the Object (LOB) which undergoes the Process with Result (RES) as a TRG.

BRD= LOB, MO= PRC, TRG= RES:
vy - kipet' – 'boil away'
vy - kurit' – 'smoke'
vy - moknut' – 'be soaked/drenched'
vy - poloskat' – 'rinse out'
vy - paxat' – 'plough/till'
vy - shagat' – 'pace out'
vy - travit' – 'exterminate/remove/take out'

Examples:
– *vs'a voda v chajnike vykipela* – 'all water in the tea - kettle is boiled away
– *ja vypoloskala vse belje* – ' I rinsed out all the linen'
– *ja vytravila p'atna na mojej jubke* – 'I removed stains from my skirt'
– *my vyshagali 10 km* – 'we paced out 10 km'

d. However, most of the instances of this type should be conceived as a blend of the last pair of opposite meaning scheme instantiations.

BRD =LPR/LOB, MO =OBJ/PRC, TRG= RES:
vy - belit' – 'whiten/bleach'
vy - brit' – 'shave'
vy - dubit' – 'tan'
vy - zolotit' – 'gild'
vy - krasit' – 'paint/dye'
vy - pravit' – 'correct'
vy - chistit' – 'clean/brush'
vy - lizat' – 'lick/clean'
vy - mazat' – 'smear/dirty'
vy - meret' – 'die out'
vy - merznut' – 'be destroyed by frost'
vy - motat' – 'drain/exhaust'
vy - rodit's'a – 'degenerate,

Examples:
– *on vybelil vs'u kvartiru* – 'he bleached the entire flat'
– *vse citrusovyje v etom godu vymerzli* – 'all citrus plants were destroyed by frost this year'

The last type includes the meanings of the verbs with the *vy*- verbal prefix only, yet it is worthwhile to mention that it is the most common one among all the considered types (up to 150 examples). At the same time it demonstrates the lowest rate of prototypical correspondences with English (about 10% of verb particle 'out'), German (nearly 30% of '*aus*-' verbal prefix) and Georgian (some 50% of the '*ga*-'' verbal prefix).

4. Conclusion

We tried to build concepts corresponding to the *v(o)-/vy-* verbal prefixes' meaning by arranging the particular domains of the supposed concepts (spatial, social, mental, processual/resultative) in accordance with the intuitive measure of their remoteness from the prototypical spatial meaning. Of course, we are aware that our attempts to find more formal criteria of distance (the (non)existence of *v(o)-/vy-* antonymous oppositions, references to the interlingual correspondences) are insufficient and unsatisfactory from many points of view. The lack of quantitative criteria, in turn, prevents us from interpreting these structures in the strict sense of Conceptual Spaces [P. Gardenfors]. Therefore, we leave the question open for further research.

It is obvious that the blended spaces of Fauconnier & Turner (1995) are indispensable for the proposed approach to meaning representation of verbal prefixes. From that standpoint we can consider the scheme in Fig. 1 and 2, to be the generic spaces, subsuming the types obtained by particular instantiations of BRD, MO, SRC, TRG, etc., blended with other input spaces (e.g., that of the verb root). The resulting blended space, being further instantiated by the means of other VP or clause components, should supply the main frame for correspondent meaning representation.

Our essentially metaphorical approach to the meaning of verbal prefixes is mostly encouraged by the set of fundamental language metaphors mentioned in the works of Taub (1996), Grady et al. (1996), Barnden (1998), Grady (1998) and others. All the above formulated non-prototypical concepts can be construed as compound metaphors built out of primitive ones (J. Grady et al., 1996).

For example, the concept of trial situation (section 3.2c) should imply primitives such as ***states are locations, action is motion, difficulties are impediments, purposes are destinations***. In terms of these metaphors, the BRD in Fig.1 designates the location of a state characterized as difficult (TRS). Its difficulties impede the motion out of this state and, indirectly, motion into it (by the anticipation of corresponding difficulties). If in spite of the difficulties a person (MO) attempts to act (e.g., move into/out of the state), the destinations of its motion will be the locations inside/outside the state/location (BRD=TRS of Fig.1).

In what follows we shall confine ourselves to a mere mention of the metaphors which are crucial for justification of the rest of the metaphoric *v(o)-/vy-* concepts. For points a, b of the social meanings of these Verbal prefixes (section 3.2), the basic metaphor ***circle/stratum/mass of persons is location*** should be indispensable. Very general support for the last point (d) of the same section 3.2 is represented by metaphors of Moral Self-Interest (Lakoff, 1996): "Well-being is Wealth", "Morality is Pursuit of Self-Interest", "Immorality is a Restraint on the Pursuit of Self-Interest", with addition of more concrete ones:"Pursuit of Self-Interest is Action" and "Action is Motion".

The mental/emotional/sensory metaphors of section 3.3 are essentially based on primitives like: ***mind as a physical space, mind as a container*** (Barnden, 1998), ***communication is sending, ideas/meanings are objects*** (Grady, 1998).

For all three points of the last subsection of section 3.4, the crucial statement is ***becoming accessible is emerging***. It is quite sufficient for the pointof subsection 3.4, where the BRD of Fig.2 is functioning as a screen dividing LS into observable and unobservable parts. The next point (b), however, needs the basic addition ***actions are locations*** (Taub, 1996) and, at the same time: ***action is motion***, implying an object (MO) moving through location corresponding to the action, and emerging as its accessible result in the end. Contrary to this, the concept in point (c) counts only if ***object is location*** through which it is moving by virtue of an action. The last point (d) blends the concepts of the two preceding points (b,c).

Finally, it should be noticed that a more complete representation of the meaning of *v(o)- /vy –* (disregarding the structuring of the LS as such) requires an analogous consideration of the whole paradigm of Russian verbal prefixes. In the first instance, one should focus on the most closely related verbal prefixes (*iz(o)-, pere-, pro-, za-, pri-, u-, pod(o)-, ot(o)-*), all of which have the crucial feature of radial direction.

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Aspects of relation algebras

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Outline

1. Introduction to relation algebras.
   Representations.
   Atom structures.
   Examples.
2. Using games to build representations.
   Axiomatising the finite representable relation algebras.
3. 'Rainbow' construction (Hirsch).
   Monk's theorem that the representable relation algebras are not finitely axiomatisable.
4. Relativised representations. Weakly associative algebras (WA, Maddux).
   Finite base property for WA by Herwig's theorem.

Why relation algebras?

- They have a long history—De Morgan Morgan 1860, Peirce Hartshorne and Weiss 1933, Schröder Schröder 1895, Tarski Tarski 1941, Lyndon Lyndon 1950, Monk Monk 1964, Andréka Andréka 1997, Németi Németi 1986, Maddux Maddux 1978, …
- They provide a simple introduction to parts of algebraic logic.
- Algebraic logic has strong connections to modal and dynamic logics. Recent Dutch–Hungarian work: e.g., **?**,van Benthem and Németi. 1998. Techniques cross over, both sides gain. *Arrow logic* Marx 1996 is the modal analogue of relation algebras.
- Relation algebras have practical applications in computing: eg. databases, artificial planning, specification theory (eg. fork algebras), concurrency Maddux 1996. RelMiCS group.
- Can develop all of mathematics by relation algebras. See Tarski and Givant 1987.

General references

Good introductions to relation algebras and more can be found in Maddux , Németi 1991., Monk and Tarski 1971, Monk and Tarski 1985.

0.1 Boolean algebras and relation algebras

Relation algebras are based on boolean algebras, and we review these first.

0.1.1 Boolean algebras

Definition 0.1.1 A boolean algebra is an algebra $\mathcal{B} = \langle\{\rangle B, +, \cdot, -, 0, 1\}$ satisfying, for all $x, y, z \in B$:

- $+$ is associative and commutative: ${}^{(}x + y) + z = x + (y + z)$ and $x + y = y + x$
- complement: $-(-x) = x$
- form of distributivity: $x = x \cdot y + x \cdot -y$
- connections: ${}^{x} \cdot y = -(-x + -y)$, $x + (-x) = 1$, $-1 = 0$.

Abbreviation: $x \leq y$ means $x \cdot y = x$ or equivalently $x + y = y$.

[1]The material in §0.2–0.3 is joint work with Robin Hirsch. Research partially supported by UK EPSRC grants GR/K54946, GR/L85978.

Representations of boolean algebras

The motivation for this definition comes from *fields of sets.* If U is a set, $\wp U$ denotes the power set (set of all subsets) of U. Suppose that $B \subseteq \wp U$ contains $\emptyset, U$ and is closed under union, intersection, complement. For example, $B = \wp U$ itself. Then $\langle B, \cup, \cap, U \setminus -, \emptyset, U\rangle$ is a boolean algebra, called a *field of sets.*

In fact, every boolean algebra is isomorphic to a field of sets (Stone's theorem, Stone 1936).

Unary and binary relations

A *unary relation* on a set U is just a subset of U. So boolean algebras are to do with unary relations. In fact, they embody all truths about unary relations (by Stone's theorem).

Can we do the same for *binary relations* (subsets of $U \times U$)?

0.1.2 Binary relations

We can consider *fields of binary relations on U.* These are subsets A of $\wp(U \times U)$. We want A to be closed under the boolean operations, as before. But there are natural operations on *binary relations* that we want to consider too.

Operations on binary relations

Which operations would we like?

Consider the binary relations *son, daughter.*

son(x,y) = y is a son of x, etc.

$$\text{x} \xrightarrow{\text{son}} \text{y}$$

From these, we know we can derive many other relations, such as:

- child
- parent
- father, mother, brother, sister
- grandchild, grandson
- grandfather, grandmother, grandparent
- aunt, uncle, niece, nephew.

Can we think of basic operations that will let us form these complex relations from *son* and *daughter*?

The relation algebra operations

Clearly, child = son + daughter. This is still boolean; we use + rather than $\cup$.

If we can form the *converse* of a binary relation:

$$R^{\smile}(x, y) \iff R(y, x)$$

then we can do parent:

- parent = child$^{\smile}$, or
- parent = (son + daughter)$^{\smile}$, or
- parent = son$^{\smile}$ + daughter$^{\smile}$.

If we can form the *composition* of two relations:

$$R\,;S(x, y) \iff \exists z(R(x, z) \wedge S(z, y))$$

then we can do more:

- grandchild = child ; child
- grandparent = parent ; parent

Need for equality

Can we say 'brother'?

How about parent ; son ?

Trouble is, parent(me,mum) $\wedge$ son(mum,me). So $\exists z$(parent(me,z) $\wedge$ son(z,me)). So by definition, [parent ; son](me,me). But I am not a brother of me.

But suppose we can express equality, by adding a constant 1' for it. Then

- brother = (parent ; son) $\cdot -$ 1' ($-$ is boolean complement)

With 1' we can express 'mother' too:

- mother = ((parent ; daughter) $\cdot$ 1') ; parent
- father = parent $\cdot -$mother.

The non-boolean operations we usually take are indeed ; , $^\smile$, and 1'.

(1) Can you express *sister* with these operations? How about *nephew*? And *aunt*?

0.1.3 Proper relation algebras

If we use these operations on binary relations, we get a new kind of algebra.

Definition 0.1.2 A *proper relation algebra* is an algebra

$$\mathcal{A} = \langle A, \cup, \cap, E \setminus -, \emptyset, E, =, ^\smile, ; \rangle$$

where $A \subseteq \wp(U \times U)$ for some set U (the 'base'), E is an equivalence relation on U, and the operations are as explained already. (A must be closed under the operations—so, e.g., $\emptyset, E, = \in A$). Usually E will be $U \times U$ itself. But for technical reasons we don't insist on this.

Eg: $\mathcal{A} = \langle \wp(U \times U), \cup, \cap, \setminus, \emptyset, U \times U, =, ^\smile, ; \rangle$, called the *full power set algebra on* U.
Question: Can we axiomatise these algebras, like we did for boolean algebras?

0.1.4 Abstract relation algebras, representations

Definition 0.1.3 (Tarski, 1940s)
A *relation algebra* is an algebra $\mathcal{A} = \langle A, +, \cdot, -, 0, 1, 1', ^\smile, ; \rangle$ where the following hold for all $x, y, z \in A$:

(R0) the axioms for boolean algebras in definition 0.1.1
(R1) $(x ; y) ; z = x ;(y ; z)$ (associativity)
(R2) $(x + y) ; z = x ; z + y ; z$
(R3) $x ; 1' = x$
(R4) $x^{\smile\smile} = x$
(R5) $(x + y)^\smile = x^\smile + y^\smile$
(R6) $(x ; y)^\smile = y^\smile ; x^\smile$
(R7) $x^\smile ; -(x ; y) \leq -y$.

These axioms are an attempted analogue, for binary relations, of the BA axioms. Do they work?

Definition 0.1.4 A *representation* of a relation-type algebra $\mathcal{A}$ is an isomorphism from $\mathcal{A}$ to a proper relation algebra. $\mathcal{A}$ is *representable* if it has a representation.

Definition 0.1.5 RA is the class of all relation algebras. RRA is the class of representable relation-type algebras.

It's easily seen that the RA axioms are sound:

(2) $\mathsf{RRA} \subseteq \mathsf{RA}$.

Equivalently, a proper relation algebra is a relation algebra.
Can we prove an analogue of Stone's theorem, showing completeness? That is, is *any* relation algebra representable?

Sadly, no [Lyndon, 1950]:

$$\mathsf{RRA} \subset \mathsf{RA}.$$

We prove this in example 0.1.12. Moreover, RRA is not finitely axiomatisable in first-order logic [Monk, 1964]: see section 0.3. In fact, RRA is not finitely axiomatisable in 2nd-order logic, 3rd-order logic, ... It can't be axiomatised by equations using a finite set of variables, nor by Sahlqvist equations. See Hirsch and Hodkinson 1999: To appear, Hodkinson 1988, Andréka 1997, Venema 1997.

At least it is recursively axiomatisable (see section 0.2).

0.1.5 Atoms and atom structures

We will mostly consider finite algebras. For these, it's easier to work with their atoms.

Definition 0.1.6 An *atom* of a (boolean algebra or) relation algebra is a $\leq$-minimal non-zero element. A relation algebra is *atomic* if every non-zero element is $\geq$ an atom.

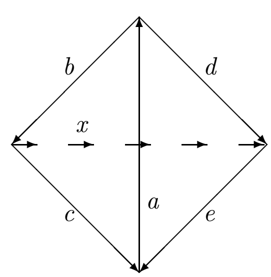
Any finite relation algebra is atomic. It can be shown that in an atomic relation algebra $\mathcal{A}$:
• 1' is determined by {atoms a : a $\leq$ 1'}.
• if a is an atom then so is $a^\smile$. And $^\smile$ is determined by its values on atoms.
• ; in $\mathcal{A}$ is determined by the set of triples of atoms (x, y, z) such that $x\,;y \geq z^\smile$. We call these *consistent triangles.*

Atom structures

So if we specify
• a finite set X of atoms,
• the set Id of atoms under 1',
• how the function $^\smile$ behaves on the atoms,
• the set C of consistent triangles,
is there a relation algebra with these atoms?

Not quite:

Definition 0.1.7 A *'relation algebra atom structure'* is a structure $(X, Id, {}^\smile, C)$ satisfying:
• $x^{\smile\smile} = x$ (for all $x \in X$)
• $x = y^\smile$ iff $\exists z \in Id((x, y, z)$ consistent)
• if (x, y, z) is consistent then so are (y, z, x) and $(z^\smile, y^\smile, x^\smile)$ ('Peircean transforms')
• diamond completion (for associativity):



If $(a, b, c), (a, d, e)$ are consistent then there is x with $(b, x, d^\smile), (c^\smile, x, e)$ consistent.

(3) Show that if $a \in Id$ then $a^\smile = a$.

Atom structures and relation algebras

Definition 0.1.8 For an atomic relation algebra $\mathcal{A}$, write $At\mathcal{A}$ for its *atom structure*:

$$At\mathcal{A} = (\{\text{atoms of } \mathcal{A}\}, Id, {}^\smile, C),$$

where $Id = \{a : a \leq 1'\}$, $^\smile$ is as in $\mathcal{A}$ (restricted to atoms), and $C = \{(a, b, c) : c^\smile \leq a\,; b\}$.

Theorem 0.1.9 (Lyndon, 1950)

1. *If $\mathcal{A}$ is an atomic relation algebra then $At\mathcal{A}$ is a relation algebra atom structure as in definition 0.1.7.*
2. *For any relation algebra atom structure $\mathcal{S} = (X, Id, ^\smile, C)$, there is an atomic relation algebra $\mathcal{A}$ with atom structure $\mathcal{S}$. If X is finite, $\mathcal{A}$ is unique.*

Proof. 1. Exercise!
2. $\mathcal{A} = \langle \wp X, \cup, \cap, \setminus, \emptyset, \wp X, Id, ^\smile, ; \rangle$ is a relation algebra with atom structure $\mathcal{S}$, where for $a, b \subseteq X$:

$$\begin{aligned} a^\smile &= \{x^\smile : x \in a\}, \\ a\,; b &= \{z : \exists x \in a\ \exists y \in b\ (x, y, z^\smile) \in C\}. \end{aligned}$$

If $\mathcal{S}$ is finite and $At\mathcal{B} = \mathcal{S}$, then $b \mapsto \{x \in X : x \leq b\}$ is an isomorphism : $\mathcal{B} \to \mathcal{A}$. ■

So we can specify a finite relation algebra by specifying its atom structure.

0.1.6 Examples of relation algebras

Example 0.1.10 The smallest non-trivial relation algebra, $\mathcal{I}$, has atoms 1' and $\sharp$, both self-converse. The consistent triangles are the Peircean transforms of (1', 1', 1') and ($\sharp, \sharp$, 1'). It is representable: take $U = \{0, 1\}$ and interpret 1' as $=$ and $\sharp$ as $\neq$.

Example 0.1.11 The *point algebra* $\mathcal{P}$ has 3 atoms, 1'$, a, a^\smile$. The consistent triangles are all Peircean transforms of $(x, x^\smile, 1')$ for all atoms x (of course), and of $(a, a, a^\smile)$. The point algebra is representable: take U to be the rational numbers, interpret 1' as $=$, and a as $<$.

McKenzie's algebra (1970)

Example 0.1.12 This is the smallest non-representable relation algebra. We call it $\mathcal{K}$. It has 4 atoms:

The consistent triangles are all Peircean transforms of:
- $(x, x^\smile, 1')$ for all x
- $(a, a, a^\smile)$ (like point algebra so far)
- $(a, a^\smile, \sharp)$, $(a^\smile, a, \sharp)$, and $(a, \sharp, \sharp)$.

This defines a relation algebra (exercise). Exercise 11 shows that it's not representable.

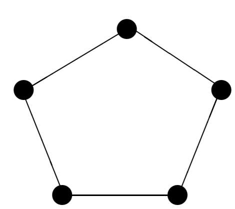
Another example

Example 0.1.13 This algebra (say $\mathcal{J}$) has 4 atoms: 1', r, b, g. They are all self-converse.
The *inconsistent* triangles are the Peircean transforms of
- identity ones (x,y,1') if $x \neq y$, as usual
- (b,b,b)
- (b,b,g)
- (b,g,g)

r is called a *flexible atom,* as all triangles with an r and no 1' are consistent, and it causes $\mathcal{J}$ to be representable (see exercise 10). As of 1996, it's not known whether it has a finite representation.

(4) If in $\mathcal{I}$ we say that $(\sharp, \sharp, \sharp)$ is consistent too, do we get a (representable) relation algebra?

(5) In the pentagon,



let $e(x, y)$ hold iff (x, y) is an edge, and $n(x, y)$ iff (x, y) is a non-edge (and $x \neq y$). Show that $\{1', e, n\}$ form the atoms of a proper relation algebra. What are the consistent triangles?

(6) Does $\mathcal{P}$ have a finite representation?

(7) Show that the atom structure for $\mathcal{K}$ in example 0.1.12 is a relation algebra atom structure.

0.2 Games

How can we tell whether a (finite) relation algebra is representable?

We use networks and games. This view developed in Hirsch and Hodkinson. 1997c based on Lyndon Lyndon 1950.

We make some assumptions, for simplicity:

- Most relation algebras here will be finite. So we can work with (relation algebra) atom structures instead of algebras.
- We will always assume 1' is an atom of relation algebras here. So $Id = \{1'\}$ in atom structures.

It follows that in a representation of a relation algebra, we can always assume that 1 is represented as $U \times U$ (not just an equivalence relation on U):

(8) Let $\mathcal{A}$ be a representable relation algebra and suppose that 1' is an atom of $\mathcal{A}$. Show that $\mathcal{A}$ has a representation h such that $h(1)$ is of the form $U \times U$.

0.2.1 Networks

A network approximates a representation.

Definition 0.2.1 (network) Let $\mathcal{S} = (A, Id, \smile, C)$ be an atom structure. An *(atomic) pre-network* over $\mathcal{S}$ is a pair $N = (N_1, N_2)$ where N_1 is a non-empty set and $N_2 : N_1 \times N_1 \to A$ is a labelling function.

N is a *network* if:

- $N_2(x, x) = 1'$ for all $x \in N_1$
- $(N_2(x, y), N_2(y, z), N_2(z, x))$ is consistent for all $x, y, z \in N_1$.

Some networks satisfy $N(x, y) = 1' \Rightarrow x = y$. We call these *strict networks.*

Some basic facts about pre-networks

Let $N = (N_1, N_2)$ and $N' = (N'_1, N'_2)$ be pre-networks (over some atom structure).

Definition 0.2.2 N, N' are *isomorphic*, written $N \cong N'$, if there is a bijection $\theta : N_1 \to N'_1$ with $N_2(x, y) = N'_2(\theta(x), \theta(y))$ for $x, y \in N_1$.

Definition 0.2.3 We write $\boldsymbol{N \subseteq N'}$ if $N_1 \subseteq N'_1$ and $N'_2 \restriction (N_1 \times N_1) = N_2$ (that is, $N_2 \subseteq N'_2$).

Definition 0.2.4 If $N_0 \subseteq N_1 \subseteq \cdots$ are pre-networks, define $\bigcup_{t<\omega} N_t$ to be the pre-network

$$(\bigcup_{t<\omega} (N_t)_1, \bigcup_{t<\omega} (N_t)_2).$$

Clearly, if the N_t are all networks then so is $\bigcup N_t$.

Notation 0.2.5 We generally write N for any of N, N_1, N_2 (sometimes write N_1 as $dom(N)$, but usually distinguish by context). We let $|N|$ denote $|N_1|$ and say N is finite if N_1 is.

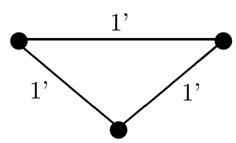
Lemma 0.2.6 *In a network N we have $N(x, y) = N(y, x)^\smile$ for all $x, y \in N$.*

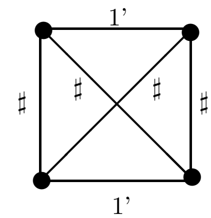
Proof. $(N(x, y), N(y, x), N(x, x))$ is consistent. But $N(x, x) = 1'$. So $N(x, y) = N(y, x)^\smile$. ■

So in diagrams, we only need label arcs one way. And if atoms are self-converse, we don't need arrows. Labels on reflexive arcs ($N(x, x)$) are 1', so we don't need to write this.

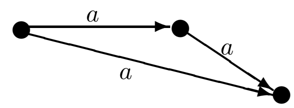
Examples of networks

Here are 2 networks over (the atom structure of) the small algebra $\mathcal{I}$ (example 0.1.10):





And a network over the point algebra $\mathcal{P}$:

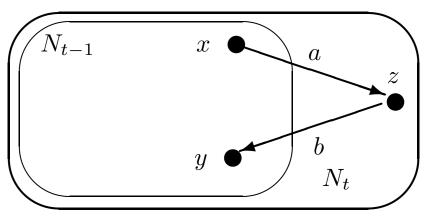


0.2.2 Game on atomic networks

Definition 0.2.7 (game) Fix $n \leq \omega$, an atom structure $\mathcal{S}$, and a pre-network N (over $\mathcal{S}$). Players $\forall$ and $\exists$ play a game $\boldsymbol{G_n(N, \mathcal{S})}$ with n rounds: $0, 1, \ldots, t, \ldots$, for $t < n$, in which pre-networks $N_0, N_1, \ldots$ over $\mathcal{S}$ are built. G_0 is a trivial game with no rounds, which $\exists$ wins by default. For $n > 0$, $G_n(N, \mathcal{S})$ is played as follows.

- In round 0, $\exists$ places a copy of N on the board (so $N_0 \cong N$).
- In round $t > 0$ (with $t < n$), if N_{t-1} has been built so far, $\forall$ chooses $x, y \in N_{t-1}$ and atoms a, b of $\mathcal{S}$ such that $(a, b, N_{t-1}(y, x))$ is consistent.
  $\exists$ must respond with a pre-network $N_t \supseteq N_{t-1}$ with a node $z \in N_t$ such that $N_t(x, z) = a$ and $N_t(z, y) = b$.

We can assume $z \notin N_{t-1}$ if we want.



Winning: After n rounds, the game is over; if all N_t played are *networks* then $\exists$ wins. Otherwise, $\forall$ wins.

0.2.3 Winning strategies

Definition 0.2.8 A *winning strategy* for a player in $G_n(N, \mathcal{S})$ is a set of rules that, if followed, always lead to a win for that player.

We need not formalise the notion of strategy. We note the following straightforward lemma:

Lemma 0.2.9 *Fix $n < \omega$ and a pre-network N over an atom structure $\mathcal{S}$. If $n = 1$ then $\exists$ has a winning strategy in $G_n(N, \mathcal{S})$ iff N is a network. If $n > 0$, the following are equivalent:*

1. *$\exists$ has a winning strategy in $G_{n+1}(N, \mathcal{S})$,*
2. *for any move $\forall$ makes in round 0 of $G_{n+1}(N, \mathcal{S})$, $\exists$ has a response leading to a pre-network N_1 such that she has a winning strategy in $G_n(N_1, \mathcal{S})$.*

0.2.4 Games and representations

Theorem 0.2.10 *Let $\mathcal{A}$ be a countable atomic relation algebra. Let I denote the unique (up to isomorphism) 1-node network over $At\mathcal{A}$.*

1. *If $\exists$ has a winning strategy in $G_\omega(I, At\mathcal{A})$, then $\mathcal{A}$ is representable.*
2. *The converse holds if $\mathcal{A}$ is finite.*

Proof. We first prove (1). Let the game commence. Let $\exists$ use her winning strategy, and let $\forall$ make every possible move at some stage of play (he can do so because $At\mathcal{A}$ is countable). Play builds a chain of networks

$$I = N_0 \subseteq N_1 \subseteq \cdots$$

Let $N = \bigcup_{t<\omega} N_t$. Define an equivalence relation $\sim$ on $dom(N)$ by

$$x \sim y \iff N(x, y) = 1\text{'}.$$

For $r \in \mathcal{A}$, let $\widehat{r} = \{(x/\!\sim, y/\!\sim) : x, y \in N,\ N(x, y) \leq r\}$. Then

$$\widehat{\mathcal{A}} = \langle\{\widehat{r} : r \in \mathcal{A}\}, \cup, \cap, \setminus, \emptyset, N/\!\sim \times N/\!\sim, =, ^{\smile}, ;\rangle$$

is a proper relation algebra, and $r \mapsto \widehat{r}$ is an isomorphism from $\mathcal{A}$ to $\widehat{\mathcal{A}}$. So $\mathcal{A}$ is representable. For more details see Hirsch and Hodkinson. 1997c, Hirsch and Hodkinson. 1997a.

Before proving (2), we need a lemma. Assume $\mathcal{A}$ is finite, and (without loss of generality) a proper relation algebra of the form $\mathcal{A} = \langle A, \cup, \cap, \setminus, \emptyset, U \times U, =, ^{\smile}, ;\rangle$ on a set U (so $A \subseteq \wp(U \times U)$).

Lemma 0.2.11
a) If $x, y \in U$ then there is a unique atom $\alpha(x, y) \in \mathcal{A}$ with $(x, y) \in \alpha(x, y)$.
b) Let $X \subseteq U$. Then $Nwk(X) \stackrel{\text{def}}{=} (X, \alpha \restriction X \times X)$ is a network over $At\mathcal{A}$.

Proof.
a) Let $a \in A$ be $\leq$-minimal with $(x, y) \in a$: a exists as $\mathcal{A}$ is finite and $(x, y) \in U \times U = 1^{\mathcal{A}}$. Check that a is an atom of $\mathcal{A}$.

If a, b are distinct atoms containing (x, y), then $(x, y) \in a \cdot b = \emptyset$, contradiction.
b) Easy. ■

Now we prove (2). $\exists$'s strategy in $G_\omega(I, At\mathcal{A})$ is to ensure that each N_t played satisfies:

- $dom(N_t) \subseteq U$
- $(x, y) \in N_t(x, y)$ for all $x, y \in U$.

That is, $N_t = Nwk(dom(N_t))$. This is a network, by lemma 0.2.11. So if she can do it, it's a winning strategy.

It's easy to do in round 0: because 1' is an atom, $\exists$ can take $N_0 = Nwk(x) \cong I$ for any $x \in U$. If she has done it as far as N_{t-1}, let $\forall$ pick $x, y \in N_{t-1}$ and atoms a, b with $(a, b, N_{t-1}(y, x))$ consistent. So in $\mathcal{A}$ we have $N_{t-1}(x, y) \leq a\,;b$. By assumption, $(x, y) \in N_{t-1}(x, y)$, so $(x, y) \in a\,;b$. So there is $z \in U$ with $(x, z) \in a$ and $(z, y) \in b$. $\exists$ defines $N_t \stackrel{\text{def}}{=} Nwk(dom(N_{t-1}) \cup \{z\})$. ■

Finite vs. infinite games

Theorem 0.2.12 *Let $\mathcal{S}$ be a finite atom structure and N a finite pre-network over $\mathcal{S}$. The following are equivalent:*
1. $\exists$ has a winning strategy in $G_\omega(N, \mathcal{S})$.
2. $\exists$ has a winning strategy in $G_n(N, \mathcal{S})$ for all finite n.

Proof. $1 \Rightarrow 2$: trivial.
$2 \Rightarrow 1$: $\exists$'s strategy in $G_\omega(N, \mathcal{S})$ is:
$(*)$ 'in each round, t, ensure I have a winning strategy in $G_n(N_t, \mathcal{S})$ for infinitely many n'.

$(*)$ is true in round 0 of $G_\omega(N, \mathcal{S})$ by assumption. Inductively assume that $(*)$ holds for N_{t-1}, and let $\forall$ make his move in round t of $G_\omega(N, \mathcal{S})$. We can evidently regard this move as $\forall$'s move in round 0 of a play of $G_{n+1}(N_{t-1}, \mathcal{S})$ for any $n > 0$. By $(*)$, $\exists$ has a winning strategy in this game for each n in some infinite set $X \subseteq \omega \setminus \{0\}$. Let the strategy's response to this move of $\forall$ in round 0 be N_t^n, say. Clearly (cf. lemma 0.2.9), $\exists$ has a winning strategy in $G_n(N_t^n, \mathcal{S})$ for each $n \in X$. As $\mathcal{S}$ and the N_t^n are finite, there is an infinite set $Y \subseteq X$ such that the N_t^n $(n \in Y)$ are all isomorphic. $\exists$ lets $N_t = N_t^n$ (any $n \in Y$). This keeps $(*)$.

$(*)$ implies each N_t is a network. So this strategy is winning for $\exists$. ■

Expressing winning strategy in logic

Theorem 0.2.13 *Let $n < \omega$. There is a first-order sentence σ_n such that for any atom structure $\mathcal{S}$, $\exists$ has a winning strategy in $G_n(I, \mathcal{S})$ iff $\mathcal{S} \models \sigma_n$. And $\{\sigma_n : n < \omega\}$ is recursive.*

Proof. For each finite set X, we write a formula φ_n^X with free variables in $\{v_{xy} : x, y \in X\}$, such that for any pre-network N with domain X,

$$\exists \text{ has a winning strategy in } G_n(N, \mathcal{S}) \iff \mathcal{S} \models \varphi_n^X(N).$$

The notation $\mathcal{S} \models \varphi_n^X(N)$ means that we evaluate φ_n^X in $\mathcal{S}$ with the variable v_{xy} assigned to the atom $N(x, y) \in \mathcal{S}$, for each $x, y \in X$.

We define φ_n^X by induction on n. We let $\varphi_0^X = \top$, and

$$\varphi_1^X \stackrel{\text{def}}{=} \bigwedge_{x \in X} (v_{xx} = 1') \wedge \bigwedge_{x,y,z \in X} C(v_{xy}, v_{yz}, v_{zx})).$$

Clearly, $\mathcal{S} \models \varphi_1^X(N)$ iff N is a network, which by lemma 0.2.9 is iff $\exists$ has a winning strategy in $G_1(N, \mathcal{S})$.

Inductively, given φ_n^X for $n > 0$, we pick $z \notin X$, let $Z = X \cup \{z\}$, and let φ_{n+1}^X be

$$\bigwedge_{x,y \in X} \forall ab \Big(C(a, b, v_{yx}) \to \exists_{w \in X} v_{wz}, v_{zz}, v_{zw} (v_{xz} = a \wedge v_{zy} = b \wedge \varphi_n^Z) \Big).$$

($\exists_{w \in X} v_{wz}$ denotes a string of quantifiers $\exists v_{wz}$ for all $w \in X$.) By lemma 0.2.9 and the inductive hypothesis, $\exists$ has a winning strategy in $G_{n+1}(N, \mathcal{S})$ iff whatever move $\forall$ makes in round 0, $\exists$ has a response that leaves a network N', which we can assume has domain Z, such that $\mathcal{S} \models \varphi_n^Z(N')$. By examining the rules governing $\forall$'s moves and $\exists$'s responses, we see that this holds iff $\mathcal{S} \models \varphi_{n+1}^X(N)$, as required. This completes the induction; the theorem follows by letting $\sigma_n = \varphi_n^{\{x\}}(1'/v_{xx})$ (any x). ■

Axioms for representability

We can now axiomatise the finite representable relation algebras (in which 1' is an atom).

Definition 0.2.14 For $n < \omega$, λ_n is the sentence obtained by translating σ_n into the language of relation algebras, by:

- relativising quantifiers to atoms ('x is an atom' is definable by $\forall y(y < x \leftrightarrow y = 0)$).
- replacing $C(x, y, z)$ by $z^\smile \leq x \,;y$.

The λ_n are essentially the 'Lyndon conditions' of Lyndon 1950. The following is immediate:

Lemma 0.2.15 *For any atomic relation algebra $\mathcal{A}$, $\mathcal{A} \models \lambda_n$ iff $At\mathcal{A} \models \sigma_n$.*

Theorem 0.2.16 (ess. Lyndon, Lyndon 1950) *Let $\mathcal{A}$ be a finite relation algebra (with $1' \in At\mathcal{A}$). Then $\mathcal{A}$ is representable iff $\mathcal{A} \models \lambda_n$ for all finite n.*

Proof. By theorem 0.2.10, $\mathcal{A}$ is representable iff $\exists$ has a winning strategy in $G_\omega(I, At\mathcal{A})$.
By theorem 0.2.12, this is iff $\exists$ has a winning strategy in $G_n(I, At\mathcal{A})$ for all finite n.
By theorem 0.2.13, this is iff $At\mathcal{A} \models \sigma_n$ for all n.
By lemma 0.2.15, this is iff $\mathcal{A} \models \lambda_n$ for all n. ■

So the equations defining relation algebras together with the λ_n (all $n < \omega$) axiomatise the finite algebras in RRA in which 1' is an atom. (For when it isn't, see Hirsch and Hodkinson. 1997b.)

0.2.5 What about infinite relation algebras?

Answer: One direction of theorem 0.2.16 still holds.

Fact 0.2.17 RRA is elementary (first-order axiomatisable). In fact, it is a variety—equationally axiomatised Tarski 1955.

Theorem 0.2.18 *If $\mathcal{A}$ is any atomic relation algebra of which 1' is an atom, and $\mathcal{A} \models \lambda_n$ for all finite n, then $\mathcal{A}$ is representable.*

Proof (sketch). By lemma 0.2.15, $At\mathcal{A} \models \sigma_n$ for all n. By theorem 0.2.13, $\exists$ has a winning strategy in $G_n(I, At\mathcal{A})$ for all $n < \omega$. By (eg) saturation (see Chang and Keisler. 1990, Hodkins 1993), there is countable $\mathcal{B} \equiv \mathcal{A}$ such that $\exists$ has a winning strategy in $G_\omega(I, \mathcal{B})$; for details, see Hirsch and Hodkinson. 1997b. By theorem 0.2.10, $\mathcal{B} \in$ RRA. By fact 0.2.17, $\mathcal{A} \in$ RRA, too. ■

The converse fails, even for atomic relation algebras (have a look at Lemma 0.2.11). But arbitrary representable relation algebras (RRA) can be axiomatised by games in a similar way. See Lyndon 1956, Hirsch and Hodkinson. 1997c, Hirsch and Hodkinson. 1997a.

(9) Show that $\exists$ has a winning strategy in $G_\omega(I, At\mathcal{P})$ where $\mathcal{P}$ is the point algebra of example 0.1.11. [Hint: if stuck, use the known representation of $\mathcal{P}$ and the proof of theorem 0.2.10(2).]

(10) Show that $\exists$ has a winning strategy in $G_\omega(I, At\mathcal{J})$ where $\mathcal{J}$ is the 4-atom algebra of example 0.1.13. [Hint: use atom r to fill in the edges.]

(11) Show that $\forall$ has a winning strategy in $G_\omega(I, At\mathcal{K})$ where $\mathcal{K}$ is McKenzie's 4-atom algebra (example 0.1.12).

0.3 Rainbow construction, Monk's theorem

Here, we are going to prove what is often called the most important negative result in algebraic logic:

Theorem 0.3.1 (Monk, Monk 1964) RRA *is not finitely axiomatisable in first-order logic.*

The idea is to construct finite relation algebras $\mathcal{A}_n$ ($2 \leq n < \omega$) such that for all n:

- $\forall$ has a winning strategy in $G_\omega(I, At\mathcal{A}_n)$
- $\exists$ has a winning strategy in $G_n(I, At\mathcal{A}_n)$.

Lemma 0.3.2 *Monk's theorem follows.*

Proof. Assume RRA is finitely axiomatised, by σ, say. Let

$$T = \{\text{RA axioms}\} \cup \{\lambda_n : n < \omega\} \cup \{\alpha, \neg\sigma\},$$

where α expresses atomicity: $\forall x(x > 0 \rightarrow \exists y(y \leq x \wedge \forall z(z < y \leftrightarrow z = 0)))$. Then by theorem 0.2.13 and lemma 0.2.15, for any finite $\Sigma \subseteq T$ there is $n < \omega$ such that $\mathcal{A}_n \models \Sigma$. By first-order compactness, T has a model, say $\mathcal{B}$. By theorem 0.2.18, $\mathcal{B} \in$ RRA, contradicting $\mathcal{B} \models \neg\sigma$. ■

0.3.1 Rainbow algebras

We build the $\mathcal{A}_n$ using a simplification of a recent idea due to R. Hirsch Hirsch 1995, called the *rainbow construction*. The atom structure $At\mathcal{A}_n$ ($n \geq 2$) is given by:

Atoms:

- 1'
- g_i for $i \leq n$ (greens)
- w (white)
- y, b (yellow, black)
- r_{ij} for distinct $i, j < n$ (red).

All atoms are self-converse, except reds, where $\mathsf{r}_{ij}^{\smile} = \mathsf{r}_{ji}$. The *inconsistent* triangles are all Peircean transforms of:
• $(x, y, 1')$ if $x \neq y^{\smile}$.
• green ones: $(\mathsf{g}_i, \mathsf{g}_j, \mathsf{g}_k)$ for any $i, j, k \leq n$
• $(\mathsf{g}_i, \mathsf{g}_j, \mathsf{w})$ (any $i, j \leq n$), (y,y,y), (y,y,b)
• any red triangle except those of the form $(\mathsf{r}_{ij}, \mathsf{r}_{jk}, \mathsf{r}_{ki})$.

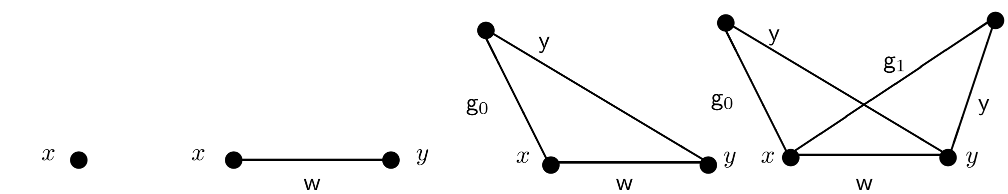
This defines a finite relation algebra There are finitely many atoms, so any relation algebra with this atom structure is finite. We check that it's a relation algebra atom structure (definition 0.1.7):

- $x^{\smile\smile} = x$ for all atoms x: clear.
- $x = y^{\smile}$ iff $(x, y, 1')$ consistent: clear.
- if (x, y, z) is consistent then so are (y, z, x) and $(z^{\smile}, y^{\smile}, x^{\smile})$ ('Peircean transforms'): by definition.
- diamond completion (for associativity): this will follow if we show $\exists$ has a winning strategy in $G_4(I, \mathcal{A}_n)$. We do this in theorem 0.3.4.

0.3.2 ∀'s winning strategy

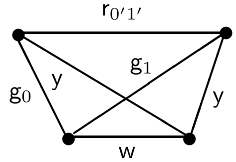
Theorem 0.3.3 *$\forall$ has a winning strategy in $G_\omega(I, At\mathcal{A}_n)$ ($\forall n$).*

Proof. His first 4 moves are:

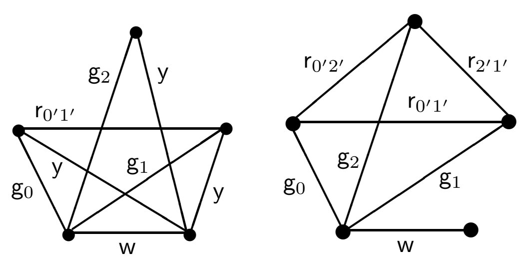


round 0 — round 1: $\forall$ picks $x, x, \mathsf{w}, \mathsf{w}$ — round 2: $\forall$ picks $x, y, \mathsf{g}_0, \mathsf{y}$ — round 3: $\forall$ picks $x, y, \mathsf{g}_1, \mathsf{y}$

Now $\exists$ has to fill in the missing edge. Which atom can she choose? Only a red—say, $\mathsf{r}_{0'1'}$ (below left). $\forall$ continues by picking $x, y, \mathsf{g}_2, \mathsf{y}$ (below middle), and $\exists$ has to fill in 2 edges with reds (below right, yellow edges omitted for clarity):



round 3



round 4

And so on: $\forall$ plays $x, y, \mathsf{g}_{i-2}, \mathsf{y}$ in round i. $\exists$ will run out of reds in round $n + 2$. ■

0.3.3 ∃'s winning strategy

Theorem 0.3.4 *$\exists$ has a winning strategy in $G_{n+2}(I, At\mathcal{A}_n)$ for each $n \geq 2$.*

Note: $n + 2 \geq 4$, giving diamond completion.

Proof. At the start of some round t, $0 < t < n+2$, suppose the current pre-network is $N = N_{t-1}$. We suppose inductively that $\exists$ has ensured that N is a network. Note that $|N| \leq t \leq n+1$. Let $\forall$ move in round t by picking nodes x, y and atoms a, b with $(a, b, N(y, x))$ consistent. We can assume $\forall$ never plays so that $\exists$ doesn't need to add a node. So

- N is strict: if $N(v, w) = 1'$ then $v = w$,
- $a, b \neq 1'$.

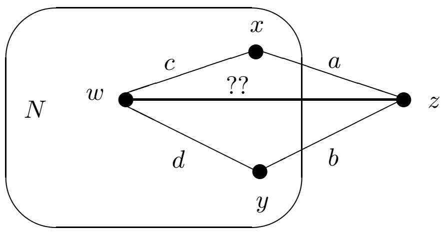
$\exists$ makes N_t by adding a new node z, defining $N_t(x, z) = a$, $N_t(z, y) = b$, $N_t(z, z) = 1'$, and then choosing $N_t(w, z)$ for all $w \in N \setminus \{x, y\}$. Of course, she defines $N_t(z, w) = N_t(w, z)^{\smile}$.

What does $\exists$ choose for the $N_t(w, z)$?

1. Use white if possible.
2. If not, use black if possible.
3. If not, use a suitable red.

Here, 'if possible' means 'if triangles w, x, z and w, y, z are rendered consistent'.

In more detail



Let $N(w, x) = c \ldots$

$\ldots$ and $N(w, y) = d$.

As N is strict, $c, d \neq 1'$.

1. If a, c are not both green, and b, d are not both green, then because $a, b, c, d \neq 1'$ she can pick $?? = N_t(w, z) = \mathsf{w}$ (white).
2. Otherwise, if a, c not both yellow, and b, d not both yellow, then, again bearing in mind that $a, b, c, d \neq 1'$, she lets $?? = \mathrm{b}$ (black).
3. Otherwise, we can assume a, c are green, and b, d, yellow. $\exists$ must choose a red for $N_t(w, z)$ for each w in the set
$$R = \{w \in N : N(w, x) \text{ green}, N(w, y) = \mathsf{y}\}.$$
If $v, w \in R$, $v \neq w$, then because N is (inductively) a strict network, $N(v, w)$ is red. Indeed, the rule for consistency of red atoms ensures that there is a function $\rho : R \to n$ with
$$N(v, w) = \mathsf{r}_{\rho(v), \rho(w)} \quad \text{(all distinct } v, w \in R).$$
(If $R = \{w\}$, we let $\rho(w) = 0$.) Note that $|rng(\rho)| \leq |N| - 2 \leq n - 1$, so there is $i \in n \setminus rng(\rho)$. $\exists$ lets $?? = N_t(w, z) = \mathsf{r}_{\rho(w), i}$ for all $w \in R$.

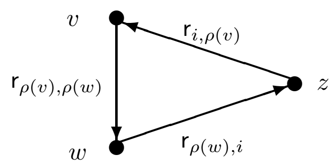
This completes the labelling and she defines N_t to be the result.

Is N_t a network?

It's enough to check that all triangles (v, w, z) with $v, w \in N$, $v \neq w$, are consistently labelled.

Triangles of the form (x, y, z), (v, x, z), (v, y, z) are clearly consistent.

Consider a triangle (v, w, z) where $v, w \notin \{x, y\}$. $\exists$ labelled (v, z) and (w, z) with w, b, or a red. All triangles with 2 edges like this are consistent, except perhaps all-red triangles. But $\exists$ only uses reds for (v, z) and (w, z) when $v, w \in R$. In this case, we have



and this is consistent. ■

0.3.4 Other results

The full rainbow construction and variations can be used to prove many more negative results, such as:

1. The class of relation algebras with representations satisfying the condition given in lemma 0.2.11(1) is not elementary Hirsch 1995, Hirsch and Hodkinson. 1997b.
2. RRA is not closed under 'completions' Hodkinson 1997.
3. (Hence) RRA is not Sahlqvist-axiomatisable Venema 1997.
4. It is undecidable whether a finite relation algebra is representable Hirsch and Hodkinson 1999: To appear.

Similar results to at least (1–3) for 'cylindric algebras' (higher-arity relations) can be proved too.

5. For all $n \geq 5$ there is a first-order sentence that can be proved with n variables but not with $< n$ Hodkinson and Maddux . It also holds for $n = 4$ Tarski and Givant 1987.
6. For $n \geq 5$, the classes RA_n, $\mathbf{S}\mathsf{RaCA}_n$, and (for $m \geq 3$, $m + 2 \leq n$) $\mathbf{S}\mathsf{Nr}_m\mathsf{CA}_n$ are not finitely axiomatisable Hirsch and Hodkinson. Submitted, 1999b.

0.4 Finite base property

Relation algebras, even non-representable ones, are surprisingly complicated. We mentioned that it is undecidable whether a finite relation algebra is representable, and other negative results (§0.1.4). Tarski proved that the equational theory of RA is undecidable: see Givant and Németi. 1997.

One of the chief 'causes' is associativity. So researchers like Maddux proposed weakening the associative law. The semi-associative law is $(x\,;1)\,;1 = x\,;(1\,;1)$. Even assuming only this, the equational theory of the resulting class, SA, is undecidable.

0.4.1 Weak associativity

Maddux Maddux 1982 proposed an even weaker law,

$$([x \cdot 1']\,;1)\,;1 = [x \cdot 1']\,;(1\,;1)$$

The class of all algebras satisfying this law and (R0), (R2)–(R7) of definition 0.1.3 is called WA ('weakly associative algebras'). The equational theory of WA is decidable. Every weakly associative algebra is representable if we allow special 'relativised representations'. And, as we now prove, finite weakly associative algebras have finite relativised representations Hodkinson and Németi. 1999.

0.4.2 Relativised representations

These provide a weak version of 'proper relation algebra' or 'ordinary' representation: all operations are relativised to (intersected with) the unit.

Definition 0.4.1 A *relativised proper relation algebra* is an algebra $\langle A, \cup, \cap, -, \emptyset, 1, 1', \smile, ;\rangle$, where $A \subseteq \wp(U \times U)$ for some set U, and for $r, s \in A$,

$$\begin{aligned} -r &= 1 \setminus r && \text{(as before)}, \\ 1' &= \{(x,y) \in 1 : x = y\}, \\ r^{\smile} &= \{(x,y) \in 1 : (y,x) \in r\}, \\ r\,;s &= \{(x,y) \in 1 : \exists z((x,z) \in r \wedge (z,y) \in s))\}. \end{aligned}$$

1 (the 'unit') need not be an equivalence relation on U.

Definition 0.4.2 A *relativised representation* of a relation-type algebra $\mathcal{A}$ is an isomorphism from $\mathcal{A}$ onto a relativised proper relation algebra.

WA and relativised representations

Weakly associative algebras are associated with relativised representations with reflexive symmetric unit.

Theorem 0.4.3 (Maddux, Maddux 1982) *Let $\mathcal{A}$ be a relation-type algebra. $\mathcal{A} \in$ WA iff $\mathcal{A}$ has a relativised representation in which the unit is reflexive and symmetric.*

We will prove a related result:

Theorem 0.4.4 (Hodkinson and Németi. 1999) *Let $\mathcal{A}$ be a* finite *relation-type algebra. $\mathcal{A} \in$ WA iff $\mathcal{A}$ has a relativised representation* with finite base *in which the unit is reflexive and symmetric.*

This is known as the finite base property for WA.

(12) Prove $\Leftarrow$ without using theorem 0.4.3.

0.4.3 Herwig's theorem

Before proving theorem 0.4.4, we need some preliminaries.

Definition 0.4.5 Let M, N be structures for a finite relational language L.

1. A subset $X \subseteq M$ is *live* if $|X| = 1$ or there are $a_1, \ldots, a_n \in M$ and n-ary $R \in L$ such that $M \models R(a_1, \ldots, a_n)$ and $X \subseteq \{a_1, \ldots, a_n\}$.
2. M is *packed* if any two elements of M form a live set.
3. A *partial isomorphism* of M is a one-one partial map $f : M \to M$ such that for any $a_1, \ldots, a_n \in dom(f)$ and n-ary $R \in L$, $M \models R(a_1, \ldots, a_n) \leftrightarrow R(f(a_1), \ldots, f(a_n))$.
   A partial isomorphism is an *automorphism* if it's bijective.
4. A *homomorphism* from M to N is a map $f : M \to N$ such that for any $a_1, \ldots, a_n \in M$ and n-ary $R \in L$, if $M \models R(a_1, \ldots, a_n)$ then $N \models R(f(a_1), \ldots, f(a_n))$.

Theorem 0.4.6 (Herwig, Herwig. 1998.) *Let M be a finite structure in a finite relational language. There is a finite structure $M^+ \supseteq M$ such that*

1. *Any partial isomorphism of M extends to an automorphism of M^+.*
2. *If $X \subseteq M^+$ is live then $g(X) \stackrel{\text{def}}{=} \{g(x) : x \in X\} \subseteq M$ for some automorphism g of M^+.*
3. *If N is a packed structure, and $f : N \to M^+$ is a homomorphism, then there exists a homomorphism $f^- : N \to M$.*

0.4.4 Finite base property for WA

Proof of theorem 0.4.4 $\Rightarrow$. Let $\mathcal{A}$ be a finite weakly associative algebra. We assume for simplicity that 1' is an atom of $\mathcal{A}$. Regarding $At\mathcal{A}$ as a binary relational language, we define a finite $At\mathcal{A}$-structure M as follows. Pick one representative of each isomorphism type of strict network over $At\mathcal{A}$. The copies are assumed pairwise disjoint. Strict networks are defined as for relation algebras. Let the domain of M be the set of all the nodes of these copies. Interpret the relation symbols of

$At\mathcal{A}$ in M by:

$$M \models a(x, y) \quad \text{iff} \quad \exists N{\in}M(x, y \in N \wedge N(x, y) = a).$$

M is thus a finite $At\mathcal{A}$-structure. Let $M^+ \supseteq M$ be as in Herwig's theorem. Let $h : \mathcal{A} \to \wp(M^+ \times M^+)$ be given by

$$h(r) = \{(x, y) : x, y \in M^+, M^+ \models a(x, y) \text{ for some atom } a \leq r\}.$$

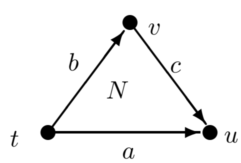
We show that h is a relativised representation of $\mathcal{A}$ (with range a relativised proper relation algebra with base M^+). Let $r, s \in \mathcal{A}$.

- h is 1–1: if $r \neq s$, take an atom $a \leq r \oplus s$ (symmetric difference). There are $x, y \in M$ with $M \models a(x, y)$. Then $(x, y) \in h(r) \oplus h(s)$.
- $h(r + s) = h(r) \cup h(s)$, $h(r \cdot s) = h(r) \cap h(s)$: clear.
- $h(-r) = h(1) \setminus h(r)$, $h(1') = \{(x, x) : (x, x) \in h(1)\}$, $h(r^\smile) = \{(x, y) \in 1 : (y, x) \in h(r)\}$, $h(1)$ is reflexive and symmetric — use liveness.

(13) Check the above.

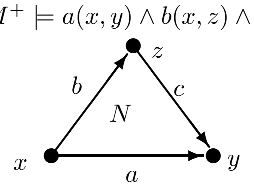
Finally we check $h(r\,;s) = \{(x, y) \in h(1){:}\exists z((x, z) \in h(r) \wedge (z, y) \in h(s))\}$.

⊆: If $(x, y) \in h(r\,;s)$, there's an atom $a \leq r\,;s$ with $M^+ \models a(x, y)$. So certainly $(x, y) \in h(1)$. Also, $\{x, y\}$ is live. Let g be an automorphism of M^+ with $g(x), g(y) \in M$. Then $M \models a(g(x), g(y))$. Now we may take atoms $b \leq r$, $c \leq s$ with $a \leq b\,;c$. So there's a network N in M with nodes t, u, v, say:



Clearly, $t \mapsto g(x), u \mapsto g(y)$ is a partial isomorphism of M. So it extends to an automorphism f of M^+. Let $z = g^{-1}f(v)$. Then $M^+ \models b(x, z) \wedge c(z, y)$. So $(x, z) \in h(r)$ and $(z, y) \in h(s)$, as required.

⊇: Let $x, y, z \in M^+$ with $(x, y) \in h(1)$, $(x, z) \in h(r)$, $(z, y) \in h(s)$. We require $(x, y) \in h(r\,;s)$. Let $b \leq r, c \leq s$, and a be atoms with $M^+ \models a(x, y) \wedge b(x, z) \wedge c(z, y)$. Then the substructure



is packed, and the inclusion map is a homomorphism: $N \to M^+$. By Herwig's theorem there's a homomorphism $x \mapsto x', y \mapsto y', z \mapsto z'$ from N into M. So

$$M \models a(x', y') \wedge b(x', z') \wedge c(z', y').$$

So by definition of M, $(b, c, a^\smile)$ is consistent. So $a \leq b\,;c \leq r\,;s$. By definition of h, $(x, y) \in h(r\,;s)$, as required.

■

Remarks

1. The size of the finite representation of $\mathcal{A}$ can be bounded recursively in $|\mathcal{A}|$.
2. The same idea proves that any universal sentence valid in finite WAs is valid in all WAs. This gives the 'finite model property' for *arrow logic* in its relativised interpretation.

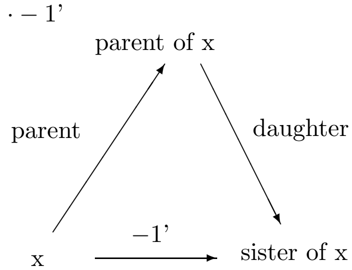
3. The method applies in several other situations. E.g., can find a finite relativised representation (of a certain kind) for any finite relation algebra, and associativity gives it more properties. See Grädel. 1998, Hirsch and Hodkinson. Submitted, 1999a.

(14) Let h be a relativised representation of $\mathcal{A} \in \mathsf{WA}$. Show that the unit, $h(1)$, is a *reflexive and symmetric* binary relation on the base of h.

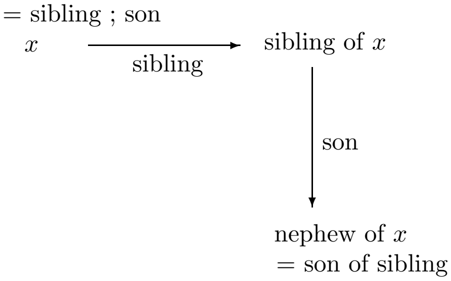
0.5 Answers to selected exercises

Exercise 1

1. sister = (parent ; daughter) $\cdot -$ 1'



2. nephew = sibling ; son



3. aunt = parent ; sister

Exercise 3

If $\mathcal{S} = (X, Id, ^\smile, C)$ is a relation algebra atom structure and $a \in Id$, show $a^\smile = a$.

We know $(a, a^\smile, b) \in C$ for some $b \in Id$. As C is closed under Peircean transforms, $(a^\smile, b, a) \in C$. But $a \in Id$, so $a^\smile = b^\smile$. So $a = a^{\smile\smile} = b^{\smile\smile} = b$. Thus, $(a, a^\smile, a) \in C$. As C is closed under Peircean transforms, $(a^\smile, a^\smile, a) \in C$. But $a \in Id$, so $a^\smile = a^{\smile\smile} = a$.

Exercise 4

The algebra with consistent triangles $(x, x^\smile, 1')$ and $(\sharp, \sharp, \sharp)$ is a representable relation algebra: one representation is on a set with three elements, interpreting 1' as $=$ and $\sharp$ as $\neq$.

Exercise 5

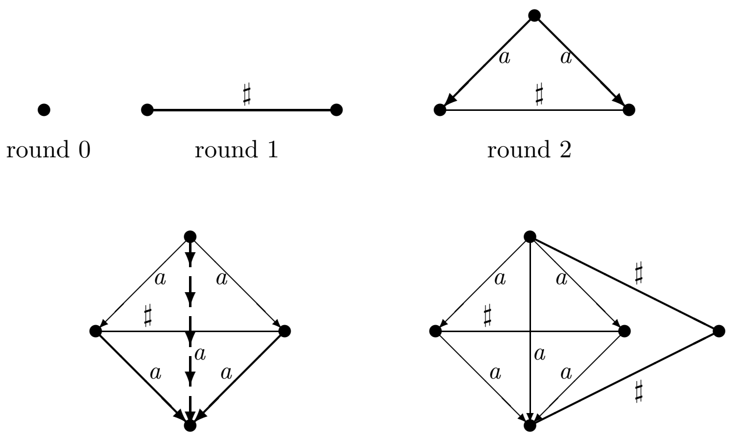
The consistent triangles are $(x, x, 1')$ for $x \in \{1', e, n\}$, (e, e, n), (n, n, e). The algebra with atoms $1', e, n$ is closed under the operations and the pentagon is a representation of it.

Exercise 6

In any representation of the point algebra, a is interpreted as an (irreflexive) dense partial order. ($a\,; a = a$ gives transitivity and density, and $a \neq a^\smile$ gives antisymmetry.) The partial order has at least 2 elements, because a is interpreted as a non-empty relation. But there is no such finite order.

Exercise 11

$\forall$ can win $G_5(I, \mathcal{K})$ as follows:



round 0 round 1 round 2

round 3: ∃ must use 'a' round 4: ∃ stuck

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A Morphological Structure and Semantics of the Georgian So-called Passive Forms

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The voice category, being one of the main verbal categories in a language, is traditionally considered to be discrete. It means that if there are two voice forms (active and passive) in a language, a verbal form may belong either to the active or to the passive voice; if there are three kinds of voice forms (active, passive and medium), then a verbal form may belong to one of these three. There is no unitary definition of the category, and even main points of the existing definitions (what determines the discrete nature of the category) are not homogeneous.

In the Georgian linguistic literature, voice is defined based on the relation between the action on the one hand, and subject and/or object on the other hand. This definition is characteristic for relational grammar Perlmutter and Postal 1977 where the essential thing for the defining voice is the change in grammatical relations (the so-called conversion); i.e. the noun, corresponding to the direct object in the transitive verbal construction, is converted into the subject. Thus in the passive, object promotion takes place because the noun, corresponding to the direct object on one level, becomes the subject on the next level. Passive forms are considered to be intransitive (Perlmutter and Postal 1977, see also Keenan 1976), though according to Comrie 1977, subject demotion takes place in the passive as well.

The definition of voice suggested by the founder of Georgian grammar, Shanidze 1953, and is also established on the relations between the action and the subject and/or object: "Voice is a verbal form which shows what kind of a relation there exists between the grammatically active person (i.e. subject) and the action expressed by the verb". A. Shanidze gives several particulars of those relations:

- the action is brought about by the subject and directed to the other person (or object);
- the subject is usually a patient;
- the action is brought about by the subject in such a way, that it is neither directed to another person (or object), nor is the subject a patient.

Thus, according to Shanidze 1953, there are various kinds of relations between the verbal action and the subject. It is clear that those relations exist on the semantic level and they could be realized in various ways on the syntactic or morphological levels. The relations between those three levels concerning passive voice are quite complicated. In order to simplify the complexity of the phenomena, a difference has been drawn between diathesis and voice: diathesis was defined as the relation of the semantic data to the syntactic level, and voice was defined as morphologically marked diathesis (Kholodovich 1974).

Unfortunately, we do not share the opinion that the differentiation has simplified the problem. In order to define voice, we intend to specify what kind of voice a verbal form belongs to based on the syntactic structure. It is, however, also important to qualify the type of semantic relations of the **subject** to the action expressed by the verbal form. Note that it is often impossible for the semantic relations to be qualified unambiguously, i.e., it is difficult to say whether the subject of a syntactic construction is an agent or a patient. As an example, let us consider the verb which conveys the notion of dying. What is the relation of the subject to the action expressed by the verb? In Georgian, the subject in this case seems to be more patient-like (rather than an agent), since the verb only appears in the passive form:

(1) Êaci Êvdeba
man (nom.case) passive, root-Êvd-to die
A man is dying.

Êvdeba is the so-called unmarked passive form (we will discuss below the Georgian passive markers), the root *Êvd-* has no active forms. The Georgian verb could be compared to the English verb *to die* with analogous semantics. This verb only occurs in the non-passive form (but not active either, as it is never used in a transitive construction[1]). According to the verb form in English, the subject is, therefore, not a patient (as it is in Georgian), but more agentive (it is not an agent). The English verb is comparable to the Russian verb (*on umiraet* "he dies").

In our opinion, it was the ambiguity of semantic relations that led to the formation of a new interpretation of voice, as proposed by M. Shibatani, whose approach relies on the prototype analysis (Shibatani 1985). It is a well-known typological fact that passive constructions are semantically manifold. In a great number of languages various semantic groups of the verbs are expressed by the passive voice constructions. M. Shibatani discusses such semantic groups of passive for a wide range of languages. The same semantic groups were also analyzed by Haspelmath 1990, who studied the morphology of passive constructions. We will not go into the details of his analysis, but it is important to mention that such semantic groups are attested also among the forms of the passive voice in Georgian.

In order to explain why those forms are expressed by the same passive construction M. Shibatani defines the prototypical passive as follows:

1. primary pragmatic function: defocusing of the agent
2. semantic properties:
   a. semantic valence: the predicate has got an agent and a patient
   b. on the semantic level: the subject is affected
3. syntactic properties:
   a. syntactic encoding: agent → Ø(not encoded)
   b. difference in the valence of the predicate: in the passive equal to $(n-1)$, n being the valence of the active form
4. morphological properties: no special marker for active, only for passive

The primary pragmatic function of defocusing the agent needs to be clarified. Using the term *focus* (from which 'defocusing' is derived from), Shibatani 1985 is afraid of confusion, as the term is used by linguists in different senses. To avoid confusion, it would perhaps be desirable to use a different term, rather than the one the author prefers.

In order to know what defocusing means, we first have to define the positive term 'focusing'. According to Shibatani's explanation, every verb has its semantic frame or valence. All the entities, which correspond to the elements of semantic frame or valence, are considered as focused (since they are singled out from all other possible entities). They are singled out as essential elements requiring the listener's attention in decoding the message. These semantically encoded elements are not equally important. Some of them are more important for the speaker and they call for more attention from the listener. Others are less important and less attention is required. Language provides various morphosyntactic devices for distributing the focus strength according to the amount

[1]For semantic reasons, we do not think that examples like *He died a slow an painful death* are genuinely transitive.

of required attention. Syntactically encoded elements have various degrees of focusing, which can be expressed in a hierarchy as follows:

Subject > direct object > indirect object > oblique object

Even along the entities that occur as subjects the degrees of focusing are different. The prototypical agent is the most focused, and it is defined as the one being high in potency and volition.

In the passive constructions, the agent is mostly omitted and therefore it is fully defocused. Opposite to it, in the active constructions the agent is maximally focused. According to M. Shibatani, the category of voice is continuous; a prototypical active and a prototypical passive could be regarded as polar points of this continuum. Certain constructions are closer to one of these points, and to that degree, the resulting construction is either active or passive (or medium, when corresponding constructions have some kind of morphological or syntactic characteristics).

Thus, the constructions traditionally regarded as passive voice constructions, actually are very close to the prototypical passive, and their main characteristic feature is defocusing of the agent.

That way of regarding the passive voice constructions enables us to explain the semantic diversity, characteristic for the Georgian passive voice forms, which ought to be included in the general typological data to make the passive voice typology more complete.

As for the Georgian passive voice forms, there are three kinds of voice in Georgian: active, passive and medium. A lot of work by the Georgian scholars is dedicated to the topic and the issue thus appears to have been thoroughly studied from both the formal and semantic point of view. The passive voice affixes have been identified and it was shown that:

- the passive and active voices have distinct models of conjugation;
- the conjugation of the medium voice is not complete in the sense that it lacks some forms which are substituted by either active or passive forms;
- only passive has the special voice marker (*i*- and *e*- prefixes, -*d* suffix and Ø), active and medium do not have any voice marker.

As mentioned above, different semantic groups are distinguished among the passive voice forms. The question has been raised whether all those constructions should be considered as passive, and in fact, whether the category of voice can be singled out in Georgian.

M. Shibatani, whose approach was briefly presented above, regards these questions as pointless. In our analysis of the Georgian passive voice forms (constructions) and, especially, the different semantic groups, we are going to follow Shibatani's approach. To begin with, we have written out over one thousand clauses with passive voice forms from Georgian prose (short stories). It is significant that only about twenty constructions were relatively close to the prototypical passive, viz the examples below (and a few others).

(2) coÔa *xnit* *gaiáhra is* *nisli*
a few, little time (instr.case) passive, past root ár- to cut, that mist (nom.case)
That mist was cut for a little while.

(3) ezoÛi imedi da dumili ertad daibada
ezo-yard Ûi-'in' hope (nom.case) and silence (nom.case) together passive, past root bad-to bear
The hope and silence were born together in the yard.

This fact gives some ideas about the chronology of forming the voice category in Georgian, which have to be further elaborated. The final result, together with the typological data of other languages, will enable us to establish the universal model of the development of voice. All the semantic groups mentioned in the Georgian linguistic literature were reflected in our data. One of those groups consists of **deponents** (passive voice forms with active meaning). Deponents may be *mono-personal* or *bi-personal*. Some examples of mono-personal deponents are:

(4) *igineba(is)* he swears, *iloceba (is)* he prays
iÚepeba(is) it barks, *iÚimeba (is)* he smiles

Obviously, these forms have an active meaning, but the subject of these constructions differs from the prototypical agent. First of all, it is not high in potency and second, it acts on itself, and the addressee of the action is unknown (to be more exact, the action could have been addressed to somebody, but the addressee is defocused). Thus, the subject of the deponents is active but not as much as the prototypical agent; therefore, the agent is defocused and a passive form is used in such cases.

Bi-personal deponents are passive forms with *e*−prefix. Some of them are semantically close to reciprocals in the sense that the action of the subjects of these constructions has its addressee, which is meant to act in the same (or analogous) way:

(5) eàidaveba (is mas)
He wrestles with somebody (who in his turn wrestles back).

(6) eÊaqmateba (is mas)
He argues with somebody (who in his turn argues back).

A part of those deponents is regarded as having the semantics of "being reverted into", e.g. *emegobreba (is mas)* "He reverts to being somebody's friend". But in our opinion, the meaning is actually different: *emegobreba (is mas)* means rather "He acts in such a way as being somebody's friend". In the forms regarded above the subject is not a prototypical agent, therefore, it is defocused, and a passive form is used.

Other semantic groups expressed by the passive forms are:

1. potentials - [2]
   (7) ak ar icxovreba
   here negative particle passive root-cxovr to live
   It is impossible to live here.
   (8) aq ar Ûeisvleba
   here negative particle passive, stem Ûesvl-to enter
   It is impossible to enter here.
   (9) ar gatetrdeba Ûorani
   negative particle passive, root tetr-white raven (nom. Case)
   It is impossible to whiten a raven.
   (10) aq cxeni gaàendeba
   negative particle horse (nom. case) passive, root àen- to gallop
   It is possible to gallop a horse here.
   (11) es puri ar iàmeva
   this bread negative particle passive, root iàam- to eat
   This bread is inedible.
   (12) mas daeãereba
   he (dat case) passive, root ãer-to trust
   You can trust him.
2. forms with the meaning "to consider" :

[2]Both negative and positive forms occur quite frequently.

(13) mas me vepataÔavebi
he (dat.case) I passive, root pataÔ-small
He considers me small.

(14) mas es Üanta emdzimeba
he this bag passive, root mdzim-heavy
He considers this bag heavy.

3. forms with the meaning "to be in the mood for/to feel like":

(15) emÙereba (mas)
passive, root mÙer-to sing he (dat.case)
He feels like singing.

(16) ra meÊocneba (me)
what passive, root Êocn-to kiss I
Why am I in the mood for kissing?

4. forms expressing an involuntary action:

(17) a.mono-personal
ecineba (mas)
passive, root cin-to smile he (dat.case)
He smiles (involuntarily).
b.bi-personal
mas puri Ûemoeàama
he (dat. case) bread (nom. case) passive, root àam- to eat
He ate bread(involuntarily/unintentionally).

5. fientive forms expressing the process of becoming:

(18) dagrdzelda (is)
passive, root grdzel-long it
It became long.

(19) dapaÔaravda (is)
passive, root paÔara small it
It became small.

6. inchoative forms:

(20) dagordeba (is)
passive, root gor- to roll it(nom. case)
It starts to roll.

(21) aÚvirdeba (is)
passive root Úvir- to shout he (nom. case)
He starts to shout.

In all these semantic groups, the subjects of these constructions are affected either externally or by themselves, and it is clear that the agent is defocused.

Therefore, all the constructions of those groups are expressed by the passive forms because they share with the prototypical passive the pragmatic function of agent defocusing.

Obviously, the Georgian data need a more thorough semantic analysis since the semantic groups of passive voice forms have to be exactly defined. The final results of the research will enrich typological data of passive voice semantics and the general theory of voice category as well.

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Joint Parameterization of Honorifics and Terms of Address in Kartvelian Languages

In the studies, aimed at disclosing the logic of the specific character of honorific speech, the attention has been mainly focused on extralinguistic factors. Such factors are, of course, relevant; but this does not imply that linguistic means themselves are just an appropriate inventory and nothing else. In this paper we argue that mechanisms and rules, provided by a language, are basic in these cases. This can be highlighted with the study of honorifics and terms of address in Kartvelian languages (here, our discussion is based only on the Georgian and Zan evidence).

Honorifics can be defined as grammatical codifications of individuals' social relations. The **T/V** distinctions in the pronominal system of Kartvelian languages are represented by the second person pronouns: Geor. *šen* /Zan *si* and *tkven /tkva*. As for their distribution according to the **T/V** distinctions, the formers are **T** pronouns, while the latters are the **V** ones. Following Braun (1988:7) «in most languages forms of address concentrate on three word classes: (1) pronoun, (2) verb, (3) noun, supplemented by words which are syntactically dependent on them». Georgian and Zan are the languages in which the use of personal pronouns, and **T/V** ones among them, is facultative, that is, the means of reference to a collocutor have not been limited only to the **T/V** pronouns, as the verb structure in Kartvelian is also capable to render the honorific meaning (hence, the first and the second classes of words, representing forms of address, can be united in this respect in Kartvelian languages). In terms of the formal representation this capability is mainly based on the originally only plural suffix **-t** which is common for both languages. And, presently, as the **-t** is both a plural and an honorific marker, there is a need to establish rules drawing clear formal boundaries between the two grammatical meanings (plural and honorific) conveyed by the suffix in question. Their identification will highlight the logic of arrangement of honorifics and terms of address in speech acts. However, these rules do exist, and for Georgian they have already been established (Kikvidze 1999). Zan, as the language very closely related to Georgian, utilizes the similar devices, though there are certain slight differences, these contributing to the manifestation of the logic of honorific speech at large.

The identification of these rules makes it possible to state that a plural and an honorific do not always coincide in terms of their structural appearance (generally, the linguistic realization of politeness semantics has been closely connected with the concept of plural; this is well illustrated with the existence of the traditional term *pluralis honorificus*). The establishment of these difference rules is also a significant task as far as the **-t** is an element of shared use, that is Georgian and Zan do have a certain inventory for the sake of grammatical expression of politeness, though it does not possess the so well elaborated system in this respect as, for example, Japanese. That is why the identification of the said rules, which exist but which have not so far been observed, is significant both from the formal and functional points of view.

Rule-1 for these languages is that, if a nominal part of a VP is in singular, while a verb is marked with **-t**, we are here dealing with an honorific only. This is a clear-cut syntactic mechanism presenting the formal boundaries between the plural and honorific **-t**'s. And this mechanism is not alien to other languages but, however, it has not been widely observed.

G-1.1 *tkven k'etil-i pirovneba-Ø-Ø x-ar-t*
you(HON) [a] kind-NOM person-NOM-SING 2-be-HON
You are a kind person.
G-1.2 *tkven k'etil-i pirovneb-eb-i x-ar-t*
you(PL) kind-NOM person-PL-NOM 2-be-PL
You are kind people.

Z-1.1 *tkva nagurapu koč-Ø-i re-Ø-t.*
you(HON) wise man-SG-NOM be-2-HON.
You are a wise man.
Z-1.2 *tkva nagurapu koč-ep-i re-Ø-t.*
you(PL) wise man-PL-NOM be-2-PL.
You are wise men.

The comprison can be rendered in G. Corbett's words describing the similar situation in Slavic languages: «The controller in question, honorific *vy* (the Russian for *tkven*), is plural but has a singular referent (cf. (1.1)). This means that plural agreement is syntactic, and singular agreement is semantic. In the case of the finite verb, the plural (syntactic) form is found, while in the case of the predicate noun, the singular (semantic) form is normal. Most writers do not even refer to these two positions, considering the forms too obvious to require comment» (Corbett 1983: 43-4).

Another rule of differentiation (Rule-2) has been presented with a certain group of verbs which are lexical honorifics themselves; for instance, Georgian -**brdzan**-/Zan -**zoj**-. They both are used in various meanings: ‘to be’, ‘to say’, ‘to come’, ‘to sit down’. Irrespective of the fact that the verbs in question are lexical honorifics themselves, they take on the said suffix (otherwise, a **-t**-less form would sound ironically). Hence, the suffix **-t** in the forms Geor. **brdzandebi-t**/Zan **zojun-t** ‘You are’, **brdzane-t**/**zoji-t** ‘Say’, **mobrdzandi-t**/**mozoji-t** ‘Come in’, **dabrdzandi-t**/**dozoji-t** ‘Sit down’ should be glossed as Verb(HON)-(±PL), that is, they are, first of all, honorifics, and they are used in a more polite context than the forms which are glossed as Verb-HON; but as polite forms, they can also be used in reference to more than one person.
Rule-3, possessed mainly by Georgian, is associated with the growing syntactic range of the suffix **-t**, as it appears with non-verb expressions; e.g. **k'argi-t** [O.K.-PL(and/or HON)]. This is a linguistic highlight of the fact that the honorific **-t** is no longer an optional phenomenon.

All these three rules are grammatical, and they, in combination with nominal terms of address, are exploited in far more complex and elaborate systems of sociolinguitic rules of address, those of alternation and co-occurrence.

The alternations are paradigmatic strings including the sets of linguistic means marking formal/informal addresses. Co-occurrence rules are of syntagmatic character, and they can show which terms of address co-occur or not with predicative honorifics (Ervin-Tripp 1969).

As a matter of fact, these co-occurrences present ‘mutual socio-pragmatic compatibilities’ of honorifics and nominal terms of address. On the one hand, a nominal term of address (for instance, Geor.: *bat'on-o* [mister-VOC] + FN [First Name]) co-occurs (because it is compatible) only with an honorific form; therefore, it triggers the appropriate changes in the verb. On the other hand, a verb-form without the suffix **-t** rules out (as it is not compatible with) the polite address term, and, hence, it can trigger either only an FN, or a DFN (Diminutive First Name), or an FN with a pejorative and/or endearment suffix (e.g. Zan: *data-sk'ua,* which is used toward a younger person as being addressed by an elder one).

With the cases where Rule-1 is in effect, the co-occurrence with an appropriate term of address is a pragmatic and a sociolinguistic reinforcement of the linguistic given.

G-1.1.1 *bat'ono davit, tkven k'etil-i pirovneba-Ø-Ø x-ar-t*
mister-VOC davit, you(HON) [a] kind-NOM person-NOM-SING 2-be-HON
Mister Davit, you are a kind person.

Z-1.1.1 *davit p'at'eni, tkva nagurapu koč-Ø-i re-Ø-t.*
davit mister-NOM, you(HON) wise man-SG-NOM be-2-HON.
Mister Davit, you are a wise man.

With Rule-2, co-occurrences determine whether a form in question is refered to an individual person or to more (as far as, in these cases, the verb-forms themselves indicate that they are honorifics, there is no need in distinguishing between a plural and an honorific):

G-2.1.1
bat'on-o davit, tkven k'etil-i p'irovneba-Ø-Ø brdzandebi-t.
mister-VOC davit, you(HON) [a] kind-NOM person-NOM-SG 2-are(HON)-HON
Mister Davit, you are a kind person.

G-2.1.2
bat'on-o davit da bat'ön-o givi, tkven k'etil-i p'irovneb-eb-i brdzandebi-t.
mister-VOC davit and mister-voc givi, you(PL/HON) kind-NOM person-PL-
-NOM 2-are(HON)-PL/HON
Mister Davit and Mister Givi, you are kind people.

Z-2.1.1
davit p'at'en-i, tkva nagurapu koč-i zojun-t.
davit mister-NOM, you(HON) wise-SG-NOM man-NOM 2-are(HON)-HON
Mister Davit, you are a wise man.

Z-2.1.2

davit p'aten-i do givi p'aten-i, tkva nagurapu k'ata zojun-t
davit mister-NOM and givi mister-NOM, you(PL/HON) wise people
2-are(HON)-PL/HON
Mister Davit and Mister Givi, you are wise men.

With Rule-3, the co-occurrence rules are necessary in the case of speech act analysis.

G-3.1 *bat'on-o davit, gamarjoba-t!*
mister-NOM davit, hello-HON
Hello, Mister Davit!
Z-3.1 *davit p'at'en-i, gomordzgua-t!*
mister-NOM davit, hello-HON
Hello, Mister Davit!
Whereas,

G-3.2 *dato, gamarjoba!*
davit(DIM), hello

Z-3.2 *data, gomordzgua!*
davit(DIM), hello

Braun (1988:11) remarks: «The distinction of pronouns and verb forms of address on the one hand vs. nominal forms on the other hand does not exactly correspond to the distinction of syntactically bound forms (integrated parts of sentences) and syntactically free forms (forms "outside" the sentence construction; preceding, succeeding, or inserted into the sentence)». This implies that the nominal forms of address like *Davit, Data(-sk'ua), Givi,* etc. are free ones. Though, judging from the Kartvelian evidence illustrating the co-occurrence between pronominal/verb forms of address and nominal ones, it seems unarguable that these forms are bound pragmatically and sociolinguistically, as far as alternations like *bat'on-o*+FN/*p'at'eni*+FN co-occur only with *tkven/si* and respective honorifics. It is noteworthy that the verb forms in question are discriminated not only between -**t**-marked and -**t**-less variants, but the -**t**-less forms, due to the specific intonational pattern and intensive stress peculiar to imperatives, occur also their contracted variants in colloquial Georgian; e.g. *ade[ki]!* ('Stand up!'), *daje(ki)!,* ('Sit down!'), *moica[de]!* ('Wait!'), *gaike[ci]!* ('Run away!'), etc. (Apridonidze 1995:64). And these contracted variants co-occur only with the nominal address forms of the pattern FN, DFN, and never with the pattern like *bat'on-o*+FN. It is a matter of the co-occurrence rule, and it is a result of the fact they are sociolinguistically bound.

It can be concluded that terms of address and grammatical honorifics are jointly parameterized systems in Kartvelian, and they not only correlate with appropriate social and pragmatic factors, but the rules of their formal arrangement and those of use are of pragmatic and sociolinguistic character, and, thus, they constitute and manifest those co-occurrence rules. This is the outcome of not only the close genetic bonds connecting these languages, but also that of the common cognitive and pragmatic background of their speakers..

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An Interpretation of Nominative Arguments, Voice-Affixes and Verb Stems in Tagalog

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In Tagalog, the main dialect spoken in the Philippines, the stem of each verb v can combine with different voice-affixes (VA). Semantically, VAs differ in three respects. First, the argument of a given phrase that is determined as subject depends on the VA with which the verb combines. Second, not each VA can combine with each stem and third the interpretation of a sentence depends on the VA. An analysis of these phenomena is developed in an extension of Dynamic Event Semantics (DES) (Naumann 1998b, Naumann 1999, Naumann and Mori 1998) that combines Event Semantics and Dynamic Logic. The basic intuition underlying DES is that non-stative verbs express changes. A change is either an object (event) or a transformation of state: a state s at which a condition Q does not hold is transformed to a state s' at which Q holds. Structures for the theory therefore contain both a domain E of events and a domain S of states. In Latrouite 1999 DES was extended by assigning to elements from E complex transformations of states instead of simple ones. It was shown how this extension can beused to explain the aspectual restrictions imposed by VAs. In the present paper it is shown how the other two semantic functions of a VA - determination of the subject (topic) and dependency of the interpretation on the VA - can be analyzed in DES.

0.1 Data and Evidence

In Tagalog each verb stem v can combine with a certain number of voice affixes(VA). Traditionally, these affixes (actor voice affixes: 'um-', 'mag-'; objective voice affixes: '-in', '-an', 'i-') have been said to identify the semantic role of the nominative argument (subject) of the sentence. Yet, as data taken from English 1997 show, they determine the aspectual interpretation of the verb.These aspectual phenomena cannot be derived from the voice affixes' characterization in terms of semantic role as shown in Latrouite 1998 and Ramos 1971. First, in many cases the interpretation of the sentence depends on the VA with which the verb combines. E.g., affixation of the Goal voice affix (GV)'-in' to the stem /kain/ ('eat') in (1:1) requires the object eaten (i.e. the fish) to be completely consumed, affixation of the locative voice affix (LV) '-an' yields apartitive reading, (1:3), whereas for the Actor voice affix (AV) 'um-' either of the two interpretations is possible, (1:1).

(1) 1.*K-um-ain ka ng isda*
Eat-AV Nom:you Gen fish (Eat at least part of the fish!)
2.*Kain-in mo ang isda*
Eat-GV Gen:you Nom fish (Eat the fish (completely)!)
3.*Kain-an mo ng isda ang plato*
Eat-LV Gen:you Gen fish Nom plate (Eat some/a part of the fish from the plate!)

On the other hand, for other stems like /kuha/ 'take' the interpretation is independent of the particular VA.

(2) 1.*K-um-uha ka sa kaniya ng lapis*
Take-AV Nom:you Dat:he Gen pencil (Get the pencil from him!)
2.*Kun-in mo sa kaniya ang lapis*
Take-GV Gen:you Dat:he Nom pencil (Get the pencil from him!)

3.*Kun-an mo siya ng lapis*
Take-LV Gen:you Nom:he Gen pencil (Get a pencil from him!)

Second, not each verb stem can combine with each VA. E.g., Point-verbs like /katok/ 'knock' only occur with the objective VA '-in' whereas the objective VA'-an' is excluded as the examples in (3) show.

(3) 1.*Katuk-in mo ang pinto*
Knock-GV Gen:you Nom door (Knock at the door!)
2.**Katuk-an mo ang pinto*
Knock-LV Gen:you Nom door (Knock at the door!)

Third, a particular class of verbs admits both the VA '-in' and the VA '-an',yet the meaning varies according to the VA. An example is given by /sunod/.

(4) 1.*Sund-an mo siya*
Follow-LV Gen:you Nom:he (!Follow him!)
2.*Sund-in mo siya*
Follow-GV Gen:you Nom:he (Obey him!)

Whereas /sunod/ means 'to physically follow' if it combines with the VA'-an', it means 'to obey' if it combines with '-in'. Fourth, the object that is determined as subject, i.e. the object denoted bythe nominative argument, in most cases the *ang*-phrase, depends on the VA, independently of the particular interpretation of the sentence.

(5) 1.*K-um-uha ka sa kaniya ng lapis*
Take-AV Nom:you Dat:he Gen pencil ((<u>You</u>) get the pencil from him!)
2.*Kun-in mo sa kaniya <u>ang lapis</u>!*
Take-GV Gen:you Dat:he Nom pencil (Get <u>the pencil</u> from him!)
3.*Kun-an mo <u>siya</u> ng lapis*
Take-LV Gen:you Nom:he Gen pencil (Get a pencil from <u>him</u>!)

If the VA is 'um', it is the actor that is denoted by the ang-phrase, whereas for the VA '-in' it is the object taken and for the VA '-an' it is the source from which some object is taken that is determined as subject.

0.2 Changes as Objects and Changes as Transformations of States

Dynamic Event Semantics (DES), Naumann 1998b, Naumann 1999 and Naumann and Mori 1998, is based on the intuition that non-stative verbs like 'eat' express changes. The intuitive notion of a change comprises at least two aspects that are complementary to each other: (i) something (an object: action, event) which brings about the change; (ii) something (a result) which is brought about by the change and which did not hold before the change occurred. In (i) 'change' is understood as the result that is brought about, i.e. in the sense that is captured by(ii), whereas in (ii) 'change' is meant as the object that brings about the result. The second aspect can be described as a transformation of state (TS). Before the change occurred, the world was in a particular state, say s, at which some condition Q did not hold, whereas after the change has occurred, the world is in a state s' at which Q does hold.

Structures for DES are based on a domain E of events, together with a partial ordering $\sqsubseteq_E$, the material part of relation, a domain S of states and a domain O of 'ordinary' objects that are related in a particular way to each other. Two functions $\alpha : E \rightarrow S$ and $\omega : E \rightarrow S$ assign to each event $e \in E$ its beginning point $\alpha(e)$ and end-point $\omega(e)$, respectively. Together, $\alpha(e)$ and $\omega(e)$ determine the execution-sequence $\tau(e) = (\alpha(e), \omega(e)) = \{s \in S | \alpha(e) \leq Ss \leq S\omega(e)\}$,where $\leq S$ is a linear ordering on the domain S. The domain E is sorted by a set $\mathbf{P} = \{Pv | v \in VERB\}$ of unary relations on E where the label set $VERB$ is a subset of the verbs in the lexicon of the underlying language, e.g. Tagalog or English. $\mathbf{T} = \{T_{pr} | pr \in PROP\}$ is a set of relations on $O \times S$, on which the relationship between E and O is based. Basically, each event-type P_v determines a set of properties. This relationship is captured by a function γ that assigns to each P_v a subset of **T:** $\gamma(P_v) \subseteq \mathbf{T}$. If $T_{pr} \in \gamma(P_v)$, then for a $d \in O$ $Q = T_{pr}(d)$ is a possible result (condition) that an event $e \in P_v$ can bring about on its execution-sequence $\tau(e)$ with respect to d. The relationship between a change as an object and a change as a TS is in general not one - one but rather one - many, i.e. an event e brings about not only one result but many. Consider an event e of type 'John eat a fish'. The initial actions executed by John, e.g. his bodily movements and opening his mouth, correspond to initial stages e' of e that are not of type 'eat'.[1] Only after John swallowed at least part of the fish an event of this type occurred. This condition is related to a decrease of the mass of the fish (due to the swallowing). Eventually, the last piece of the fish is swallowed and the fish has disappeared. Thus, to an event of eating a fish by John correspond three transformations of states: (a) the result effected by the initial actions performed by John: Q_1, (b) partial decrease of the mass of the fish (due to swallowing part of the fish): Q_2,(c) complete decrease of the mass of the fish: Q_3. Each of the three results is evaluated on the execution-sequence $\tau(e)$ of events e of type P_{kain} which bring them about in a particular way: (i) if Q_1 holds at a state s in $\tau(e)$, then it also holds at all states s' such that $\alpha(e) <_S s' \leq_S s$; (ii) if Q_2 holds at a states in $\tau(e)$, then it holds at all states s' such that $\alpha(e) <_S s' \leq_S s$ and s' is the end-point $\omega(e')$ of an initial stage e' in P_{kain} of e, (iii) if Q_3 holds at a state in $\tau(e)$, then it holds for no state s' such that $\alpha(e) <_S s' \leq Ss$. These ways in which the results are evaluated characterize different types of results. In (6) these results are formally defined (note that a result is of a particular type only relative to an event-type P_v).[2]

(6) 1.$Prefix_{<_S}$ -closedness of a result Q relative to an event-type P_v

$$\forall Q[Prefix_{<_S} - closed_v(Q) \leftrightarrow \forall s, e[e \in P_v \wedge s \in \tau(e) \wedge s \in Q \rightarrow \forall s'[\alpha(e) <_S s' \leq_S s \rightarrow s' \in Q]]]$$

2.$Prefix_v$-closedness of a result Q relative to an event-type P_v

$$\forall Q[Prefix_v - closed_v(Q) \leftrightarrow \forall s, e[e \in P_v \wedge s \in \tau(e) \wedge s \in Q \rightarrow \forall s', e'[\alpha(e) <_S s' \leq_S s' \wedge prefix(e', e) \wedge \omega(e') = s' \wedge e' \in P_v \rightarrow s' \in Q]]]$$

3.$Non - prefix_{<_S}$ -closedness of a result Q relative to an event-type P_v

$$\forall Q[Non - prefix <_S -closed_v(Q) \leftrightarrow \forall s, e[e \in P_v \wedge s \in \tau(e) \wedge s \in Q \rightarrow \forall s'[\alpha(e) <_S s' <_S s \rightarrow s' \notin Q]]]$$

In the sequel the following more intuitive names for the types of results will be used: $prefix_{<_S} - closed$ results are called $s - minimal$; $prefix_v$-closed results $w - minimal$ and non-prefix$_{<_S}$ -closed results maximal. [3]If a result Q of a particular type is required to be true at the end-point of an event $e \in P_v$, it is evaluated on $\tau(e)$ in a particular way. The way Q is evaluated corresponds to a (variant of a) dynamic mode from Dynamic Modal Logic, de Rijke 1993. In DES a dynamic mode is defined as a function from $\varsigma(E) \times \varsigma(S)$ to $\varsigma(E)$ (for details on the exact relationship between dynamic modes in DML and the way they are defined in DES see Naumann 1999a, Naumann 1999). In (7), three examples of dynamic modes are given and the relationship between types of results and dynamic modes is shown in Table 1.

[1]Note that it is not necessary to assume that the event e' is a material part of e. Alternatively, one can take e' to be a presupposed event that preceeds e.

[2]In some cases the definitions given in (6), in particular that of a non-prefix$_{<_S}$ -closed result,are too weak. In Latrouite 1999 an appropriate definition is given that proceeds by first defining the corresponding types at the level of properties Q, in terms of which the types of results are defined in a second step; see Naumann 1999 for details.

[3]See also the Appendix.

(7) 1.$Min-BEC_{<_S} = def.\lambda P\lambda Q\lambda e[e \in P \cap \alpha(e) \wedge Q \wedge \omega(e) \in Q \wedge \forall s[\alpha(e) <_S s <_S \omega(e) s \notin Q]]$
2.$Con-BEC = def : lambdaP\lambda Q\lambda e[einP \wedge \alpha(e) = \in Q \wedge \omega(e) \in Q \wedge \forall e'[prefix^*(e', e) \wedge e' \in P \rightarrow !(e')inQ]]$
3.$Con - BEC_{<_S} = def : \lambda P\lambda Q\lambda e[einP \wedge \alpha(e) = \in Q \wedge \omega(e) \in Q \wedge \forall s[\alpha(e)_{<_S} s <_S \omega(e) \rightarrow s \in Q]]$

(8) $\forall e; e'[prefix^*(e'; e)[e'vEe \wedge \alpha(e) = \alpha(e') \wedge \omega(e') <_S \omega(e)]]$

Table 1

type of result	dynamic mode
$prefix_s$-closed	$Con - BEC_{<_S}$
$prefix_v$-closed	$Con - BEC$
$non - prefix_{<_S}$-closed	$Min - BEC_{<_S}$

The relationship between types of results and dynamic modes is only a correspondence because a result can be of a particular type but it is not evaluated according to the corresponding dynamic mode. If the execution sequence of an event $e \in P_v$ is a singleton, then $\alpha(e) = \omega(e)$ holds. But the three dynamic modes require the result to be evaluated differently at the beginning- and the end-point of e. Formally, a TS can be defined as a pair $(Q; (s; s'))$ consisting of a result Q and a pair of states $(s; s')$ such that $s \notin Q$ and $s' \in Q$. To each dynamic mode DM in (7) corresponds a variant DM* that is a function from $\varsigma(E) \times \varsigma(S)$ to $\varsigma(S \times S)$.

(9) 1.$Min - BEC^* <_S = def : \lambda P\lambda Q\lambda ss'\exists e[e \in P \wedge \tau(e) = (s; s') \wedge \alpha(e) \notin Q \wedge \omega(e) \in Q \wedge \forall s[\alpha(e) <_S s <_S \omega(e) \rightarrow s \neq Q]]$
2.$Con - BEC^* = def : \lambda P\lambda Q\lambda ss'\exists e[e \in P \wedge \tau(e) = (s; s') \wedge \alpha(e) \notin Q \wedge \omega(e) \in Q \wedge \forall e'[prefix^*(e'; e) \wedge e' \in P \rightarrow !(e') \in Q]]$
3.$Con - BEC^* <_S = def : \lambda P\lambda Q\lambda ss'\exists e[e \in P \wedge \tau(e) = (s; s') \wedge \alpha(e) = 2Q \wedge \omega(e) \in Q \wedge \forall s[\alpha(e) <_S s <_S \omega(e) \rightarrow s \in Q]]$

For an event $e \in P_v$ with $\tau(e) = (s; s')$ one has: $(Q; e) \in DM(P_v)$iff$(Q; (s; s')) \in DM^*(P_v)$.

0.3 The Interpretation of Verbs in DES

0.3.1 The Classification of Verbs

Verbs v can be classified on the basis of (i) the *types* of results that are determined by the corresponding event-types P_v, (ii) the number of results of a given type that are determined by P_v and (iii) the sort to which the event-types P_v belong. E.g., accomplishment-verbs like 'kain' determine all three types of results. P_{sunod_f} , the subset of P_{sunod} that corresponds to the meaning of 'to follow', does not define a maximal result but only an $s-$minimal and a $w-$minimal one. Three different sorts of event-types are distinguished: an event-type P_v is P-atomic if no proper initial stage (prefix) e' of an event e belonging to P_v is of this type too: $P - Atomic(P_v), \forall e[e \in P_v \rightarrow \neg\exists e'[prefix^*(e'; e) \wedge e' \in P_v]]$; an event-type P_v is instantaneous if each of its elements has an execution sequence that consists of a single state (i.e., the beginning point is identical to the end point: $\alpha(e) = \omega(e)$), Naumann 1999b. Instantaneous and P-atomic event-types together form the atomic event-types. Finally, an event-type P_v is non-atomic if it is not P-atomic and if the execution sequence of each of its elements is not a singleton. Event-types of sort Accomplishment and Activity are non-atomic. Examples for P-atomic event-types are those corresponding to Transfer-verbslike 'kuha' and 'buy'. Point- and Achievement-verbs like 'katok' and 'reach', respectively, correspond to instantaneous event-types. From the definition of P-atomicity it follows that for this sort the distinction between $w-$minimal and maximal results collapses: each $w-$minimal result is maximal and vice-versa. This property distinguishes the (non $s-$minimal) results assigned to verbs of this class from those results assigned to verbs belonging to the class of Accomplishments or Activities where neither $w-$minimal results are maximal nor maximal results $w-$minimal. Maximal results that are not $w-$minimal (and not $s-$minimal) are called strongly maximal (s-maximal) whereas maximal results that are also weakly minimal (and not $s-$minimal) are called weakly maximal (w-maximal). A result that is $w-$minimal but not maximal (and not $s-$minimal) will be called w*-minimal in the sequel. Transfer-verbs determine two w-maximal results. For instantaneous event-types, i.e., the event-types corresponding to Achievement- and Point-verbs, the distinction between the three basic types collapses: they are indistinguishable because the execution-sequences are singletons. If it is required that a result Q hold at the end-point of an event e belonging to an instantaneous event-type, it vacuously holds at all points in between $\alpha(e)$ and $\omega(e)$ (because there are no such points); thus, the result is $s-$minimal. By the same argument Q is maximal because it vacuously fails to hold at all intermediate points. Finally, Q is $w-$minimal because it holds, again vacuously, at the end-points of all proper prefixes e' of e. Results that are $s-$minimal, $w-$minimal and maximal will be called min-max. The classification based on the three criteria is given in Table 2.

Table 2	s-min.	w^*-mina	min-max	w-max.	s-max.
Acco. (kain)	+	+	-	-	+
Act. ($sunod_f$)	+	+	-	-	+
Transfer (kuha)	+	-	-	(+)2	-
Point/Ach. (katok)	+	-	+	-	-

0.3.2 Objects and Changes

Each basic event type P_v determines for each of its elements e a set $Res(P_v; e)$ of results that e can possibly bring about. These results are linearly ordered interms of (i) the temporal order in which they are brought about based on the relation 'not before' and (ii) implicational relations. The ordering that results if only the first criterion is applied is $\leq_v$. The ordered set of results constitutes $e's$ event structure (ES). Each result that is an element of $Res(P_v; e)$ is brought about with respect to at least one object that participates in e. E.g., if e is of type 'John eat a fish', the results are brought about with respect to John and the fish. John is assigned both the $s-$minimal result (e.g. his mouth is open) and a $w*$-minimal one (part of the fish is in his stomach) whereas the fish is assigned a $w*$-minimal result (its mass partly decreased) and the s-maximal one (its mass is zero). In the case of an event e of type 'Bill push the cart' Bill is assigned the $s-$minimal result (his actions towards the cart which bring about a change of location with respect to Bill or a part of Bill's body, e.g. his arms) and possibly a $w*$-minimal one (Bill traverses a non-empty path) whereas the cart is assigned only a $w*$-minimal result (the cart traverses a non-empty path). For an event e of type 'John give the book to Mary' both the book and Mary are assigned a w-maximal result and no $s-$minimal one such that it is not possible to distinguish them on this basis. The difference between these two objects is that the book is assigned two w-maximal results (John does not have the book; Mary does have the book) whereas Mary is assigned only one (Mary does have the book), Naumann and Mori 1998. The Actor John is assigned the $s-$minimal result (corresponding to his movements towards Mary which bring about a change of location with respect to John) and a w-maximal one (John does not have the book). Furthermore, both results are w-maximal, and are therefore equivalent with respect to the ordering $\leq_v$. They can be distinguished on the basis of implicational relations. 'Mary does have the book' (Q) implies 'John does not have the book' (Q') but not vice versa, i.e., $Q \subseteq, Q' \wedge Q' \not\subseteq Q$ holds. Using implicational relations in addition to the temporal relation 'not before' gives rise to a more fine-grained ordering $\leq_v^*$ (compared to $\leq_v$): $Q \leq_v^* Q'$ iff $Q <_v Q' \vee [Q =_v Q' \wedge Q' \subseteq Q]$. If for two w-maximal results Q and Q' $Q = vQ'$ and $Q <_v^* Q'$ holds, Q is called the w-max1 result whereas Q' is the w-max$_2$result. With the more fine-grained ordering $\leq_v^*$ one gets: Actor (John): s-min and w-max$_1$; Recipient (Mary): w-max$_2$ and Object transferred (book): w-max$_1$ and w-max$_2$. Similarly, it is possible to distinguish the two w-maximal results defined for an event e in P_{kuha}. If d' is possessed by d'', this implies that $d(\neq d'')$ does not have d'. The other direction does not hold because d can have given d ' to some $d''' \neq d''$ or simply have thrown d' away. Thus, $Q' \subseteq Q$ holds,where Q' corresponds to the result that d is possessed by d''. The aspectual relevance of this inclusion lies in the fact that the stronger result but not the weaker one can be used as the minimal result that events of type P_{kuha} must bring about. If for an event e, Q but not Q' holds at $\omega(e)$, e cannot be of type P_{kuha}. The relationship between an event-type P_v, an event e in P_v, an object $d \in O$ participating in e and a result Q that e can bring about with respect to d is captured by a function $\Delta : \Delta(P_v)(e)(d)$ is the set of results that e can bring about relative to $d(\Delta(P_v)$ is abbreviated to $\Delta_v)$. For each $Q \in \Delta_v(e)(d)$ it is required that $Q = Q(d)$ for some $Q \in \lambda(P_v)$. The set $Res(P_v, e)$ can be defined in terms of $\Delta : Res(Pv; e) = \{Q|\exists d[Q \in \Delta_v(e)(d)]\}$.

0.4 The Interpretation of the Voice Affixes

In section (1) it was shown that a VA has at least two semantic functions. It determines both the subject of a sentence and the particular interpretation the sentence (or the corresponding VP) has (e.g. partive vs. non-partitive). The set of results $Res(P_v; e)$ 'spawns' a maximal possible transformation of state that can be brought about by e if all elements of this set are realized. This maximal transformation of state is the 'join' of the transformations that correspond to the elements of $Res(P_v; e)$ according to the dynamic modes that are assigned to the results with respect to their type.

The two functions of a VA can both be defined in terms of $Res(P_v; e)$. Intuitively, they can be formulated as follows.

1. each VA determines for an event e in P_v a subset of $Res(P_v; e)$ as admissible results and requires e to bring about at least one element Q from this subset. This means that at the level of P_v a voice affix maps P_v to one of its subsets: $\forall d_1 ... \forall d_n \forall e[!d_1, ..., d_n; e >\in [[VA]](P_v) \rightarrow [[VA]](P_v)(d_1)...(dn)\theta P_v$
2. each d participating in e is uniquely determined by a subset of $Res(P_v; e)$ that can be defined in terms of (i) the types of results that are assigned to it and (ii) a (temporal) maximality or a minimality condition with respect to condition (i); the subject determined by a VA is singled out by a particular subset (plus, possibly, a maximality or minimality condition).

0.4.1 The Determination of the Result

The differences in interpretation depending on the VA with which a stem combines show that an event need not bring about all the results that belong to $Res(P_v, e)$. On the other hand, each event $e \in P_v$ is required to bring about at least a result that is sufficient for it to be of type P_v. Such results are called $v-$closed $(prefix(e'; e) = def : prefix^*(e', e) \vee e = e')$.

(10) $\forall Q[closed_v(Q) \Leftrightarrow \forall e \forall e' \forall d[\Delta(e)(d)(Q) \wedge prefix(e', e) \wedge \rightarrow (e') \in Q \rightarrow e' \in P_v]]$

What types of results are v-closed depends on the event-type. For Accomplishments, Activities and Transfer-verbs $w-$minimal and maximal results are $v-$closed, whereas $s-$minimal ones are not. For these verbs the requirement therefore excludes that only an $s-$minimal result is brought about. In the case of Achievement- and Point-verbs, for which the distinction between the three basic types collapses, all types of results are v-closed (see Latrouite 1999 and Naumann and Mori 1998 for details). For the three VA's discussed in this article one gets the following requirements.

(i) 'um-' requires an event e at least to be of type P_v; this requirement is satisfied if e brings about at least one result that is v-closed. This VA does therefore not restrict P_v because this requirement is satisfied by each element of P_v. On the other hand, no v-closed result is excluded so that'um-' does not uniquely determine a particular result.

(ii) '-in' requires an event e to bring about a maximal result Q. It does not matter whether Q is s-maximal, w-maximal or min-max. From this requirement it immediately follows that verbs of sort Accomplishment,Transfer, Achievement and Point admit this VA because the corresponding event-types determine this type of result. Stative verbs and Activity-verbs, on the other hand, do not admit 'in-' because no maximal result is determined (an example is given by P_{sunod_f} ; see section (5) for details)

(iii) '-an', finally, requires an event e to brinng about a result Q that is (a) non-prefix-closed$_v$, i.e. w-minimal or maximal (b) not prefix$_{<_S}$-closed$_v$, i.e. s-minimal and (c) minimal with respect to $\leq_v$ among the set of results that satisfy conidtions (i) amd (ii). The second condition admits $w*$-maximal, w-maximal, s-maximal and min-max results. If condition (i) is taken into account, min-max resutls are excluded because they are also s-minimal. From this requirement it immediately follows that '-an' in not admissible for verb stems of sort Achievement and Point because their corresponding event-types only determine the latter type of results. The third condition excludes s-maximal results because whenever an event-type determines this type of result, it also determines a $w*$-minimal one (examples are (incremental) Accomplishment-verbs), which (properly) precedes the s-maximal one according to $\leq_v$. The necessity to exclude $s-$maximal result as admissible arises from the partitive reading one gets for '-an' for Accomplishment-verbs like 'kain'. As a consequence, the admissible results for this voice affix are $w*$-minimal and w-maximal result (the formal names of which are $prefix^*_v - closed$ and $w - non - prefix_{<_S} - closed$, respectively; see the appendix for formal definitions.)

These requirements are formulated in terms of function NS^{VA} that map an event-type P_v and a set of states Q to a subset of $P_v(NS^{VA}(P_v)$ is abbreviated to NS^{VA}_v; $\delta^*_v(e)(Q) = def.\exists d.\delta_v(e)(d)(A))$

(11) $1.NS^{um}_v(Q) = \{e|[\delta a^*_v(e)(Q) \wedge closed_v(Q)]\}$
$2.NS^{um}_v(Q) = \{e|[\delta a^*_v(e)(Q) \wedge non - prefix_{<_S} - closed_v(Q)\}$
$3.NS^{um}_v(Q) = \{e|[\delta a^*_v(e)(Q) \wedge [prefix^*_v - closed_v(Q) \vee w - non - prefix_{<_S} - closed_v(Q)\}$

In terms of (11:3) the admissibilty condition imposed by '-an' can be formulated in the following way. Recall that $w*$-minimal and w-maximal results are both w-minimal and not s-minimal. As each basic event-type P_v determines one result that is $prefix_{<_s} - closed$, it follows that '-an' is admissible only for event-types that determine at least two types of results that do not coincide, ie., that are not $\leq_v$-equivalent. If the relationships between types of results determined by a P_v are taken into account, this condition is equivalent to that given in (12).

(12) $\forall Q[Q \in Res(P_v, e) \rightarrow [closed_v(Q) \Leftrightarrow prefix^*_v - closed_v(Q)]] \wedge \exists Q \exists Q'[Q \in Res(P_v, e) \wedge Q' \in Res(P_v, e) \wedge closed_v(Q) \wedge \neg prefix_{<_S} - closed_v(Q \wedge closed_v(Q')) \wedge \neg(Q' =^*_v Q)$

The subset $Res^{VA}(P_v, e)$ of $Res(P_v, e)$ that corresponds to $NS^{VA}(P_v, e)$ is defined in (13).

(13) $\forall Q[Q \in Res^{VA}(P_v, e) \Leftrightarrow e \in NS^{VA}(P_v, Q)$

The requirement that is imposed by a VA on the results is formulated in (14).

(14) $\exists Q[Q \in Res^{VA}(P_v, e) \wedge \omega(e) \in Q]$

According to (14), only those events $e \in P_v$ are denoted by a complex predicate consisting of a verb stem and a voice affix that bring about at least one result from the subset of results $Res^{VA}(P_v, e)$ determined by the affix.

0.4.1.1 The Determination of the Subject

The second function of a VA consists in determining the subject of a sentence.

The subject cannot be defined in terms of thematic relations. For instance, the VA '-in' determines both incremental themes, for the stem /kain/, transferred objects for the stem /kuha/ and goals for the stem /akyat/ as subjects (for details, see Latrouite 1998, Latrouite 2000).

(15) Kun-in mo sa kaniya ang lapis! (Get the pencil from him!) subject= transferred object

Kain-in mo ang isda! (Eat the fish!) subject = incremental themec. Akyat-in mo ang kanya-ng kuwarto! (Go up to his room!) subject = goal

As will be shown below, in DES it is possible to uniquely discern participants of an event e in P_v in terms of the dynamic structure that is spawned by $Res(P_v, e)$ together with maximality or minimality conditions.A first criterion that is used to uniquely single out a participant d of e is based on the types of result to which d is related to e. This first criterion could equally be formulated in terms of dynamic modes that correspond to particular types of results because results of the same type are assigned the same dynamic modes. Let $[d]_i^v 1 \leq i \leq 3$, be the set of objects participating in $e \in P_v$ that are assigned a result of type i, where T_i is one of the three types defined in (6)).

(16)1.$[d]\theta_{v_i} e = \{d | \exists Q[Q \in \Delta_v(e)(d) \wedge T_i(Q)]\}$

$[d]_i^v; e$ need not be a singleton such that on the basis of (single) types of results or even sets of types of results objects participating in e need not be distinguishable. For instance, for an event $e \in P$ push both the object that is pushed, e.g. a cart, and the Actor who does the pushing are assigned a w^*-minimal result (both traverse a non-empty path). Consequently, these two participants cannot be distingushed in terms of w*-minimal results. It is not possible to argue that the cart can be distinguished from the Actor by not being assigned an $s-$minimal result, i.e., the Actor is that object which is assigned both an $s-$ and a w^*-minimal result whereas the cart is assigned only a w^*-minimal result. Such a definition is not applicable to many other Activity-verbs like 'shave' for instance which admit of reflexive uses as in 'John shaved himself'. In this particular case the Actor is identical to the Undergoer. As a consequence the Undergoer is assigned an $s-$minimal result too. But this is excluded on the purported definition given above.

The solution to this problem consists in the observation that the Undergoer is that object which is assigned a w^*-minimal result and which is involved latest in the event among those objects that are assigned a w^*-minimal result. These condition imposes a maximality condition on the object denoted by the internal argument. This condition is formulated with respect to the temporal-dynamic dimension. It can be made precise in terms of results that are assigned to participants. If an object d is involved as the latest with respect to some condition (e.g. being assigned a w^*-minimal result), then to all other objects d' that satisfy this condition is assigned a result Q that precedes all results Q' assigned to d, where 'precedes' is defined in terms of the ordering $\leq_v^*$ on the set of results. As the object denoted by the internal argument is assigned only the $w*$-minimal result whereas the Actor is assigned the $s-$minimal result besides the $w*$-minimal one and as the $s-$minimal one precedes the $w*$-minimal one according to $\leq_v^*$, it follows that the Actor is excluded because it does not satisfy the maximality condition. This condition is satisfied by the object denoted by the internal argument such that it is the Undergoer. If both objects are identical, as in the case of a reflexive construction like 'shave oneself', there is only one object that is assigned the $w*$-minimal result, which trivially satisfies the maximality condition and which therefore, then, is the Undergoer. As this object is also assigned the $s-$minimal condition, it is in addition the Actor.

In (17) a set of semantic roles is defined according to the two criteria from above (these roles are relativized to an event-type P_v such that $e \in P_v$; an unrelativized role is defined as the union of the relativized ones, see the appendix for a formal definition).

(17)

1.$Role_v^1(e,d)$ just in case d participates in e and (i)d is assigned an $s-$minimal result and (ii) d is minimal with respect to condition (i)

2.$Role_v^2(e,d)$ just in case d participates in e and (i) d is assigned a $w*$-minimal result and (ii) d is maximal with respect to condition (i)

3.$Role_v^3(e,d)$ just in case $Role_v^2(e,d)$ and d is assigned an s-maximal result

4.$Role_v^4(e,d)$ just in case d participates in e and (i) d is assigned two $w-$maximal results, a w-maximal$_1$ and a w-maximal$_2$ one

5.$Role_v^5(e,d)$ just in case d participates in e and (i) d is assigned exactly one w-maximal result Q, (ii) Q is w-maximal$_2$ and (iii) d is maximal with respect to conditions (i) and (ii)

6.$Role_v^6(e,d)$ just in case d participates in e and (i) d is assigned exactly one w-maximal result Q, (ii) Q is w-maximal$_1$ and (iii) d is maximal with respect to condition (i) and (ii)

The roles in (17) define what is called the *dynamic component* of a *semantic role* (SR) such that it can be viewed as a partial definition of the SR. It is therefore not excluded that besides the dynamic ones other properties, like causation or volition for instance, define the SR. The definitions are given in (18).

(18) $1.\theta_{Act_v}(e,d) \rightarrow Role^1_v(e,d)$
$2.\theta_{UG_v}(e,d) \rightarrow Role^2_v(e,d)$
$3.\theta_{UG_{inc_v}}(e,d) \rightarrow Role^3_v(e,d)$
$4.\theta_{BO_{max_1} \wedge max_{2_v}}(e,d) \rightarrow Role^4_v(e,d)$
$5.\theta_{BO_{max_{2_v}}}(e,d) \rightarrow Role^5_v(e,d)$
$6.\theta_{BO_{max_{1_v}}}(e,d) \rightarrow Role^6_v(e,d)$

According to (17a), the Actor is that participant who is assigned the $s-$minimal result and who is minimal with respect to this property. As the $s-$minimal result is assigned to exactly one participant of an event, the latter condition can in fact be dropped because the first condition already singles out a unique object (see Naumann and Mori 1998 for details). The internal arguments of (transitive) Accomplishment- and Activity-verbs are both Undergoers in the sense of (17b). Those of Accomplishment-verbs satisfy in addition (17c), i.e., they are incremental UGs (Themes)[4].. Note that the maximality conditionin the definition of $Role^2_v$ excludes the Actor if (s)he is distinct from the UG because if both are assigned a $w-$minimal result that is not maximal (and, possibly, a maximal result), the Actor is assigned in addition the $s-$minimal result which means that (s)he is involved in the event before the UG and therefore fails to satisfy the maximality condition. The roles assigned to the direct and oblique arguments of Transfer-verbs like /kuha/ and /bigay/ are called *maximal boundary object* (BO_{max}), (17d-f), respectively. They can be distinguished on the number of w-maximal results they are assigned and the sort of the w-maximal result. The object that is transferred, e.g. the book given or taken, is assigned both w-maximal results determined by the corresponding event-type whereas the object denoted by the oblique argument is assigned only one (see the appendix for examples). According to these definitions, each object that bears some TR_v to an event e in P_v is assigned a result, i.e., it undergoes a change. This follows from the definitions of the different roles $Role^i_v$ in (17). Thus, in a sense each object that is assigned a result is an UG in the traditional sense. In terms of the SRs defined in (17) further SRs can be defined that generalize those in (18).E.g., a generalized Undergoer can be defined as the union of the relations $UG, BO_{max_1} \wedge max_2, BO_{max_2}$ and BO_{max_1}. For verbs v with corresponding event-types P_v that are instantaneous, the objects participating in e are discerned on the basis of the presupposed event e'. They cannot be discerned on the basis of results that hold on the execution-sequence of e because all types of results collapse such that one rests, in effect, with a single type that does not admit to distinguish more than one object.If the objects are discerned with respect to e', this means that the semantic roles that are assigned to participants of e are the value of those roles for thepresupposed event e', i.e. the θ_{v_i} are defined in terms of the corresponding roles t, i for the event-type of the presupposed event. This yields (19).[5]

(19) $\theta_{v_i}(e,d) iff \exists e'[presup(e,e') \wedge P_v(e) \wedge \theta_i(e',d)]$

(19) can be generalized to all basic event-types by assuming that for events that do not presuppose other events $presup(e,e)$ holds. There is the following relationship between the set of admissible results de-termined by a VA and the object d denoted by the ang-phrase ($Res_d(P_v,e) = \{Q|Delta_v(e)(d)(Q)\}$ is the set of all results assigned to d).

(20) $Res^{VA}(P_v,e) \cap Res_d(P_v,e) \neq \theta$

[4]SR is short for θ_{SR}

[5]Note that in effect $\theta_i(e',d)$ must be relativized to a basic event-type P'_v . This can beachieved by using $P(P) \wedge P = P'_v \wedge \theta_{v'_i}(e,d)$ for some v' that is determined by e and $P_v(P = \{P_v|v \in VERB\})$. An alternative solution consists in defining semantic roles not as functional relations on $O \times E$ but on $\varsigma(E) \times O \times E$. This makes first the dependence on an event-type explicit and can be used even if the event-type of e' is not a basic one.

According to (20), the object d that is denoted by the ang-phrase must be assigned a result that is admissible for the VA.

The object d participating in an event $e \in P_v$ that is determined as subject by the voice affixes are determined by the following condition.

(21) 1.'um-': the object d that is assigned the $s-$minimal result, i.e., the object that is assigned the semantic role $Role^1_v(\theta_{Act_v})$ with respect to e

2.'-in': the object d that is assigned all v-closed results

3.'-an': the object d such that (i)d is assigned a maximal element Q from $Res(P_v, e)$ with respect to $\leq_v$, (ii) d is assigned the least number of maximal elements from $Res(P_v, e)$ among the objects participating in e that are assigned a maximal element from $Res(P_v, e)$ and (iii) d is maximal with respect to condition (i)

0.4.2 The Interpretation of the VA

In Tagalog, a verb stem does not subcategorize for an n-tuple of arguments, Himmelmann 1987. A verb stem v is therefore interpreted in DES as the corresponding event-type P_v, (22:1).Voice affixes are interpreted as mapping basic event-types P_v to relations on On times E (i.e., they are polymorphic, for v an n-place verb, the result of applying the interpretation of a VA to the interpretation of v is a relation on $O^n \times E$). In (22), $\mu(P_v)$ is the set of semantic roles which is defined for P_v, for F see below and Naumann 2000.

(22) 1.$[[v]] = P_v$

2.$[[um-]](P_v) = \lambda d_1 ... \lambda d_n \lambda e \exists Q \exists d[e \in P_v \wedge \theta_{v_i}(e, d_i) \wedge \omega(e) \in Q \wedge e \in NS^{sum}_v) \wedge Subject(P_v, e, d) \wedge d = \iota d'.F_{um}(\mu(P_v))(e) = d']$

3.$[[-in]](P_v) = \lambda d_1 ... \lambda d_n \lambda e \exists Q \exists d[e \in P_v \wedge \theta_{v_i}(e, d_i) \wedge \omega(e) \in Q \wedge e \in NS^{in}_v(Q) \wedge Subject(P_v, e, d) \wedge d = \iota d'.F^{in}(\mu(P_v))(e) = d']$

4.$[[-an]](P_v) = \lambda d_1 ... \lambda d_n \lambda e \exists Q \exists d[e \in P_v \wedge \theta_{v_i}(e, d_i) \wedge \omega(e) \in Q^e \in NS^{an}_v(Q) \wedge Subject(P_v, e, d) \wedge d = \iota d.F^{an}(\mu(P_v))(e) = d' \wedge \forall Q'[NS^{*-an}_v(Q')(e) \rightarrow \omega(e) \notin Q']]$

The additional condition in the interpretation of '-an' makes sure that for(incremental) Accomplishment-verbs like /kain/ the s-maximal result is not brought about such that one gets a partitive reading. $NS^{*-an}(P_v)(Q)$ is defined as follows $(Q \in \Delta^* v(e) \text{iff} \exists d \neq \Delta v(e)(d)(Q)) \neq NSd^a, an(P_v)(Q) = fe|Q \in \Delta^*_v(e) \wedge non-prefix-closed_v(Q) \wedge \forall Q''[NS^{an}_v(Q'')(e) \rightarrow \neq (Q'' =^*_v Q)]\}$. According to the first condition, Q is either w^*-minimal, w-maximal or s-maximal.

The last condition requires, in addition, that Q is not v^*-equivalent to an element of the set of admissible results determined by '-an'. As all elements of $Res^{an}(P_v, e)$ are either w^*-minimal or w-maximal, it follows that exactlythe s-maximal results are singled out (if this type of result is determined at all). Thus, the requirement $\forall Q'[NS^{an}_v(Q')(e) \rightarrow \omega(e) \notin Q']$ in the interpretation of '-an' excludes that an s-maximal result is brought about. The requirement $\forall Q'[NS^*_v(Q')(e) \rightarrow \omega(e) \notin Q']$ can equivalently be formulated as $\forall Q'[Q' \in (Res^* - an(P_v, e) \cap Res^{an}(P_v, e)) \rightarrow \omega(e) \notin Q']$.

Following Latrouite 1998, Latrouite 2000 it is assumed that the order of arguments in (22) is the same for each voice affix. This means that if the maximal number of arguments is n, then there is an n-tuple $< d_1, ..., d_n >$ such that $[[VA]](P_v)(d_1)...(dn)(e) = 1$, independently of the particular voice affix VA. As shown in A. Latrouite 2000, $< d_1, ..., d_n > = < \theta_{v_1}(e), ..., \theta_{v_n}(e) > (where \theta_{v_i}(e) = \iota d.\theta_{v_i}(e, d))$ and the order of the t, v_i is identical to the orderingof the semantic roles that are defined for P_v. *Subject* is a non-functional relation on $\varsigma(E) \times E \times O$ that holds between basic event-types P_v and events $e \in P_v$ and $ad \in O$ if d is a participant of e and satisfies the condition imposed by a voice affix VA on the subject. For details of how F_{VA} is defined and on the exact relationship between semantic roles andthe set of results determined by a VA see Naumann 2000.

0.5 Verbs with Meaning Variance

In contrast to P_{kain} and P_{kuha}, P_{sunod} defines two different types of (maximal) execution-sequences: one that is P-atomic and one that corresponds to that defined by event-types of sort activity. Elements of P_{sunod} that are denotedby expressions with meaning 'to obey' are assigned the first type (they belong to P_{sunod_o}) whereas events denoted by expressions with meaning 'to follow' are assigned the second type (they belong to P_{sunod_f}). One has: $P_{sunod} \cup [P_{sunod_f} = P_{sunod}$ and $P_{sunod} \cap P_{sunod_f} = \theta$. Meaning variance of a verb stem like /sunod/ is explained as resulting from the possibility of bringing about a result in different, dynamically non-equivalent ways. For 'eat' the result that must be brought about only holds at the endpoint of an event, i.e., it can be assigned a unique type: it is an s-maximal result. This uniqueness need not always be the case. Consider the result 'the actions of d are dependent on those of d'' $(= Q)$. Bringing about this result can be done by either deciding to carry out the instructions (orders) of d' $(= Q_1)$ or by traversing a path that has the same direction as that traversed by d' $(= Q2)$.These two instances of Q differ in the way they are brought about. Q_2 is similar to the result that characterizes verbs of motion: a non-empty path must be traversed. In addition, it is required that the path has the same direction as the path traversed by d'. This result holds at intermediate states of the execution sequence of an event that brings it about such that it is $w-$minimal. Q_1, on the other hand, is an s-maximal or a min-max result because it only holds at the end point of an event (the decision process). Tagalog and English differ in the way the conditions on the result that must be brought about are determined. In English the result is required to be maximal with respect to $\leq_v$ and to have a unique type. This requirement is violated if a verb in English were assigned the result Q the type of which depends on the particular instance that is brought about. Therefore, the two ways of bringing about Q are assigned not to one but to two different verbs, namely 'obey' $(Q_1)$and 'follow' (Q_2). It is assumed that P_{sunod} only determines the general result Q such that the meaning of the stem comprises both instances, i.e., it is underspecified with respect to the two instances Q_1 and Q_2. Which instance is selected depends on the requirement that is imposed by a VA. As '-in' requires the result to be maximal (non-prefix$_{<_S}$ -closed), it follows that Q must be realized by its subtype Q_1 because then that this requirement is satisfied. The VA '-an', on the other hand, selects the subtype Q_2 because it is w*-minimal, a type of result that is admissible for this VA, whereas Q_1 is excluded because it is neither $w*$-minimal nor $w-$maximal. In (23) the interpretation of /sunod/ for the voice affixes '-in'and '-an' are given using the types of the admissible result.

(23) 1.$[[in]](P_{sunod}) = \lambda d_1 \lambda d_2 \lambda e \exists Q[e \in P_{sunod} \wedge \theta_{sunod_i}(e, d_i) \wedge non-prefix_{<_S} - closed(Q) \wedge \omega(e) \in Q]$

2.$[[an]](P_{sunod}) = \lambda d1 \lambda d \in \lambda e \exists Q[e \in P_{sunod} \wedge \theta_{sunod_i}(e, d_i) \wedge [prefix^*_v - closed_v(Q) \vee w - non-prefix_{<_S} - closed_v(Q)] \wedge \omega(e) \in Q]d$

0.6 Appendix

0.6.1 Formal Definitions of further Types of Result

In (24) further types of results are defined in terms of the three basic ones defined in (6) above.

(24) 1.$\forall Q[prefix^*_{<_S} - closed_v(Q) \Leftrightarrow prefix_{<_S} - closed_v(Q) \wedge \neg non - prefix_{<_S} - closedv(Q)]$

2.$\forall Q[prefix^{**}_{<_S} - closed_v(Q) \Leftrightarrow prefix_{<_S} - closed_v(Q) \wedge non - prefix_{<_S} - closed_v(Q)]$

3.$\forall Q[prefix^*_v - closed_v(Q) \Leftrightarrow \neg prefix_{<_S} - closed_v(Q) \wedge prefix_v - closed_v(Q) \wedge : non - prefix <_S - closed_v(Q)]$

4.$\forall Q[w - non - prefix_{<_S} - closed_v(Q) \Leftrightarrow \neg prefix_{<_S} - closed_v(Q) \wedge prefix_v - closed_v(Q) \wedge non - prefix_{<_S} - closed_v(Q)]$

5.$\forall Q[s - non - prefix <_S - closed_v(Q) \Leftrightarrow \neg prefix_{<_S} - closed_v(Q) \wedge \neq prefix_v - closed_v(Q) \wedge non - prefix <_S - closed_v(Q)]$

6.$\forall Q[non-prefix-closed_v(Q) \Leftrightarrow prefix^*_v-closed_v(Q)w-non-prefix_{<_S}-closed_v(Q) \vee s-non-prefix <_S -closed_v(Q)]$

In the table below the correspondence between the offical names and the more intuitive names used in the text is given.

s-minimal	prefix$_{<_s}$ -closed
s^*-minimal	prefix$^*_{<_S}$ -closed
w^*-maximal	prefix$^*_v$-closed
min-max	prefix$^{**}_{<_S}$ -closed
w-maximal	w-non-prefix$_{<_S}$ -closed
z-maximal	s-non-prefix$_{<_S}$ -closed

0.6.2 Formal Definitions of the Semantic Roles

In a first step a set of functional relations on E times O is defined: these relations,called semantic roles, are relativized to an event-type P_v.

(25) 1.$Role^1_v(e,d) \Leftrightarrow \exists Q[\Delta_v(e)(d)(Q) \wedge prefix^* <_S -closed_v(Q)]$
2.$Role^*_v(e,d) \Leftrightarrow \exists Q[\Delta_v(e)(d)(Q) \wedge prefix^*_v - closed_v(Q)]$
3.$Role^2_v(e,d) \Leftrightarrow Role^{2*}_v(e,d) \wedge Max_{Role^{2*}_v}(e,d)$
4.$Role^3_v(e,d) \Leftrightarrow Role^2_v(e,d) \wedge \exists Q[\Delta_v(e)(d)(Q) \wedge s-non-prefix_{<_S} - closedv(Q)]$
5.$Role^4_v(e,d) \Leftrightarrow \exists Q \exists Q'[\Delta_v(e)(d)(Q) \wedge \Delta_v(e)(d)(Q') \wedge w-non-prefix_{<_S} - closed_v(Q) \wedge w-nonprefix_{<_S} - closed_v(Q') \wedge : \neg(Q =^*_v Q')]$
6.$Role^{5*}_v(e,d) \Leftrightarrow \exists Q[\Delta_v(e)(d)(Q) \wedge w-non-prefix_{<_S} - closed_v(Q) \wedge \forall Q'[\Delta_v(e)(d)(Q') \wedge w-non-prefix_{<_S} - closed_v(Q') \rightarrow Q' = Q] \wedge \forall Q''[\Delta^*_v(e)(Q'') \wedge w-non-prefix_{<_S} - closed_v(Q'') \rightarrow Q'' \leq^*_v Q]]$
7.$Role^5_v(e,d) \Leftrightarrow Role^5 *_v (e,d) \wedge MaxRole^{5*}_v(e,d)$
8.$Role^{6*}_v(e,d) \Leftrightarrow \exists Q[\Delta_v(e)(d)(Q) \wedge w-non-prefix_{<_S} - closed_v(Q) \wedge \forall Q'[\Delta_v(e)(d)(Q') \wedge w-non-prefix_{<_S} - closed_v(Q') \rightarrow Q' = Q] \wedge \forall Q''[\Delta^*_v(e)(Q'') \wedge w-non-prefix <^*_v -closed_v(Q'') \rightarrow Q \leq^* vQ'']]$
9.$Role^6_v(e,d) \Leftrightarrow Role^{6*}_v(e,d) \wedge Max_{Role^{6*}_v}(e,d)$

$Max_{R^{n*}_v}$ is defined as follows:

(26) $Max_{R^n_{v_v}}(e,d) = def : \forall d' \forall Q'[\Delta_v(e)(d')(Q') \wedge R^n_v(e,d') \rightarrow \exists Q''[\Delta_v(e)(d')(Q'') \wedge \forall Q'''[\Delta_v(e)(d)(Q''') \rightarrow Q'' \leq_v Q''']]]$

Intuitively, $Max_{R^n_v}(e,d)$ means that for all participants d' of e that are assigned the results determined by Rn^*_v is involved not earlier than d'. As was said above, a role $Role^n_v$ is relativized to an event-type P_v. The corresponding unrelativized role is defined as the union of the relativized ones.

(27) $Role^n = [\cup_{v \in VERB} Role^n_v$

•examples for the different types of results

(28) 1.d_1 eats d_2

d_1: is assigned the prefix* $<_S$ -closed result ($Role^1_{eat}$)

d_2: is assigned the prefix$^*_v$-closed result and the s-non-prefix$_{<_S}$ -closed result ($Role^3_{eat}$), this is also true of d_1, but d_2 is maximal w.r.t. this property; therefore d_2 is assigned Role$^3_{eat}$ w.r.t. to e (in the definition only the maximality w.r.t. the prefix$^*v$-result is required which is sufficient to discern d_2 from d$_1$)

2.$d1$ pushes d_2

d_1: is assigned the prefix* $<_S$ -closed result ($Role^1_{push}$)

d_2: is assigned the prefix$^*_v$-closed result ($Role^{2*}_{push}$); this result is also assigned to d_1 but only d_2 is maximal w.r.t. this property ($Role^2_{push}$)

3.d_1 takes d_2 from d_3 (the objects are different)
d_1: is assigned the prefix* $<_S$ -closed result ($Role^1_{take}$)
d_2: is assigned two $w-$non-prefix$<_S$ -closed results ($Role^4_{take}$)
d_3: is assigned only one w-non-prefix$<_S$ -closed result that is not maximal with respect to $\leq^*_v$ ($Role^6_{take}$)d.
4.d_1 gives d_2 to d_3 (the objects are different)
d_1: is assigned the prefix$^*_{<_S}$ -closed result ($Role^1_{give}$)
d_2: is assigned two w-non-prefix$_{<_S}$ -closed results ($Role^4_{give}$)
d_3: is assigned only one w-non-prefix$<_S$ -closed result that is maximal with respect to $\leq^*_{xv}$ ($Role^5_{give}$)

0.6.3 Formal Definition of the Subject determined by a Voice Affix

In (28)-(30) formal definitions of the subject determined by the voice affixes are given ($\Delta'_v(e)(d) iff \exists Q : \Delta_v(e)(d)(Q)$).

(29) 'um-': $\iota d \exists Q[\Delta_v(e)(d)(Q) \wedge prefix^* <_S -closed_v(Q)]$

(30) '-in': $\iota d[\Delta'_v(e)(d) \wedge \forall Q[Q \in Res(P_v, e) \wedge closed_v(Q) \rightarrow \Delta_v(e)(d)(Q)]]$

(31) '-an': $\iota d \exists Q[\Delta_v(e)(d)(Q) \wedge \forall Q'[Q' \in Res(P_v, e) \rightarrow Q' \leq_v Q] \wedge \forall d' \forall Q'[\Delta_v(e)(d)(Q') \wedge \forall Q''[Q'' \in Res(Pv, e) \rightarrow Q'' \leq_v Q'] \rightarrow |Res_{d_{max}}(P_v, e)| \leq |Res_{d_{max}}(P_v, e)|] \wedge \forall d'' \forall Q''[\Delta_v(e)(d'')(Q'') \wedge \forall Q'''[Q''' \in Res(P_v, e) \rightarrow Q''' \leq_v Q''] \wedge \forall d''' \forall Q'''[\Delta_v(e)(d''')(Q''') \wedge \forall Q''''[Q'''' \in Res(P_v, e) \rightarrow Q'''' \leq_v Q'''] \rightarrow |Res_{d''_{max}}(P_v, e)| \leq |Res_{d'''_{max}}(P_v, e)|] \rightarrow \exists Q'[\Delta_v(e)(d'')(Q') \wedge \forall Q''[\Delta_v(e)(d)(Q'') \rightarrow Q' \leq_v Q'']]]]$

In (30) $Res_{d_{max}}(P_v, e) = \{Q | \Delta_v(e)(d)(Q) \wedge \forall Q'[Q' \in Res(Pv, e) \rightarrow Q' \leq_v Q]\}$.

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A Subsystem Analyzing Georgian Word-Forms and Its Application to Spellchecking

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Homonyms essentially complicate the analysis of word-forms. Additional complications are created by the prefixes (besides suffixes), that are particularly characteristic for Georgian word-forms, and hinder the use of the abundant information contained in word stems. It must be emphasized that most of the prefixes are multi-homonymic: on the one hand, one and the same prefix may be of different kind – noun, as well as verb type. On the other hand, a prefix of one kind (mostly of verb type) may frequently have more than one (three, four) meanings.

Sometimes a prefix is not a homonymic one, but all the same creates a homonymic situation. Such a prefix may become a source of mistakes. In such cases, the resolution of a homonymy means the correction of a mistake as well. This comment refers mainly to verb prefixes, particularly, it is possible that prefix of a verb may be mistaken for the beginning of a stem or, vice versa, the stem beginning - for the end of prefix, or the stem beginning may be mistaken for prefix. The problem may be settled by dividing the vocabulary into zones and by a subsystem analyzing the Georgian word-forms.

This paper describes a subsystem for analyzing the prefix part of a word-form, as one of the constitutive parts of an analyzing processor. It may be used in spellcheckers, as well as in the word-form synthesizing processors. The prefix analyzing subsystem derives from the vocabulary, integrated in synthesizing/analyzing processors and word stems, which are necessary components of word-form synthesizing processors.

The creation of a Georgian spellchecker is connected with some specific problems. The basis of a "rigorous" spellchecker is an analytical processor, with the help of which the correctness/incorrectness of the content and structure of phrases and word-forms are established.

Prior to the final development of such processor (the morphological part of which is at the stage of correction), a spellchecker of a mixed type, constructed on the basis of two-directional processor (analysis-synthesis is meant) is proposed.

The proposed version of a spellchecker is able to correct only the mistakes of morphological level, the number of which is very great. It manages that through an interactive system, one of the main parts of a spellchecker, in which the typical mistakes made in the language are classified and a mechanism for their correction is included.

Key words: analysis, synthesis, processor, spellchecker, prefix

The analysis of word-forms is complicated by the problem of homonyms. The homonymic situation is aggravated, and hence specific obstacles are created also by prefixes and suffixes, which are particularly characteristic for Georgian word-forms, and hinder the use of the abundant information contained in the stems. For example while in the case of prefix-free forms “tb-ebi” (you warm yourself), “kats-ebi” (men) the isolation of stems and the identification of suffixes using the stem information is very easy, for analyzing such forms as “me-zghva-uri” (sailor), “ga-v-a-ket-eb” (I'll make), first of all the identification of the prefixes is needed. At the same time, we should remember that most of prefixes are multi-homonymic. On the one hand, one and the same prefix may be of the noun or verb type (me-zgvauri - sailor, me-zrdeba - it's grows for me), on the other hand, verb prefixes frequently (even most of the time) may express more than one meaning (“a” is: a verb prefix - “a-sheneba” (to built), a version indicator - “a-shenebs” (he is building), a descriptor of the situation - “a-khatavs”, - he is drawing over smth., etc.). In the above example “gavaketeb” (I'll make) the “ga-“ element, before considering the following components, must be characterized as a verb prefix (“gaketeba” - to make), an object indicator, and also, as a direct object, or indirect one, as a neutral version, and an indicator of surface directed situation (cf. “gakebs” (he is praising you), “gakhatavs” - he is drawing over you). Sometimes a prefix is not homonymic, but creates a homonymic situation all the same. Such prefixes may become a source of mistake. In these cases the resolution of the homonymic situation means the correction of the mistake as well. This may be helpful in the functioning of a spellchecker. These comments refer mainly to verb prefixes. There exist two kinds of mistakes. The first may occur with one group of verb prefixes (to which most of the verb prefixes belong). The error consists in mistaking stem letter-sounds following the verb prefix as a prefix of another kind. The second mistake is the following: due to mixing up “agh”, “gan”, “tsar” verb prefixes with “a”, “ga”, “tsa” verb prefixes, the stem elements “gh”, “n”, “r” are allocated to “a”, “ga”, “csa” verb prefixes, or the same verb prefixes “gh”, “n”, “r” are erroneously attributed to the stem.

The division of stem vocabulary into zones is very helpful for correction of mistakes. The mistakes of first kind may be overcome by dividing the vocabulary in nine zones, because there are nine possible prefixes starting with “a”, “g”, “e”, “v”, “i”, “m”, “n”, “s”, “u” letters. The second kind of mistakes may be avoided by help of a so-called “small vocabulary” where we have placed stems, starting with “gh”, “n”, “r” letters and compatible with “a”, “ga”, “tsa” verb prefixes (in difference with "agh", "gan", "tsar" verb prefixes). It must be noted that these "small" vocabularies are very small in size. To show how a subsystem for analyzing prefixes functions, a diagram of one unit of the subsystem is included (see diagram Prt1). The following comments refer to the diagram: if in a small vocabulary (li) there exists a stem starting with indicated letter (one of the "gh", "n", "r" letter-sounds), this would mean that we have the case "verb prefix – stem", hence verb prefixes are "a", "ga", "tsa" ("ageba", "ganadgureba" - to destroy, "tsartmeva" - to take away…), and naturally there is no need to search for other prefixes and the word form is sent to the table of suffixes (see output Spt1), and in case if the small vocabulary does not contain such a stem, then the presence of "agh", "gan", "tsar" verb prefixes is proved and it is taken into account that between the verb prefix and stem there may be other prefixes ("aghvadgen" – I'll destroy, "ganimarteba" – it explained , "tsarsadgeni" – which should be submitted …) as well, and, the word form will be sent to another table of prefixes (see output Prt2).

Almost all affixes (prefixes or suffixes) are homonymic in their nature, and, subsequently, the problem of identifying and "deciphering" of homonyms is solved mostly at the morphological level, but frequently the problem of homonyms must be solved at other levels (syntactic, semantic) as well.

In interpreting homonyms different means may be used: stem information (the information placed in the vocabulary at the word stem), suffix information for prefixes and prefix information for suffixes, and, generally, combinatorial analysis of stems and affixes. The basis for the use of these means is the fact that a word-form is a structure, and, naturally, shows the properties characteristic for a structure, which yields important information for studying word-forms – the structure is created by the compatibility or incompatibility of certain (and not all) affixes. Connection of an affix with an affix/a stem is restricted, by the regularities of structure that in turn is based on the principle of economy. As a result structural elements (in this case stem-affixes) are distributed in an extremely rational and economical way, and the language maintains this principle, raising it to the rank of perfect regularity, while skillfully using the methods of connecting the elements. In this way, it can assign to the same element different functions. In order to be more concrete, we will dwell on the question of the homonymy of "a", which may be an affix-marker of neutral version (agebs – he is building), a verb prefix (ageba – to build), a marker of surface directed situation (akhatavs – he is drawing over smth.), a creator of a causative situation (atirebs – he makes smb. cry), an indicator of direction (akhta – he jumped), and a derivative prefix expressing a negative nuance (alogikuri). According to our scheme, the noun prefix "a" is distinguished by stems (with a noun stem "a" is a derivative prefix) from the verb prefix "a", which still remains homonymic even after exclusion of the noun information. In the forms agebs/ageba two different "a"'s are distinguished by distribution of suffixes: with the stem complex "-ebs", prefix "a" is a neutral version prefix, with the stem complex "eba" – a verb prefix, if a stem is not of a passive voice (tbeba - it is getting warm, khmeba - it is drying). The neutral version may be distinguished from the surface-directed one by the stem type ("g" and "khat" stems are different types of stem). This information is placed with the stem. Because of this, in the form "agebs" prefix "a" is characterized as an indicator of a neutral version, and in the form "akhatavs" – as a marker of surface-directed situation. One fact more should be emphasized: the verbs that choose the category of surface directed situation and use the prefix "a" to mark it, make use of a zero marker for the neutral version (khatavs – he is drawing, cf. agebs - he is building). As for the verbs having "a" as a neutral version marker, they have no surface directed situation category at all (they have no need of it, for semantic reasons).

Below it will be shown how a subsystem, analyzing the prefix part of a Georgian word-form can be used in a spelling checker.

The creation of a Georgian spelling checker is connected with some specific problems, the main of which is the absence of protection of the literary language from the mistakes, which are very frequent and therefore very detrimental to the language. The most urgent task, needing radical measures, is the creation of systems which will, if not completely remove, at least somewhat decrease the number of mistakes and deviations from the literary language. One of such systems is a spelling checker, as a mechanism regulating and ascertaining the language norms, next to finding the normal mistakes.

The basis for the system of a "rigorous" spellchecker is an analyzing processor, by which the correctness/incorrectness of the content and structure of phrases and word-forms can be established.

Until the accomplishment of a such "high degree" analyzing processor (the morphological part of which is now in the stage of final correction), a spelling checker of a mixed type, based on a bi-directional processor (the analysis and synthesis) is proposed. The description of this version will be given below.

This spelling checker is able to remove only the morphological mistakes which occur in great number. The main part of the spelling checker is a processor synthesizing Georgian word-forms. It consists of a vocabulary of stems (roots) and of an algorithm, integrated with it, that constructs from affixes and stems correct (established in the literary language) word-forms (Margvelani, 1997). The algorithm operates in several modes. One of them is a mode constructing a paradigm and assigning to its members a complete set of adequate morphological characteristics, i.e. parameters of all kinds, reflecting categories, of which there are many for Georgian verb-forms and noun-forms.

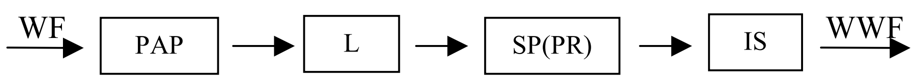
Word-forms obtained in this way may be used by spellcheckers as samples. A word-form at the input of the system, the correctness/incorrectness of which is to be proved, is compared with the reference. As a result the correctness/ incorrectness of the word-form can be established. It is obvious the above, that for solving of the problem, it is necessary to perform the following steps: identification of the stem, construction of a sample, and proposing of a correct form, if identification fails. As it was mentioned above, the system constructing the samples has been implemented and (if a stem is available), can perform its task. For separation of a stem, a subsystem analyzing the prefix parts of the word-forms is proposed (Margvelani, 1999). To get the stem of a complex word-form (e.g. "ga-ma-ket-eb-d-e-s" – if he had made me, where the stem is "ket"-doing) is a rather complicated problem.

The subsystem cuts off the prefix part, and sends the remainder (which may consist of a stem and suffix part) to the vocabulary of stems. As a result the stem becomes separated from the suffix part. The stem, obtained in this way, may be included in the above-mentioned system for constructing references.

As for the last question – comparison of word-form delivered at the input of the spellchecker, with the reference and replacement of the incorrect version by a correct one, we are in the process of development of an interactive system representing aggregation of subsystems.

Independently from the fact, whether the processor will perform the identification of a word-form only or a bidirectional (synthesizing-analyzing) process, an interactive system is equally necessary in both cases. That is why great attention is paid to its creation. An interactive spellchecker system will be designed for handling the linguistic mistakes. It will be constructed from subsystems. Each subsystem will deal with the field for which it is designed. The mistakes of morphological character will be corrected by a morphological interactive subsystem, syntactical ones – by a syntactical interactive subsystem and so on. It must be underlined here that the spelling checker may have to correct two kinds of morphological mistakes. First, the ones that take place within the word itself, and that are corrected within the same word (an obvious mistake). Second the ones that may be mistakes or not (a hidden mistake). The confirmation/correction of the latter ones is possible only after the analysis of the sentence. But the morphological subsystem must pass the notification about the possible mistake to the syntactic level.

The above version of the Georgian spellchecker may be presented diagrammatically in the following way:



The meanings of the symbols: WF – a word-form to be checked, WWF – the correct word-form, PAP – a processor analyzing the prefix parts, L – the vocabulary, SP(PR) – a synthesizing processor in the mode of paradigm, IS – an interactive system.

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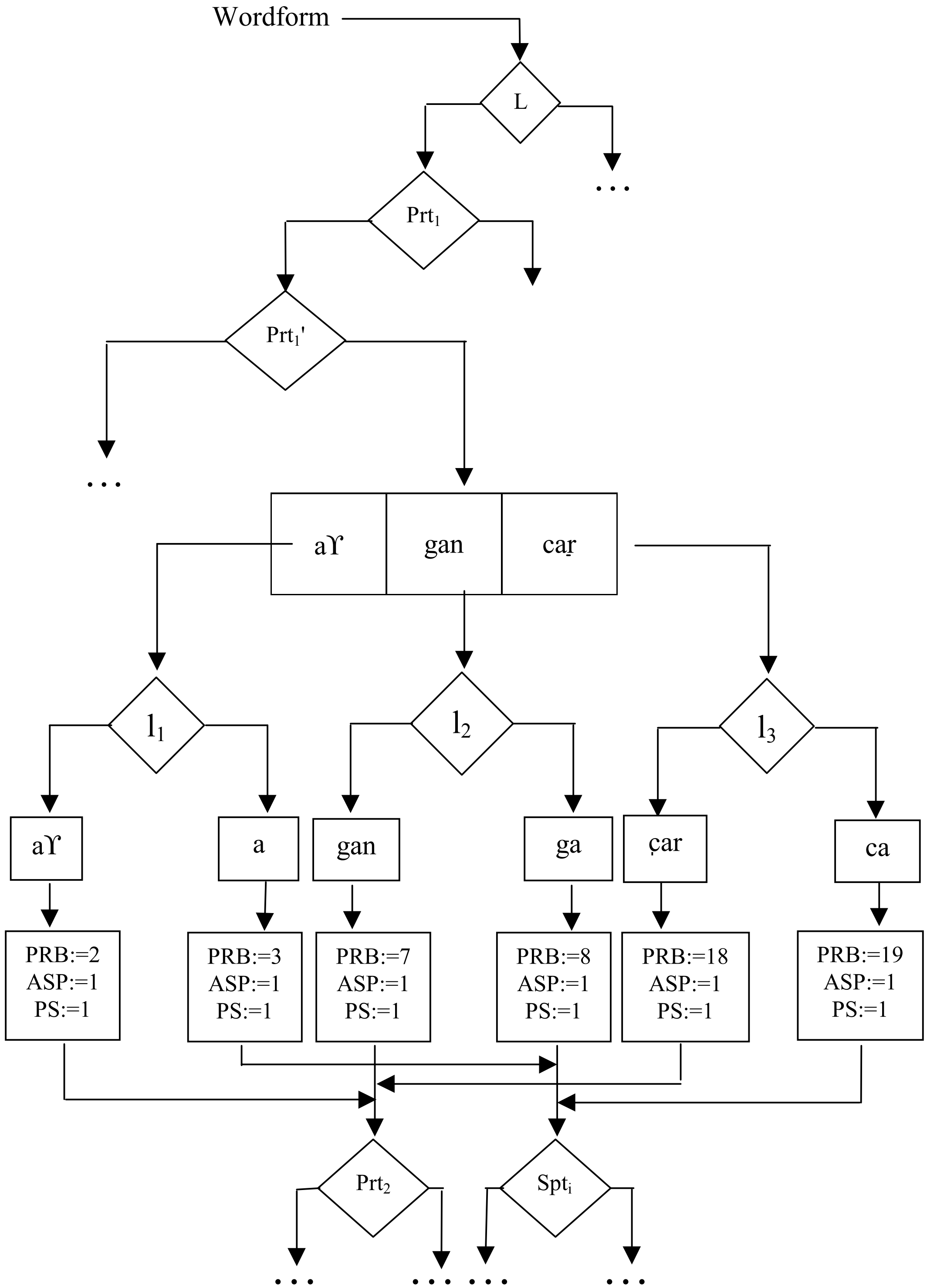
Scheme (Prt_1')



Wordform
L
...
Prt1
Prt1'
...
aϒ
gan
caṛ
l1
l2
l3
aϒ
a
gan
ga
ċar
ca
PRB:=2
ASP:=1
PS:=1
PRB:=3
ASP:=1
PS:=1
PRB:=7
ASP:=1
PS:=1
PRB:=8
ASP:=1
PS:=1
PRB:=18
ASP:=1
PS:=1
PRB:=19
ASP:=1
PS:=1
Prt2
Spti
...
...
...
...

Zero Sign (in Morphology

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Kurum, ma, magana ce
<Silence, too, is speech>.
A Hausa Proverb (Allen and Hill 1979[123]).

0.1 Notion of Zero Linguistic Sign and the Principle for the Postulation Thereof

Strange as it may seem, in spite of wide-spread and fruitful use of zero signs in linguistics, there is, as far as I know, no universally accepted good definition of the concept itself of linguistic zero sign: a maximally general one which would cover all possible types of zero signs and, at the same time, be rigorous and logically satisfactory. Paradoxically, such a definition is easy to supply, proceeding from the basic concept of linguistic sign as a triplet ⟨<Signified> ; /Signifier/ ; Σ(Syntactics)⟩. Here, 'Signified' (= signatum, *signifié*) and 'Signifier' (= signans, *signifiant*) are taken in their Saussurian sense, and 'Syntactics' is a set of combinatorial properties, or features, of the sign that are determined neither by the signified nor by the signifier (these are such features as part of speech, grammatical gender, government pattern, etc.). Formally, then:

Definition 1: Zero linguistic sign

A *zero linguistic sign* **X** is a sign whose signifier is empty: $\mathbf{X} = \langle < X >; /\Lambda/; \Sigma_X \rangle$
(Λ stands for 'the empty set;' a zero sign is written as **Ø**; cf. Introduction, p. 00.)

Let it be emphasized that the signifier of a zero sign is by no means a perceptible phonetic pause - it is strictly the ABSENCE of a particular signifier in a particular position where such signifiers must be expected (Janda and Manandise 1984[234] against the identification of linguistic zeroes with phonetic pauses). The addressee of an utterance with a zero sign - such as, e.g., Rus. $ruk + \emptyset < of hands >$, where the zero suffix expresses <plural, genitive> -identifies the zero not because of a silence (the next wordform may follow *ruk* without any slightest pause), but because the perceptible form *ruk* is not followed by one of possible inflectional suffixes.

The concept of zero sign is thus obtained as a natural extension of the concept of linguistic sign-by allowing one of its three components, namely the signifier, to be empty.

However, even if Definition 1 is clear and rigorous, it is absolutely not sufficient: it does not constrain the use of zero signs by linguists. Zero is a powerful device, so that a linguist can be easily tempted into postulating zeroes everywhere, as soon as a zero helps to make his description more consistent or elegant. An unrestricted use of zeroes empties them of any positive content; they may become a sort of a convenient stopgap- LINGUIST'S zeroes, instead of being genuine linguistic signs, i.e. LANGUAGE zeroes. Zero signs should be an apt generalization of 'more normal' non-zero, i.e. overt, signs. As Haas 1968a[34] put it:"The novel ... use of the term ['ZERO sign'-IM] must not be allowed to interfere with the established ... uses [of the term 'OVERT sign'-IM]." Therefore, we need a stringent Principle that will guide our choices in the process of introducing zeroes, so that some presumed zeroes would not be admitted. Such a Principle is proposed immediately below. It is aimed at avoiding zeroes wherever you can avoid them:"If you can do without a zero, you **should** do without a zero" (Plungjan [149] translation is mine-IM).

Reasoning within the framework of the Meaning-Text theory entails, among other things, considering introduction of zero signs exclusively from the viewpoint of text synthesis, or of speaking-that is, in the process by which the speaker moves from meanings to texts. In simple terms, I study how some meanings present in the Semantic Representation of the intended utterance are expressed on a higher level - syntactic or morphological - by the absence of ‘physical’ signals. Metaphorically speaking, a zero linguistic sign is a meaningful absence. The Principle presented below is an attempt to capture the essence of ‘meaningful absence.’

0.1.1 Principle for the introduction of zero linguistic signs [= IZLS Principle]

Let there be an expression E -a clause or a wordform-of language L; a zero sign found at the clause level is a SYNTACTIC zero (= a zero wordform), while a zero sign found at the wordform level is a morphological zero (= a zero morph or a zero morphological operation).

The presence of a zero linguistic sign X in E is admitted if, and only if, the following three conditions are simultaneously satisfied:

1. Expressiveness: E carries a meaning $< X >$ or the value γ of a syntactic feature Σ[1] such that $(X)/\gamma$ has to be ascribed to X as (a part of) X’s signified/as (a part of) X’s syntactics.
2. Exclusiveness: E does not contain a non-zero signifier to which $< X > /\gamma$ could be ascribed in a systematic and natural way at any level of representation.[2]
3. Contrastiveness: E admits, in the corresponding position, a semantic contrast between X and another non-zero sign X' [in other terms, X has a distinctive value].

A zero sign must always do a clearly circumscribed job (= express some content really present in the utterance, i.e. carry an information payload); it must do it in the absence of other contenders (= be exclusive on the job, i.e. the very last resort of our description); and it must be opposed to a non-zero sign (= distinguish two utterances, i.e. participate in a semantic contrast).

0.2 Different Types of Zero Signs

The above formulation of the IZLS Principle allows for various types of linguistic zeroes.

- First, along with better known morphological zeroes, there are NON-MORPHOLOGICAL ZEROES, that is, ZERO WORDFORMS or ZERO LEXEMES, which can be called syntactic, or lexical, zeroes. In most cases, zero wordforms are zero megamorphs: for instance, the Russian copula BYT <[to] be> in the present of the indicative: Ø$^{BYT'}_{pres.ind,3sg}$; cf. *Ivan bolen* <Ivan [is] sick> (where no overt copula is possible) vs. *Ivan* **byl** ⟨**budet**⟩ *bolen* (Ivan was ⟨will be⟩ sick>. Examples of zero lexemes include the Russian lexemes Ø^{PEOPLE} and Ø^{ELEMENTS} which appear as subjects in the syntactic structure of the sentences *Ivana ubili* <Some people killed Ivan> vs. *Ivana ubilo* ≈ <Something mysterious killed Ivan>: see Mel’čuk 1997[178ff].
- Second, among morphological zeroes, we can also get NON-GRAMMATICAL zeroes, i.e. zero radicals in wordforms having non-zero affixal parts. I could cite three examples of zero radicals.

  (1) Kirundi

  Deictic demonstratives in Kirundi in different noun classes

[1](IZLS Principle, p. 00) As an example of a zero sign carrying only some values of syntactic features but having an empty signified I consider empty zero signs in 5, p. 00ff.

[2]This requirement (“Don’t introduce a zero sign in the presence of an explicit formal difference”) was formulated, in a very clear manner, in Nida 1948[256]). It was later vigorously elaborated in Haas 1968b[35]; cf., for instance: “Two obvious carriers of a semantic distinction ... [should not be] ... ousted by the introduction of two ghosts-presence of zero and absence of zero” [speaking of a viewpoint according to which *go* and *went* are allomorphs of the same morpheme and are distinguished by a zero of the past tense in *went* ‘contrasting’ with an absence of a tense marker in *go*]. Haas called a fictitious zero used instead of a perceptible distinction a ‘quid pro quo’ zero.

noun class:	I	II	III	IV	V	V
1. <this-close to 1st person> (Sp. este)	uwu	aba	uwu	iyi	iri	aya
2. <this-close to 2nd person> (Sp. ese)	uwo	abo	uwo	iyo	iryo	ayo
3. <this-close to 3rd person> (Sp. aquel)	uryá	bárya	uryá	iryá	rírya	aryá
4. <that-very far from 1st& 2nd persons>	uríýa	báriíya	uríýa	iríýa	ríriíya	aríýa

In lines 2 - 4, we see the radicals **-o, -rya** and**-ríýa**, preceded by class prefixes, which show the agreement with the modified noun: **u-, ba-, u-, i-, ri-** and **a-**. In the actual forms, we have the following three alternations: consonantization /u/ ⇒ /w/ and /i/ ⇒ /j/ (spelled *y*) before a vowel; truncation of /a/- before a vowel; and epenthesis of /w/ and /j/ between vowels. Moreover, if the form obtained is monosyllabic, the class prefix is preceded by an epenthetic vowel identical to its own vowel, for instance: *ba+o* ⇒ *bo* ⇒ *abo* (class II); *ri*+o ⇒ *ryo* ⇒ *iryo* (class V); etc.

Now, what is the radical of the wordforms in line 1? They consist of a class prefix preceded by an epenthetic vowel (because of forbidden monosyllabicity): *u* ⇒ *uu* ⇒ *uwu*, *ba* ⇒ *aba*, etc. But a class prefix is a PREFIX - it must precede a radical, which has to carry the deictic meaning. Therefore, these wordforms contain a zero radical: $-\mathbf{Ø}^{\boldsymbol{THIS}}$ a sign of the following structure:

$-\mathbf{Ø}^{\boldsymbol{THIS}}$= ⟨ <this-close to the 1st person> ; /Λ/ ; Σ = radical, demonstrative Adj, ... ⟩

(2) Serbo-Croatian
3rd person pronominal clitics (MiliŤeviŤ 2000)
They exist only in the genitive-accusative and the dative; I show their masculine and neuter forms in parallel with the corresponding full forms:

	singular (masculine and neuter)		plural	
	full form	clitic	full form	clitic
genetive = accusative	*nj+ega*	*ga*	*nj +ih*	ih
dative	*nj+emu*	*mu*	*nj+ima*	*im*

The radical of the pronoun {ON <he>} in the full forms of all cases of the two numbers, except for the nominative, is **nj-** /N/; **-ega**, **-emu**, **-ih** /ix/ and **-ima** are cumulative suffixes of gender, number and adjectival case. These are the same suffixes as those found in all the adjectives of the corresponding declensional type, e.g., VRUŚ <hot>:

[SG, MASC/NEU] GEN *vruŤ* +**eg(a)**, DAT *vruŤ* +**em(u)**,
[PL] GEN *vruŤ* +**ih**, DAT *vruŤ* +**im(a)**.

The clitic wordforms have only suffixes (abridged); the meaning <he> is expressed by the absence of a radical which would 'support' these suffixes, that is, by a zero radical allomorph of the morpheme ON <he>. The morphic representation of these clitics is as follows (for economy's sake, I omit the genitive forms, homophonous with those of the accusative):

ga =	$Ø^{HE}$ =	⟨<he>	; /Λ/ ;	Σ = radical, clitic pronoun, 3rd person, ⟩
	⊕			
	ga =	⟨<masc, SG, acc>	; /ga/ ;	Σ = suffix, of a clitic pronoun, ⟩
mu =	$Ø^{HE}$ =	⟨<he>	; /L/ ;	Σ = radical, clitic pronoun, 3rd person, ⟩
	⊕			
	mu	⟨<masc, SG, dat>	; /mu/ ;	Σ = suffix, of a clitic pronoun, ⟩
ih =	$Ø^{HE}$=	⟨<he>	; /Λ / ;	Σ = radical, clitic pronoun, 3rd person, ⟩
	⊕			
	ih =	⟨<pl, acc>	; /ix/ ;	Σ = suffix, of a clitic pronoun, ⟩
im =	$Ø^{HE}$ =	⟨<he>	; /Λ / ;	Σ = radical, clitic pronoun, 3rd person, ⟩
	⊕			
	im =	⟨<pl, dat>	; /im/ ;	Σ = suffix, of a clitic pronoun, ⟩[3]

Zero radicals are paradoxical entities, and as such they are rare in human languages. This is understandable: radicals are meant to designate a huge number of poorly organized signifieds (= lexical meanings), and it is difficult to use an absence to signify something if there is no fixed position in which a limited number of elements is supposed to appear, so that this absence could readily contrast with one of few 'presences'. For this reason, both of the above zero radicals are found in a strongly grammaticized part of the lexicon: among pronouns, where the number of possible signifieds is small and the oppositions well marked. Another example of a zero radical known to me is the radical of the verb meaning <give> in a Papuan language, Awa:

(3) Awa:

a. Ø +nuw +éhq = Nuwéhq
give my PAST.3_{SG}
([He] gave something mine>.

b. Néne sòn nuwéhq
my garden give.my.PAST.3_{SG}
([He] gave my garden>.

c. Keki +nuw +éhq = Kekinuwéhq
burn my PAST.3_{SG}
([He] burnt something mine>.

d. Néne òn kekinuwéhq
my garden burn.my.PAST.3_{SG}
([He] burnt my garden>.

Comparing (3a, 3b) to (3c, 3d), we see that the meaning <give> is expressed by the absence of a radical before the inflectional ending **-nuwéhk**; this means a zero radical.

- Third, the grammatical morphological zeroes are all inflectional, i.e. they are inflectional zero affixes (= they always express grammemes). As stated in Section 12, derivational zeroes are not admitted.
- Fourth, the IZLS Principle does not limit us to zero segmental signs (= to zero morphs and zero megamorphs). It allows also for zero operation signs: zero reduplications, zero apophonies, and zero conversions. These are operations whose output is identical to their input; such 'null-modifications' are introduced only in opposition to non-zero operations; cf., for instance, Eng. foot, where the singular is expressed by a zero $A^{\Lambda\Lambda_{SG}}$ apophony opposed to the $A^{/u/}_{PL}$ apophony, which expresses the plural in feet. (For more on zero morphological operations, see Mel'čuk 1982[51, 101-102] and Mel'čuk 1993-2000[vol. 4: 286, 304, 321]

0.3 The Requirement of Non-Zero Alternants

The IZLS Principle does not require that a zero sign should necessarily have a non-zero alternant, that is, that any zero sign should have a fully synonymous non-zero (= overt) partner; a zero sign can be a unique allomorph of its morpheme or a unique lex of its lexeme. Thus, in the wordform book the singular is expressed by a zero suffix $-\emptyset_{SG}$, opposed to the plural suffix **-s**; this can be maintained even without having recourse to foreign overt singular markers as those in alumn+**us**, phenomen+**on** or virtuos+**o**, which could be quoted as non-zero alternants of $-\emptyset_{SG}$. After all, I admit the singular zero suffix in Spanish, where no non-zero alternant can be found: *libro*+$\emptyset_{SG}$ <book>, <*rbol+* $\emptyset_{SG}$ <tree>, etc.

Haas 1968b[45-57] strongly rejects 'unsupported' zero signs, i.e. zero signs having no overt alternants. For him, the only justification for associating a meaning with a zero must be that the same meaning is also associated with a non-zero: since the meaning <singular> is never expressed by an overt form in English, "we should leave it merged in the total semantic values of forms like *cat, boy,* etc." (p. 47). The last statement raises, however, two serious objections:

- First, if we say that the meaning <singular> is included in the signified of the radical cat-, then the meaning <plural> of the suffix **-s** must be replacive for all English nouns: it will have to push out the meaning <singular>, which is already present in the radical, and take its place. However, 'unsupported' zeroes are widespread in different languages:
  $-\boldsymbol{\emptyset}_{\boldsymbol{3_{SG}}}$ in Serbo-Croatian (verb: <ita+Ø <[he] reads> vs. <ita+**m** <[I] read>, <*ita+á* <[*you*$_{SG}$] read>, ...);

  $-\boldsymbol{\emptyset}_{\boldsymbol{MASC}}$ in Russian (verb: *spa+l*+Ø <[he] slept> vs. *spa+l*+**a** <[she] slept>, *spa+l+o* <[it] slept>; or predicative adjective: *gotov*+Ø <[he is] ready> vs. *gotov*+**a** <[she is] ready>), *gotov+o* <[it is] ready)>;

  $-\boldsymbol{\emptyset}_{\boldsymbol{PRES.IND}}$ in Spanish and French (*canta*+**Ø**+*mos* <[we] sing> vs. *cantá*+**ba**+*mos* <[we] sang>, *canta*+**r**+*emos* <[we] will sing>,

  etc., just to name a few among the best known cases. As a result, if we agree with Haas, numerous inflectional meanings will turn out to be replacive. Although I admit replacive meanings in special situations (Mel'čuk 1991 and Mel'čuk 1993-2000, vol. 4: 45, 332, 402), I am not prepared to say that most grammemes are replacive. This would change the picture of linguistic morphology in too drastic a manner to be easily digested.
- Second, and more importantly, the radicals **cat-, boy-, book-**, etc. do not carry the meaning <singular>. This is clearly seen in compounds: a **mousetrap** is for catching mice, not one mouse; the **toothbrush** is for teeth, not for one tooth; and a **bookbinder** binds books rather than one book. What expresses the meaning <one book> is the complete wordform **book** rather than the (homophonous) radical *book-*; and this wordform does contain a singular zero suffix. The same consideration is valid for Spanish: the person who is **oj+inegro** <black-eyed> has black eyes (*ojos*), not one black eye (*ojo*); **pat+ituerto** <crooked-legged> has two crooked legs (*patas*), not just one (pata); etc.

The meaning <singular> cannot be associated directly with the radical in the case of English nouns (and in all similar cases); therefore, unsupported (= lacking overt alternants) zero signs must be admitted. An immediate corollary of this is the existence of zero -emes (sets of signs): a zero morpheme/zero lexeme that contains only a zero element (a zero morph/allolex). For instance, the Spanish morpheme SINGULAR is a zero morpheme, while the Russian lexeme $\emptyset^{people}$ and the Spanish lexeme $\emptyset^{impers}$ are zero lexemes. (Cf. Bazell 1949s remark on the possibility of zero morphemes: Bazell 1949[1966: 225, fnt. 26].) The morpheme {3SG} in Sierra Totonac is another good example of a zero morpheme (from Nida 1948[46]: “The third person singular is never indicated overtly,” i.e. this combination of grammemes is expressed only by a zero suffixal morph, which is the only allomorph of the corresponding zero morpheme):

1_{SG}	**k-**	1PL	-wi
2_{SG}	**-ti**	2PL	-tit
3_{SG}	-Ø	3PL	-**q**d≫

0.4 Empty Zero Signs

Condition 1 of the IZLS Principle allows, among other things, for an EMPTY SYNTACTIC, or LEXICAL, zero sign, i.e. a sign whose both the signified AND the signifier are empty and which exists only because of its syntactics: it controls particular agreement somewhere in the clause. Such is the ’impersonal’ zero pronoun in Spanish or Russian, which is found, for instance, in sentences with a meteorological verb, such as *Llueve* <[It] rains> or *Svetaet* <[It] dawns>: here, the empty wordform $Ø^{IMPERS}_{3sg}$ requires the agreement of the Main Verb in the 3rd person singular (in Russian, also in the neuter gender in the past). The wordform $Ø^{IMPERS}_{3sg}$ corresponds to the Fr. *il*, Germ. *es*, and Eng. *it*, these three being equally empty, but non-zero. On the other hand, the zero wordform $Ø^{ELEMENTS}_{neuter}$ in Russian, as in *Kräu sorval+***o** *vetrom*, lit. <[It] tore the roof away with the wind>, is not empty: it means <mysterious/natural forces> and contrasts with $Ø^{PEOPLE}_{pl}$ (*Kryäu sorval+***i** *vetrom*, lit. <[They ≈ Some people] tore the roof away>), as well as with non-zero subjects. (Each of these zero wordforms constitutes a one-lex zero lexeme, see above.)

An alternative description would be to say that meteorological (and other impersonal) verbs do not agree with anything, but appear always in the 3_{SG} form. Such a description is logically possible; however, it would destroy an obvious parallelism 1) between impersonal verbs and all other verbs (within a given language), 2) between impersonal and personal uses of the same verb (*Slony sorvali* ... <The elephants tore away ... > vs. *Sorvali* ... ([<They ≈ Some people] tore away ... >), and 3) between sentences with impersonal empty non-zero pronouns in some languages and structurally identical sentences, but without such pronouns in other languages (Fr. **Il** *pleut* vs. Sp. *Llueve*). Looking for a more homogeneous treatment, I prefer to stick to zero empty wordforms/lexemes.

Nevertheless, I have to point out that an empty zero subject wordform/lexeme is-unlike all other zero signs-non-contrastive; it is imposed by the syntactic context (the necessity of a subject in a clause for the Main Verb to agree with) and has an empty signified, so it cannot contrast semantically with anything. To accommodate it formally, we need an additional condition in the IZLS Principle. (I am indebted to N. Pertsov, who drew my attention to this fact.)

0.5 Zero as a Last Resort

Condition 2 of the IZLS Principle protects us against the proliferation of zeroes in all those cases where the information involved (= the meaning <X> or the value γ of a syntactic feature) is carried by another, non-zero sign. This means that, generally speaking, one should not look for a zero marker where one could see a real, ‘physical’, i.e. overt, difference: a zero sign must be exclusive as a possible carrier of the information in question or there is no zero. In other words, “ceteris paribus, accounts that do without zeroes are always to be preferred over ones that include them” (Janda and Manandise 1984[231]). A zero sign should be introduced only if there is no other linguistic means available to take care of the observed chunk of meaning to be expressed. Let it be emphasized that linguistic means include more than segmental signs, i.e. morphs; there are also reduplications, apophonies and conversions, and all these overt operation signs are valued higher than zero signs: they should be preferred over a zero. Let us consider the following simple example.

(4) German

The wordform **Mütter** <mothers> has no plural zero suffix $-\emptyset_{PL}$, because Mütter contains a non-zero signifier to which the meaning <plural> can be ascribed in a natural and systematic way: this is the Umlaut alternation /u/ ⇔ /ü/, applicable to the corresponding singular wordform Mutter (without Umlaut). German has many plurals of this type:

*V***a***ter*	<father>	V**ä***ter*	<fathers>
A*pfel*	<apple>	**Ä***pfel*	<apples>
*F***a***den*	<thread>	*F***ä***den*	<threads>
*V***o***gel*	<bird>	*V***ö***gel*	<birds>
O*fen*	<oven>	**Ö***fen*	<ovens>
*T***o***chter*	<daughter>	*T***ö***chter*	<daughters>
*Br***u***der*	<brother>	*Br***ü***der*	<brothers>

All these pairs show an obvious phonemic difference with which the signified <plural> can be naturally associated. Therefore, the plural formation in (4) must be described by the apophonies $\boldsymbol{A_{PL}^{/a//\epsilon/}}$, $\boldsymbol{A_{PL}^{/o//ö/}}$ and $\boldsymbol{A_{PL}^{/u//ü/}}$. The signified (singular) is expressed in corresponding nouns by the absence of any modification-in our terms, by the zero apophony $\boldsymbol{A_{SG}^{\Lambda\Lambda}}$. On the other hand, since the wordform **Mütter**, which represents the nominative, the genitive and the accusative in the plural, contrasts with the plural dative form **Mütter+n**; therefore, the wordform **Mütter** contains a zero case suffix: $\boldsymbol{-\emptyset_{NOM/GEN/ACC}}$, which is opposed to the -n of the dative. As a result, the wordform **Mütter** has the following morphic representation:

$$\boldsymbol{Mutter \oplus A_{PL}^{/u/ü/}\emptyset_{NOM/GEN/ACC}},$$

namely, a radical morph, a plural apophony and a zero case suffix. The wordform Mutter has of course a different morphic representation:

$$\boldsymbol{Mutter \oplus A_{SG}^{\Lambda\Lambda}\emptyset_{NOM/GEN/DAT/ACC}}$$

We cannot say that the singular of **Mutter** is expressed by the radical itself (rather than by a zero apophony): in compounds, this radical does not necessarily imply <singular>, as, for instance, in **Muttertag** <Mothers' Day> or **Mutterschutz** <mother protection>, cf. 3.

0.6 Zero Signs and Parasitic Formations

Condition 2 helps us make a decision in the cases where one morphological form is built on another complete form-what is known as **parasitic formations** (Mel'čuk 1991 and Mel'čuk 1993-2000[vol. 4: 46-47]); as an example, I will present a well-known parasitic formation-secondary cases in Daghestanian languages.

(5) Archi (Kibrik 1997[27-28]; the zero suffixes are my addition: in the left column, the first zero marks the singular, while the last zero in the first line marks the nominative)

The noun GEL <mug, tankard>

	singular	plural
nominative	$gel{+}\emptyset + \emptyset$	$gel{+}um{+}\emptyset$
ergative	$gel{+}\emptyset + li$	$gel{+}um{+}\check{c}aj$
genitive	$gel{+}\emptyset + li + n$	$gel{+}um{+}\check{c}e\ {+}n$
dative	$gel{+}\emptyset + li + s$	$gel{+}um{+}\check{c}e\ {+}s$
comitative	$gel{+}\emptyset + li + llu$	$gel{+}um{+}\check{c}e\ {+}llu$
comparative	$gel{+}\emptyset + li + xur$	$gel{+}um{+}\check{c}e\ {+}xur$
...	...	...

Beginning with the genitive, all the cases are expressed by suffixes added to what seems the complete form of the ergative, marked by **-li** in the singular and by **-čaj/-če** in the plural. This situation can be described in two opposite ways:

- Either we say that the genitive, the dative, etc. are built on the complete form of the ergative; then we have to admit replacive signifieds: when the suffix of the genitive **-n** is added after the suffix of the ergative **-li**, the signified <genitive> replaces the signified <ergative>, previously brought into the wordform by **-li**. This is my viewpoint (cf. Mel'čuk 1991 and Mel'čuk 1993-2000, vol. 4: 47); however, I cannot enter now into the justification thereof. (Technically, in the form of the ergative, the suffix **-li** is selected for its signified; in the form of any other oblique case, it is selected automatically-together with the suffix of the corresponding case.)
- Or we say that all the oblique cases-the genitive, the dative, etc., including the ergative itself!-are formed from the *oblique stem* of the noun; the suffixes **-li** in the singular and **-čaj/-če** in the plural are then not markers of the ergative, but those of this oblique stem. This is the viewpoint of Kibrik 1992[81-82]Kibrik 1997[27-28][4]

If we accept the second viewpoint, we have to admit that the ergative is marked by a zero suffix, which would be in contrast with all other case suffixes, except, of course, for the nominative: the nominative also has a zero suffix. Then we have two forms-*gel* NOM and *gelli* ERG-that are said to differ only by two different zero suffixes, one of the nominative, the other of the ergative, while the perceptible difference (**-li**) is considered to be meaningless; cf. *man* vs. *mes* or *take* vs. took. However, such an ergative zero suffix would violate Condition 2 of our Principle: the signified <ergative> CAN be associated with the suffixes **-li** and **-čj/-če**, therefore it should-and thus we have to accept the first viewpoint. As a consequence, if we keep Condition 2 in the IZLS Principle, we have to agree to a description of secondary cases I.1b that admits case formation from a complete case form (and of course replacive signifieds).

0.7 Irrelevant Physical Distinctions Accompanying Zeroes

Condition 2 contains two important provisos: one which requires that the expression of the information in question be SYSTEMATIC AND NATURAL; and another one which requires that a possible candidate for the carrier of this information be absent at all levels of representation, i.e. including the deeper ones.

To illustrate the first proviso, let me consider a situation where there exists a physical distinction δ between two wordforms showing a semantic distinction $< \sigma >$, but where, in spite of this, the researcher has to posit a zero sign which expresses $< \sigma >$, ignoring δ: it is impossible to associate $< \sigma >$ with δ in a systematic and natural way. Here is an example.

(6) Russian The paradigm of the noun SESTR(-*á*) <sister> has, among others, the following forms:

	singular	plural
nominative	/s'istrá/	/s'óstri/
genitive	/s'istrí/	/s'is't'ór/
dative	/s'istr'é/	/s'óstram/

The morphological structure of these forms is transparent enough: each of them is composed of two morphs, a radical and a number-case suffix.

- The underlying radical has the signifier /s'os't'or/. This signifier never appears as such on the surface; in the process of synthesis, it is modified by the following five morphonological rules,

[4](5, p. 00) In most Daghestanian languages, the existence of a special oblique stem in the declension of the noun cannot be doubted. Thus, in Tsakhur the ergative is expressed by a suffix added to this oblique stem - just like all other case suffixes are. The above reas oning applies only to Archi. Note that under my analysis, the Archi ergative is quite different from other cases: it has a cumulative case-number marker, while all other cases are expressed agglutinatively.

which based on it, construct predictable allomorphs:

-substiution
/o/ ⇒ /i/
(after a palatalized consonant in an unstressed syllable, notated as);
-truncation
/o/ ⇒ Λ
(a fleeting /o/ falls in a radical marked as undergoing this rule, before a suffix that begins with a vowel);
-substitution
/r/ ⇒ /r'/
(a consonant becomes palatilized before the suffix **-e**);
-substitution
/t'/ ⇒ /t/
(a dental consonant looses its palatalization before another consonant, with which it comes in contact as a result of the fall of a fleeting vowel);
-substitution
/s'/ ⇒ /s/
(a fricative consonant looses its palatalization before a non-palatalized consonant).

- The cumulative suffixes of number and case are *-á, -í, é, . . . , -i,* $-\emptyset$*;, -am*

It is the zero suffix of the genitive plural that is problematic: we postulate it in spite of the fact that the forms /s'i**st**r+á/ [SG.NOM] and /s'i**s't'**ór/ [PL.GEN]-if we ignore the suffix **-a**-show a physical difference: **/st/** ∼ **/s't'ó/**. This difference, however, is a result of the application of some morphonological rules, which are extremely productive in Russian: they apply to thousands of nouns depending on morphological/phonological context, but without any direct link to the plural or the genitive. It is simply impossible to say that in /s'is't'ór/ the combination of grammemes (plural, genitive) is expressed by the substitution operation /st/ ⇒ /s't'ó/. Even if truncation (or insertion) of a fleeting /o/ is very frequent in Russian (it affects thousands of radicals), it is not at all related to the expression of the plural or the genitive: thus, this fleeting /o/ appears in the nominative of the singular in masculine nouns (*úg***o***l* <angle, SG.NOM> ∼ *ugl+á* <angle, SG.GEN>) or in denominative adjectives (*ok***ó***n+n+yj* <window [pane]> ∼ *okn+ó* <window, SG.NOM>). The presence/absence of a fleeting /o/ in Russian nouns depends only on morphonological conditions (a fleeting /o/ is truncated in an unstressed syllable before a vocalic morph). Moreover, Russian does not use any morphological operations to express any grammemes; therefore, the statement. In /s'is't'ór/, the plural and the genitive are expressed by the substitution /st/ ⇒ /s't'ó/' is anti-systematic and anti-natural to the highest degree. If we try to link the signified <plural, genitive> to the /st/ ⇒ /s't'ó/ operation, we get an even muddier picture: the string /s't'ór/ belongs to the signifier of the basic radical allomorph, so that we have to say that <plural, genitive> is expressed by a zero substitution, while the string /st/ marks all the other forms different from the genitive plural! This is clearly unacceptable.

The proviso under discussion loosens Condition 2 of the IZLS Principle ('No zero in the presence of perceptible distinctions') a bit-in order to ensure a more systematic and natural description in such cases as (6).

The second proviso, concerning different levels of representation, foresees different cases of ellipsis-i.e. situations where the information is carried by a non-zero sign present at a given level of representation, but eliminated closer to the surface by special rules (all sorts of deletion, such as that of personal pronouns in Pro-Drop languages, etc.). Thus, the Spanish sentence *Estoy leyendo* <[I] am reading> does not have the zero subject $*\emptyset_{1SG}$ <I>: in the surface-syntactic structure, this sentence has the overt subject *YO* <I>; rules of Spanish syntax delete it in the passage to the morphological string- after it has specified the agreement of the verb. In other words, this proviso requires distinguishing ellipses (= elimination of non-zero signs) from zeroes. It plays a special role in the analysis of the Georgian example (7), which is considered in Section **10**.

0.8 No Non-Contrastive Zeroes

Condition 3 of the IZLS Principle stipulates that a zero sign should contrast semantically with at least one non-zero sign capable of occupying the same position; this requirement applies of course only to full zero signs, i.e. zero signs that have a non-empty signified. (NB : For empty zero signs such a contrast is, of course, impossible.) Note that this condition does not forbid two zero signs 'contrasting' in the same position, provided this position can also contain a non-zero sign (cf. example (5) below). Thanks to Condition 3, 'useless,' i.e. non-distinctive, or 'stopgap,' as Haas (1968)b called them, zero signs are avoided in two types of situations:

- where the absence of a sign is not significative, because the meaning involved is actually carried by a different non-zero sign;
- where the absence of a sign is significative, but it is a result of a morphological ellipsis-that is, of the deletion of a non-zero sign introduced on a deeper level of representation.

I will deal with the first case immediately and keep the problem 'zero vs ellipsis' for the next section. If the presumed zero sign cannot contrast (in the given position) with a non-zero sign, this can happen because the meaning observed is expressed by a different sign, for instance, by the radical of an invariable form. In such a case, the absence of an affix is not significative and, consequently, a zero sign postulated here would be a linguist's zero rather than a zero of the language. Three examples follow.

(7) English
The wordform **sheep**, as in *The sheep were grazing …* , where, as the agreement of the verb shows, it is in the plural, does not include a plural zero suffix $*\emptyset_{PL}$, because this $*\emptyset_{PL}$ does not contrast with a non-zero suffix: the noun SHEEP is invariable. The wordform **sheep** must be characterized in the lexicon as either singular or plural, that is, we deal here with two different signs:

sheep = ⟨<domestic mammal of the genus *Ovis*, SG >; /á'p/ ; Σ = Noun, … ⟩
and
sheep = ⟨<domestic mammal of the genus *Ovis*, PL> ; /ä'p/ ; Σ = Noun, … ⟩

Both signs **sheep** are megamorphs (see Mel'čuk 1982[61, 105] and Mel'čuk 1993-2000[vol. 4: 353*ff*], lexes of the lexeme SHEEP; each of them realizes simultaneously two morphemes:

{SHEEP}, {SG} ⇔ **sheep** and {SHEEP}, {PL} ⇔ **sheep**

Several other English nouns of the same type (*deer, elk, moose, grouse, trout, …*) are described in the same way. (Janda and Manandise 1984[232], who emphatically reject a zero plural suffix in the plural form **sheep**.)

(8) French
An identical treatment is reserved for French nouns of the type **cas** <case>, which are invariable in written French:

cas = ⟨<case, SG> ; /ka/ ; Σ = Noun, masculine, … ⟩
and
cas = ⟨<case, PL> ; /ka/ ; Σ = Noun, masculine, … ⟩.

In spoken French, the situation is different. Since [un] cas intéressant <[an] interesting case> and [des] cas intéressants <interesting cases> may be pronounced differently-/kaε=teresa=/ [without *liaison*] vs. /kazε=teresa=/ [with a possible, although by no means obligatory, *liaison* in the plural]-the two forms have different morphic representations: **cas**+$\emptyset_{SG}$ in the singular and **cas+z** in the plural. The same situation obtains with virus <virus>: **virus**+$\emptyset_{SG}$ and **virus+z** (*[un] virus affreux* **/virüsafr**$\varnothing$**/** <[a] horrible virus> vs. *[des] virus affreux* **/virüszafr**$\varnothing$**/** <horrible viruses>), etc.

(9) Russian
A so-called invariable Russian noun distinguishes numbers and cases, as all Russian nouns do, but these are not expressed by zero suffixes. Thus, for instance, consider the declension of the noun PAL'TO <coat>:

a.

ot	*èt* +**ogo**	*pal'to*	<from this coat>
from	this	NEUT.SG.GEN	coat(NEUT)-SG.GEN
vs.			
k	*èt* +*P***omu**	*pal'to*	<to this coat>
to	this	NEUT.SG.DAT	coat(NEUT)-SG.DAT
vs.			
k	*èt* +**im**	*pal'to*	<to these coats>
to	this	PL.DAT	coat(NEUT)-PL.DAT

Nouns of this type cannot have any non-zero declensional suffix; therefore, Condition 3 of the IZLS Principle does not allow them to have zero suffixes. It is the radical megamorph that carries the grammemes of number and case:

b.

<coat, SG.NOM>	⇔	*pal'to*	<coat, PL.NOM>	⇔	*palto*
<coat, SG.GEN>	⇔	*pal'to*	<coat, PL.GEN>	⇔	*palto*
<coat, SG.DAT>	⇔	*pal'to*	<coat, PL.DAT>	⇔	*palto*
...		...	...		...

0.9 Different Zero Signs in the Same Position and Adjacent Zero Signs

The IZLS Principle bars neither two alternating zeroes in the same morphological position ('contrasting' zeroes), nor simultaneous zeroes in two adjacent positions. The first situation can be illustrated from English, the second, from Hungarian.

Consider verbs of the type PUT, CUT and HURT. The wordform **put** in Alan put his hand on ... has the past tense zero suffix $-\emptyset_{PAST}$, which contrasts with the present tense suffix **-s** *Alan puts his hand on ...* and with the **-ing** suffixes. Now, what about *I put my hand on ...* ? With other English verbs, the wordform such as walk in *I* ⟨*you, we*⟩ *walk* contains a present tense zero suffix $-\emptyset_{PRES}$ (not in the 3rd person singular), opposed to **-s** in the present and to **-ed** in the past.

By analogy, I would say that in *I put* in the present (*I put my book on the table and take my coat*), the wordform **put** has the zero suffix $-\emptyset_{PRES}$-as in *[I] walk*; on the other hand, in *I put* in the past (*I put my book on the table and took my coat*), put has the zero suffix $-\emptyset_{PAST}$. Moreover, **put** in *I have put ...* has the zero suffix $-\emptyset_{PPART}$, and put in *I want to put ...* has the zero suffix $-\emptyset_{INF}$, as all English verbs do. Thus, we can have a 'contrast' between four zero signs in the same position. This simply means that 'homophonous' zero signs (actually all zero signs are 'homophonous'!) can co-exist in the same paradigm. At the same time, the IZLS Principle admits several zero signs in one wordform, including adjacent zeroes.

(10) The Hungarian wordform **könyv** /kön'v/ <book> contains three successive zero suffixes:

-the zero suffix of (non-belonging> $-\boldsymbol{\emptyset_{NON-BEL}}$, which contrasts with the non-zero suffix **-e** of <belonging>:

könyv+Ø <book non-belonging [to anyone]> vs. **könyv+e** <book belonging to . . . >;
-the zero suffix of <singular> $-\boldsymbol{\emptyset_{SG}}$, which contrasts with the non-zero suffixes **-i** and **-k** of <plural>:

könyv+Ø <book non-belonging . . . > vs. **könyv+(e)k** <books non-belonging . . . > or

könyv+e+i <books belonging to . . . >;
-the zero suffix of <nominative> $-\emptyset_{NOM}$, which contrasts with numerous non-zero case suffixes:

könyv+Ø <book, NOM> vs. **könyv+ben** <book, INESSIVE>,

könyv+re <book, SUPERLATIVE>,

könyv+nek <book, DATIVE>, etc.

Thus, the following wordforms can be contrasted:

könyv + $\emptyset_{NON-BEL}$ + $\emptyset_{SG}$ + $\emptyset_{NOM}$ < *book* >
and
könyv + e + i + ben <in books belonging to . . . .>.
Each one of the three morphological positions available in a nominal wordform-belonging, number, and case-and every combination thereof can hold a zero suffix:

könyv + $\emptyset_{NON-BEL}$	**+ ek + ben**	<in books>	(i*könyvekben*)
könyv + e	+$\boldsymbol{\emptyset_{SG}}$ **+ ben**	<in [the] book belonging to >	(*könyvben*)
könyv + e	**+ i +** $\emptyset_{NOM}$	<books belonging to >	(*könyvei*)
könyv + $\emptyset_{NON-BEL}$	+ $\boldsymbol{\emptyset_{SG}}$ **+ ben**	<in [the] book>	(*könyvben*)
könyv + $\emptyset_{NON-BEL}$	**+ ek** + $\emptyset_{NOM}$	<books>	(*könyvek*)
könyv + e	+$\boldsymbol{\emptyset_{SG}}$ + $\emptyset_{NOM}$	<[the] book belonging to >	(*könyve*)

It is easy to see that all these zeroes satisfy the IZLS Principle.[5]

0.10 A Zero Sign or an Ellipsis?

In some cases, it is impossible to associate a grammeme that is expressed by a wordform with an overt marker; however, a zero sign cannot be invoked either, because the morphological position under consideration does not allow a contrast between a zero sign and a non-zero sign (and such zeroes are rejected by the IZLS Principle). A possible solution in this situation is MORPHOLOGICAL ELLIPSIS -deletion of a non-zero sign that appears on a deeper level of representation. Let me illustrate this case with a summary description of a fragment of Georgian conjugation.[6]

[5] To avoid unnecessary complications, I do not show the suffix marking the number-person of the possessor:

knyv+e+i+**m**+$\emptyset_{NOM}$	(book + belonging to . . . + PL	+ 1_{SG}	+ NOM)	= (my books)
knyv+e+i+**mk**+ben	(book + belonging to . . . + PL	+ 1_{PL}	+ INESS)	= (in our books)

etc.

[6] Georgian verbal morphology has been discussed in a series of relatively recent publications, of which I mention here only Anderson 1986 [6-14] (an analysis of the pluralizer **-t**) and Anderson 1992[137-156], Spencer 1991[219-223], Aronoff 1976, Halle and Marantz 1993[116-120,] and Carmack 1997; these sources provide all further relevant references.

In Georgian, a form of a transitive verb in the present of the indicative expresses the person and the number of both the Subject and the Direct Object [= DirO], using the following non-zero markers (for simplicity's sake, I ignore the fact that some verbs can, in addition, express in their forms the person and the number of the IndirO):

(11) Georgian

a.

	Subject	Direct Object		Subject	Direct Object
SG			PL		
1	**v-**	**m-**	1	**v-** . . . **-t**	**gv-**
2		**g-**	2	**-t**	**g-** . . . **-t**
3	**-s**		3	**-en**	

The subject suffixes 3sg **-s**, 3pl **-en** and the object prefixes 1sg **m-** and 1pl **gv-** are cumulative: they express the person and the number together. The other three affixes are 'agglutinative:' they express either the person (1st person, subject: **v-**; 2nd person, object: **g-**) or the number (plural of the Subject or the DirO: **-t**). The paradigm of the transitive verb XAT́V(-a) <[to] draw, paint> (in the present of the indicative, the active voice) will illustrate the distribution of these markers (prefixes and suffixes).

b.

				Direct Object			
			singular plural			Subject	
		1	2	3	1	2	3
sg	1	-	**g**+xat́av	v+xat́av	–	**g**+xat́av+**t**	v+xat́av
	2	**m**+xat́av	–	xat́av	**gv**+xat́av	–	xat́av
	3	**m**+xat́av+**s**	**g**+xat́av+**s**	xat́av+**s**	**gv**+xat́av+s	**g**+xat́av+**t**	xat́av+**s**
pl	1	–	**g**+xat́av+**t**	v+xat́av+**t**	–	**g**+xat́av+**t**	v+xat́av+**t**
	2	**m**+xat́av+**t**	–	xat́av+**t**	gv+xat́av+**t**	–	xat́av+**t**
	3	**m**+xat́av+en	**g**+xat́av+en	xat́av+en	**g**v+xat́av+en	**g**+xat́av+en	xat́av+en

[Blanks in the table show the impossibility of forms with the same person of the Subject and the DirO: *<I - me>, *<I - us>, *< you$_{SG}$ - you$_{SG}$ >, . . . For the signifieds of this type, Georgian uses a reflexive construction with the noun TAVI <head> in the role of reflexive pronoun.]

Table (11:1) shows multiple discrepancies between the grammemes expressed in surface forms and the corresponding non-zero markers. Thus, in *gxat́av* <I draw you$_{SG}$ > (the first form of the second column), the prefix **g-** expresses the 2nd person of the DirO, but we do not find the marker which expresses the singular of this DirO (<you$_{SG}$ > rather than <you$_{PL}$ >), nor the marker for the meaning (I). Similarly, in *gxat́aven* <they draw you$_{PL}$ > (the last form of the fifth column), the same prefix **g-** expresses again the 2nd person of the DirO, while the suffix **-en** shows the 3rd person plural of the subject; but what expresses the plural of the DirO (<you$_{PL}$ > rather than <you$_{SG}$ >)? This should be the suffix **-t**, but it is not there. This type of question can be asked about most forms of table (11:2). A logically possible answer could be the introduction of zero affixes in all cases where we lack 'material' markers: a zero suffix to mark the singular of the object in *gxat́av*, another one to mark the plural of the object in *gxat́aven*, and so forth. But let us have a closer look at these eventual zeroes-to see whether they will be admissible from the viewpoint of the IZLS Principle. We will begin with the form *gxat́av* <I draw you$_{SG}$ >, which I have already mentioned.

1. x<1st person> of the subject must be expressed by the prefix **v-**, but we do not see it in the form. GenerI presuppose the order of prefixes **v-+g-** (rather than ***g-+v-**) because in the forms with the 3rd person IndirO prefix **s-**, for some speakers the sequence **vs-** is possible: *mi+v+***s***+cem* (I will give [this] to him); thus, the subject marker precedes the object

marker. (In the normative language, **s-** must be elided, so that the correct form is *mivcem.*) Cf. Footnote 9ally speaking, if a Georgian verbal form contains a non-zero object prefix (in this case, the 2nd person **g-**), no other non-zero prefix can be present in it. Therefore, we have no right to postulate here a subject zero prefix: it would never contrast with a non-zero prefix, so that it would be non-contrastive; Condition 3 of the IZLS Principle disallows such zeroes. The morphic representation of the form in question cannot be $*\emptyset_{1p}$ **+ g + xatav-.iv** [7]

The correct description is different: the 1st person of the subject is expressed by the prefix **v-** , which, closer to the surface, is evicted by the prefix **g-**; this is a typical morphological ellipsis:

$$[\textbf{v- + g-}] \Rightarrow \textbf{g-}$$ [8]

(I use square brackets to indicate sequence of linguistic signs which is ill-formed form the viewpoint of surface constraints.) This rule produces the correct (part of the) verbal form:

$$\textbf{v + g + xaťav-} \Rightarrow \textbf{g + xaťav-.}$$

Note that the initial phonemic cluster **vg-** is possible in Georgian: *v+gv+i* <[I] sweep>, *v+gzavn+i* <[I] send>, *v+glez* <[I] tear>, etc.; therefore, the substitution **v-+g-** ⇒ **g-** cannot be described as phonemic alternation of cluster simplification.
In a general form, the ellipsis rule under discussion may be written as follows:

c. [pref'∈ ... SUB, pref"∈ ... OBJ] ⇒ pref"

The morphotactic constraints of normative Georgian do not allow more than one non-zero subject/object marking prefix (and, as we will see in most cases, more than one subject/object suffix). It is this fact that I am trying to capture withellipsis rules such as (11c).

2. The form *gxaťav* <I draw you$_{SG}$ > contrasts with the form *gxaťavt* <I draw you$_{P}L$ >, where the suffix **-t** expresses the plural of the DirO; this proves the presence, in *gxaťav,* of a singular DirO zero suffix. We can then write, for <I draw you$_{SG}$ >, the following incomplete morphic representation:

$$\textbf{v + g} + xaťav + \emptyset_{SG}\text{-}.$$

3. The form *gxaťav* contrasts as well with two other forms *gxaťavt*:

- the form *gxaťavt*, which means <we draw you$_{SG}$) and where **-t** expresses the plural of the Subject;
  and
- the form *gxaťavt*, which means <we draw you$_{PL}$> and where **-t** expresses the PLural of both the Subject and the DirO.

From this, two conclusions follow:

1. -The suffix **-t** is an 'unselective' pluralizer: it can express the plural of the Subject, or the DirO, or both (and also the IndirO, which I do not consider here); its signified is simply (plu-

[7] However, Georgian seems to have a 1st person subject zero prefix $\emptyset_{1p}$- (an allomorph of the same morpheme as v-), which appears in one verb only: *mo+∅+val* <[I] will come>, *mo+∅+vedi* <[I] came> (vs. *mo+x+val* <[you$_{SG}$] will come>, *mo+x+vedi* <[you$_{SG}$] came)>. The absence of the prefix **v-** in these forms cannot be explained phonologically, since this **v-** appears before the stem-initial **v-** in other verbs without problem: *v+varcxni* <[I] comb someone's hair>, *v+vaärob* <[I] trade>, *v+vaxmob* <[I] eat supper>. (These facts were pointed out to me by L. Margvelani.)

[8] I presuppose the order of prefixes **v-+g-** (rather than ***g-+v-**) because in the forms with the 3rd person IndirO prefix **s-**, for some speakers the sequence **vs-** is possible: *mi+v+***s***+cem* (I will give [this] to him); thus, the subject marker precedes the object marker. (In the normative language, **s-** must be elided, so that the correct form is *mivcem.*) Cf. Footnote 9

ral), without specifying whether it 'pluralizes' the Subject or an Object. By analogy, we can decide that in the singular, the zero suffix is equally unselective in the same sense: -$\emptyset_{SG}$ is for the Subject, the DirO, or both. The wordform meaning <we draw you$_{PL}$ > cannot have two plural suffixes **-t**-one for the Subject, and the other for the DirO; by analogy, the wordform meaning <I draw you$_{SG}$ > cannot have two singular zero suffixes one after another, nor the wordform meaning <I draw you$_{PL}$ >) the combination of **-t** with $-\emptyset_{SG}$:

[**-t** + **-t**] ⇒ **-t**; [$-\emptyset_{SG}+ -\emptyset_{SG}$] ⇒ $-\emptyset_{SG}$; [**-t** + $-\emptyset_{SG}$] ⇛ **-t**; [$-\emptyset_{SG}$ + **-t**] ⇒ **-t**

The final phonemic cluster of two dentals is possible in the Georgian verb: *v+ple***t+t** <[we] wear out, tear>, *v+le***ṫ+t** <[we] exterminate>; therefore, the substitution **-t+-t** ⇔ **-t** cannot be described as cluster simplification. -The morphic representation of the form *gxaṫav* <I draw you$_{SG}$ > contains another zero suffix, which marks the singular of the subject (<(I) rather than <we>). As a result, the complete morphic representation of this form is as follows: v + g + xaṫav + $\emptyset_{SG}$ + $\emptyset_{SG}$.

2. The form *gxaṫav* <I draw you$_{SG}$ > is also opposed to the forms *gxaṫav***s** <**he** draws you$_{SG}$ > and *gxaṫaven* <they draw you$_{SG}$ >. But here the opposition is expressed-at the level of the morphic representation-by the 1st person subject prefix **v-** (in the morphic representation **v+***g+xaṫav-*), which contrasts with the 3rd person singular subject suffix **-s** and the 3rd person plural subject suffix **-en**. (Closer to the surface, as has been already stated, **v-** is evicted by the prefix **g-**.) So again there is no zero affix-more specifically, no zero prefix *$\emptyset_{1p}-$; Condition 2 of the IZLS Principle bars the introduction thereof.

Now let me turn to the second form mentioned above: *gxaṫaven* <they draw you$_{PL}$ >, where the problem arises in connection with the 'absent' pluralizer of the DirO **-t**. Table (11:1) shows that this suffix does not combine with any other suffix; but it behaves differently with respect to subject suffixes of the 3rd person **-s** [SG] and **-en** [PL]. Namely, **-t** evicts **-s**, but is itself evicted by **-en**:

<he draws you$_{PL}$ >	⇒	g + xaṫav + s	+ t	⇐	*gxaṫavt*	<*gxaṫavst>
<they draw you$_{PL}$ >	⇒	g + xaṫav + en	+ t	⇐	gxaṫaven	< **gxaṫavent*>

To express this fact, I introduce two further morphological ellipsis rules:

[**-s** + **-t**] ⇔ -t; [**-en** + **-t**] ⇔ **-en**[9]

Again, these are morphological, rather than phonological, rules: the final clusters *-st* and *-nt* are possible in Georgian: *v+sreP***s+t** <we rub him/them>, *a+lxe***n+t** <you$_{PL}$ amuse him>, *v+a+***r¡'en+t** <we support him/them>. Finally, I suppose that the non-zero suffixes **-s** and **-en** always evict adjacent zero suffixes; therefore, two more morphological ellipses are needed:
[**-s** + $-\emptyset_{SG}$] ⇔ **-s**; [**-en** + $-\emptyset_{SG}$] ⇔ **-en** Given this complex combinatorics of Georgian verbal affixes, many verbal forms in the present indicative active manifest multiple ambiguities; for instance :

[9]In these rules, the order of suffixes 'subject marker + object plural marker' in the morphic representation is accepted - because in the forms where such markers cooccur on the surface they are arranged exactly in this order:

g+ćq̇ur	+i+a3$_{SG.Subj}$	+t$_{PL.Obj}$	<You$_{PL}$ are thirsty>, lit. <It is thirsty to you$_{PL}$ >;
g+xedav	+d+a3$_{SG.Subj}$	+t$_{PL.Obj}$	<He saw you$_{PL}$ >.

Moreover, in colloquial/dialectal Georgian, the suffix sequence **-s +-t** is actually heard *xaṫavst*, instead **pf** the normative *xaṫavt*).

verbal form	signified			morphic representation	
gxat́avt	<I draw you$_{PL}$ >	⇔	v+	g + xat́av + $\emptyset_{SG}$	+ t
	<we draw you$_{SG}$ >	⇔	v+	g + xat́av + t	+$\emptyset_{SG}$
	<we draw you$_{PL}$ >	⇔	v+	g + xat́av + t	+ t
	<he draws you$_{PL}$ >	⇔		g + xat́av + s	+ t
gxat́aven	<they draw you$_{SG}$ >	⇔		g + xat́av + en	+ $\emptyset_{SG}$
	<they draw you$_{PL}$ >	⇔		g + xat́av + en	+ t

To sum up: If we take into account only the form *gxat́av* and its oppositions with other forms of the (partial) paradigm of the Georgian verb, just one verbal zero suffix is found in Georgian: an unselective singularizer $-\emptyset_{SG}$. In particular, forms of the type *gxat́av* <I draw you$_{SG}$> or *gxat́avt* <we draw you$_{SG}$ > do not contain the 1st person subject zero prefix: these forms are obtained as a result of morphologicalellipsis-elimination of the 'regular' 1st person subject prefix **v-**. There is no 3rd person singular subject zero suffix in *gxat́avt* (he draws you$_{PL}$> either: this form is also produced by theellipsis of the subject suffix **-s**. However, the paradigm in (11:1) shows the presence of another 'unquestionable' zero prefix: the 2 person subject prefix $\emptyset_{2p}$, seen in the forms *Ø+xat́av+Ø* <you$_{SG}$ draw him/them> and *Ø+xat́av+t* <you$_{PL}$ draw him/them>, as opposed to *v+xat́av+Ø* <I draw him/them> and *v+xat́av+t* <we draw him/them>.

I will stop my analysis of the Georgian verb here, even if there remain some other interesting zero-related problems: for instance, the existence of the 3rd person direct object zero prefix.[10]

[10]**The 3rd person DirO zero prefix in the Georgian transitive verb**

Aronoff 1976 argues against the 3rd person DirO zero prefix; in conformity with the tradition, he maintains that the verb does not agree with a 3rd person DirO (while obligatorily agreeing with 1st/2nd person DirOs). His main argument is that on some occasions, what is basically a transitive verb has no DirO in the sentence, so that there cannot be 3rd person agreement. Aronson refers to two types of verbs:

1. Verbs having two syntactic modifications, such as <[to] hit N> vs. <[to] hit on N> or <[to] point N> vs. <[to] point to N>; when used in the second, prepositional modification, they do not have a DirO.
2. Verbs having so-called 'absolute' use, such as <She smokes>, <I paint when I have time>, <You read better than you write>, etc. I, however, do not find this argument convincing. 'Inherently' transitive verbs appearing in a prepositional or 'absolute' modification are not transitive any more; they function as separate lexical unit - intransitive verbs, which cannot have DirOs. When Aronson says (p. 3) that "a Georgian verb form such as *ćers* (writes> gives no more and no less information about the presence or absence of a direct object than does the corresponding Russian verb form *piäet* <writes>", he is not correct: in Russian, *piäet* can have a DirO of the 1st or 2nd person (*piäet menja/tebja* <[he] writes me/thee> is grammatically perfect, leaving aside the semantic implausibility), while in Georgian, **ćers me/äen* <he writes me/you$_{SG}$ > is ungrammatical (the correct forms being *mćers* and *gćers*). Yet even if his central argument is not sufficient, Aronson is right in his main claim: the transitive verb in Georgian does not agree with the 3rd person DirO. This is shown by the impossibility of using the pluralizer **-t** for a 3rd person DirO. Thus, the form meaning (he draws them) theoretically could be *xat́avt* ⇔ Ø *3p+xat́av+s+t*; but in fact this form does not have the indicated meaning. The meanings <he draws him/her> and (he draws them> cannot be dsitinguished: both are expressed as *xat́avs* (without the pluralizer). At the same time, **-t** can 'pluralize' a zero marker: it does exactly that for the 2nd zero person prefix. Therefore, we have to say that a Georgian transitive verb does not agree with its DirO in number. Then, why insist that it agrees with the DirO in person? By all means, this 'agreement' would be shown by a zero. Taking everything into account, it is simpler to admit that a transitive verb does not agree with a 3rd person DirO at all. As a result, the Deep-Morphological representation of a transitive verb finite form does not include the grammeme (3rd person, DirO) (and in the case of the DirO of the 3rd person, no grammemes of the number of the DirO, either). Consequently, Georgian does not have a 3rd person verbal zero prefix: such a prefix would have nothing to signal and thus it would violate Condition 1 of the IZLS Principle. Still, I have to point out three facts that argue against the decision 'no agreement with a 3rd person DirO:'The Georgian verb agrees with its IndirO even in the 3rd person, the corresponding agreement prefixes being **h-/s-/Ø-**; this phenomenon contradicts the typologically important hierarchy DirO > IndirO. In other words, the 3rd person IndirO should not impose agreement on the verb if the 3rd person DirO does not. Note, however, that such type of agreement can be naturally explained by the IndirO (often) being higher in animacy than the DirO; and typologically, the preferred agreement with animate actants is well known. (I thank D. Beck, who pointed out this fact to me.)

What we have just seen is sufficient to illustrate the fact that absence of an affix, even it this absence is significative, is not necessarily a zero affix-it can well be a morphological ellipsis.

0.11 Morphological Ellipsis

Ellipsis in syntax, i.e., at the clause and sentence level, is relatively well studied; it is much less popular in morphology, that is, at the wordform level. I think, however, that in morphology, ellipsis functions as well and fully deserves the linguist's attention.[11]

0.11.1 The Notion of Morphological ellipsis and Related Notions

In order to put morphological ellipsis into perspective, I will start with a few definitions (X is any linguistic item; C is the set of contexts, or conditions, for the application of the rule).

Definition 2: Deletion

A *deletion* is an operation described by a rule of the form "X $\Leftrightarrow$ Λ $\|$ C". Deletion rules fall into two major types as a function of what X is:

- if X is a non-significative item, i.e. a phonemic string or a prosodemic complex, we have a truncation alternation;
- if X is a significative item, i.e. a linguistic sign, a configuration of signs or a set of contextually distributed signs (= an -EME, that is, a lexeme or a morpheme), we have an ellipsis.

Definition 3: Truncation A *truncation* is a deletion in which X is a phonemic string or a prosodemic complex. The context C of a truncation can in Principle be anything: a phonemic string (= phonologically controlled truncations) or a particular sign or signs (= morphologically controlled truncations). From the viewpoint of its semantic role, truncation can be either meaningful (being the signifier of a sign- an apophony) or meaningless (being then an empty alternation, imposed by the context).

(12) Meaningful Truncation: Alabama (Anderson 1992[66])

As Aronoff 1976[6] indicates, there are quite numerous cases where the 3rd person IndirO prefix **s-** is 'erroneously' used for agreement with a DirO (the following sentences are taken from literary texts):

a.
Is páiesa [= DirO] sami čľis čìnat da+s+ćera
this play.$_{SG.NOM}$ three years ago *wrote*.3$_{SG}$
(This play [s/he] wrote three years ago>

b.
Momaḱvdavis otaxi [= DirO] da+s+t́ova
.dying.person.$_{SG.GEN}$ room.$_{SG}$.NOM left.3$_{SG}$
([S/he] left dying person's room>

c.
Sicilma ḱamara [= DirO] e+h+ḱra
laughter.$_{SG.ERG}$ vault.$_{SG.NOM}$ surrounded.3$_{SG}$
(The laughter surrounded the vault>.

In colloquial Georgian, some verbs admit the pluralizer of the DirO:

d. *Es mat* [= DirO] *aćuxeb+t* <This upsets them>, instead of the normative *aćuxeb+s*.
e. *Is mat* [= DirO] *abruneb+t* <S/he rotates them>, instead of the normative *abruneb+s*.
f. Is mat [= DirO] *agoreb+t* (S/he rotates them>, instead of the normative agoreb+s.

The frequency of non-standard forms such as in (i) - (vi) seems to indicate a strong tendency in Georgian to mark agreement with the 3rd person DirO as well.

[11] The concept of morphological ellipsis and the term itself were introduced in Mel'čuk 1973[53-55, 75-78], as applied to Alutor.

	semelfacitive	**iterative**
	(<once>)	⟨<repeatedly>⟩)
<lie down>	*bal, +li*	*bal+li*
<hit>	*batat +li*	*bat+li*
<cut>	*kolof +li*	*kol+li*

The sign observed in the right-hand side members of these pairs is a truncation apophony A applied to a verbal stem /ΦV(C)/- (where /Φ/ is any string of phonemes) and expressing the iterative aspect: $\boldsymbol{A_{ITERAT}}$ = <(repeatedly) ; /ΦV(C)/- ⇔ /Φ/- ; Σ = applies to verbs, … >

Such signs are 'anti-diagrammatical,' or 'anti-iconic' (Dressler, on many occasions; see, e.g., Dressler 1987): they violate the Principle of maximal parallelism between the signified and the signifier, since they express addition of meaning by subtraction of the form. Therefore, meaningful truncations are rather rare in languages of the world (cf. Mel'čuk 1991).

(13) Meaningless Truncation: Latin

	genitive	nominative
<bridge>	*pon***t** *+is*	*pon*+s
<mountain>	*mon***t** *+is*	*mon*+s
<swamp>	*palu***d***+is*	*palu*+s

Meaningless truncation alternations are too well known to need further discussion.

Definition 4: Ellipsis

An *ellipsis* is a deletion in which:

1. X is a linguistic sign, a configuration of linguistic signs or a set of contextually distributed signs (an -eme); and
2. *C* is a linguistic sign, a configuration of linguistic signs or a set of contextually distributed signs.

Thus, ellipsis is also, like truncation, a particular case of deletion, but a very special case: deletion of whole signs in the context of other whole signs. Note that:

In contrast to truncation, an ellipsis is necessarily meaningless, because context-imposed: elimination of signs cannot be meaningful. Ellipsis does not change the meaning of the linguistic unit it is applied to (clause/sentence or wordform). Thus, for instance, French and German eliminate the sequence of two identical prepositions, as in *les lettres* *** de de** *Saussure* <the letters of de Saussure> ⇒ *les lettres de Saussure* or … *ist* ***von von** *Wartburg geschrieben* <is written by von Wartburg> ⇒ *ist von Wartburg geschrieben*; this is a good example of syntactic ellipsis which does not affect meaning at all. To put it differently, an ellipsis eliminates a sign or a configuration of signs X only in such a significative context in which X is redundant to a sufficient degree, so that its meaning can be recovered, perhaps not unambiguously, from the context.

- An ellipsis can be optional or obligatory.
- An ellipsis applies only to a segmental sign, i.e. a sign having as its signifier a string of phonemes.

Definition 5: Morphological Ellipsis A *morphological ellipsis* is an ellipsis which is applicable only within the limits of a wordform. Put in simple terms, a morphological ellipsis is triggered by the surface incompatibility of two signs within a wordform. Ellipsis should be carefully distinguished from, on the one hand, truncation, and on the other hand, zero signs.[12] What unites these three

[12]Cf. A logical analsys of various types of significative absence in natural language at the sentence level in Apresjan 1978[304-308] (the triple opposition 'zero ∼ ellipsis ∼ non-saturation of an abligatory valence slot'). In this connection, see also Panevová 1998

phenomena is that all of them imply some relevant absence in the text; however, they are quite different in their nature:

1. Ellipsis and truncation are operations, while a zero sign is an entity.
2. Ellipsis is always meaningless, zero signs always meaningful,[13] and truncation can be both.
3. In the process of text synthesis, morphological ellipsis is normally triggered, as I have said, by the context (morph incompatibility); a zero sign is selected, as all signs are, for its signified; and truncation is sometimes used to express a meaning and sometimes in order to satisfy contextual requirements.

0.11.2 Illustration of Morphological Ellipses

Let me give more examples of morphological ellipses (curly brackets ({}) denote morphemes; square brackets include the set of incompatible signs).

(14) In Alutor, the following incompatibilities are observed within a verbal wordform at the morphic level:

a. [**suf'** ∈ … SUB, **suf"**∈ … OBJ] ⇒ **suf"**

Suffixal person-number markers of the Subject and those of the DirO cannot co-exist in the same verbal wordform; the object suffix 'evicts' the suffixal part of the subject circumfix, so that only the prefixal part of this circumfix remains.
For instance:

*met	+uvvat	+Ø	+**m'ek**	+**tek**	⇔ met+uvvat+tek ⟨*metuvvanmektek⟩
1DU.SUBJ.IND	kiss	AOR	1DU.SUBJ.IND	2DU.OBJ	

<We$_{DU}$ kissed you$_{DU}$ >.

The rule (14a) can be formulated neither in terms of 'pure' grammemes (since subject and object markers not only can, but actually must cooccur in the verbal wordform), nor in terms of phonemes (the sequence *-mktk* and all the other ones of this type do not violate any phonological constraints of the language): it is is a typical morphological ellipsis. The reduction it describes is parallel to what is seen in syntax when the speaker omits one of the two consecutive identical prepositions (see above), the verb in the construction of the type [*Alain gave an apple to Helen*,] and Leo a pear to Marga or the noun in the construction of the type Fr. *Je prends la bleue*, lit. <I take the blue> (as an answer to the question *C'est quelle robe que tu prends ?*, lit. <Which dress do you take?>, etc.

b. [**suf'**∈{PL.SUB}, **suf"**∈{PL.OBJ}/{DU.OBJ}] ⇒ **suf"**

A (suffixal) subject pluralizer cannot co-exist in the same verbal wordform with an object pluralizer or a dualizer (equally suffixal); the object pluralizer evicts the subject pluralizer, so that only the object pluralizer/dualizer remains. As we see, the object is again 'stronger' than the subject. [14]

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[13] With the exception of empty zero lexeme-dummy subjects with the impersonal verbs in languages with obligatory Main Verb ∼ Subject agreement.

[14] (11.2, (14b), p. 00) Subject vs. Object Markers in the Verb

Neither in Georgian nor in Alutor Subject and Object markers are compatible on the surface in one wordform; the Object marker always evicts the Subject one. However, the situation in Principle can be different. Thus, in Wichita (Rood 1996, 1996), Subject and Object markers cooccur obligatorily:

1.

ta	**+s**	**+ki**	+? ,j +s	
IND	2p.SUB	1p.OBJ	see	IMPF

<You$_{SG}$ saw me>.

ta	**+t**	**+,**	+? ,j +s	
IND	1p.SUB	2p.OBJ	see	IMPF

<I saw you$_{SG}$>.

Here is an example:
subject pluralizer object pluralizer
*met +uvvat **+la** **+tk**e +na **+wwi**
1DU.SUB.IND kiss PL.SUB PRES 3.OBJ PL.OBJ
⇒ met+uvvate+tke+na+wwi ⟨*me tuvval**la**tke nawwi⟩X
<We$_{PL}$ kiss them$_PL$ >.
(In the wordform to be constructed, I have already omitted-according to rule (14a)-the suffixal part of the subjectal circumfix **met-** ... **-mek** ***met**+uvvat+la+tekni**+mek**+na+wwi.)
Here, as before, the rule cannot be stated in terms of grammemes (both plural grammemes can be expressed by the resulting wordform mtuvvattknawwi) or in terms of phonemes; this is another typical morphic rule, namely a morphological ellipsis.

c. [-n∈POSTER(iority), **-t**∈PL.SUB(ject)] ⇒ **-n**

The suffixal part of the circumfix of the posteriority (roughly speaking, of the future) **ta** ... **-n** cannot co-exist in a verbal wordform with the suffixal part of the subject plural circumfix **la** ... **-t**; **-n** evicts **-t**. For instance:
*Ø +ta +arat +Ø **+la** **+n** **+t** ⇔ t+aral+la**+**n <*tarallane**t**>
3.SUB.IND POSTER fall AOR PL.SUB POSTER PL.SUB
<They$_{PL}$ will fall>.
Compare, however, the corresponding form in the dual of the Subject:
Ø +ta +arat +Ø **+n** +t ⇔ t+aran**+**ne**+t**
3.SUB.IND POSTER fall AOR POSTER DU.SUB
<They$_{DU}$ will fall>

---

b.
ta **+t** +Ø **+**? ,j +s
IND 1p.SUB 3p.OBJ see IMPF
<I saw him>.
c.
ta **+i** +Ø **+**? ,j +s
IND 3p.SUB 3p.OBJ see IMPF
<He$_i$ saw him$_j$> [⇒ ti**+**? ,s].
d.
ta +Ø **+ki** **+**? ,j +s
IND 3p.SUB 1p.OBJ see IMPF
<He$_i$ saw me>.
Similarly to Georgian and Alutor, Wichita has separate pluralizers for Subject and Object markers, and, unlike Georgian and Alutor, they can cooccur as well, but restricte only when the Subject is 1st/2nd person and the Object, 3rd person:
2. 3
ta **+s** +Ø **+r,k+?ak** **+**? ,j +s
IND 2p.SUB 3p.OBJ PL NON-SG.OBJ see IMPF
<You$_{PL}$ saw them$_{DU/PL}$ >.
b.
ta **+t** +Ø **+r,k+?ak** **+**? ,j +s
IND 1p.SUB 3p.OBJ PL NON-SG.OBJ see IMPF
<We$_{PL}$ saw them$_{DU/PL}$ >.
If the Subject is 3rd person or the Object 1st/2nd person, only Object can be pluralized (Rood 1996[603]); the number of the Subject cannot be expressed at all, so tdhat all forms of the type <he/they - ... > and < ... - me/you/us> are ambiguous:
c.
ta +Ø **+ki** +r,k+ ? ,j +s
IND 3p.SUB 1p.OBJ PL see IMPF
<He/They saw us$_{DU/PL}$ >.
It is also worth noting that **-r,k** pluralizes both Subject and Object (dependning on the context), while **-?ak** is reserved for the Object. Thus, as we see, after all, the Object is somehow priviledged in Wichita as well.

where **-n** of the posteriority co-exists without problem with the subject dualizer **-t**. It is clear that rule (14c), in the same vein as (14a) and (14b), cannot be formulated in terms of grammemes (since posteriority and the subject plural are paradigmatically compatible), nor in terms of phonemes (since **-n** can be followed by the phoneme /t/-provided this /t/ is not part of a plural marker).

Expressions (14a-c) are morphological filter rules of Alutor; they express obligatory morphological ellipses of this language.[15].

The next two examples are slightly more contentious.

In some Australian languages (Dench and Evans 1988 examples (74) and (87)-(88)), we find incompatibilities between case morphs within a nominal wordform. (In these languages, a noun N can include two case markers: one represents the governed case, imposed on N by its syntactic governor; the other represents the agreeing case, coming from the syntactic head of the noun phrase to which N belongs.)

(15) In Dyiwarli, the following filter rule is operational at the morphic level:

$$[\mathbf{suf}{\in}\{\mathrm{DAT}\} + \mathbf{suf}{\in}\{\mathrm{DAT}\}] \Rightarrow \mathbf{suf}$$

Two identical dative suffixes cannot follow each other in a nominal wordform; one of them is obligatorily deleted. For instance, consider the wordform purat>iyi in sentence (15a):

a. D†uma +t>i +Ø d†iril +ari aṭuṭu +**wu** urat>iyi
child PL NOM be.afraid INCHOAT PRES dog DAT1 woman.DAT1.DAT2
<Children are afraid [lit. (became afraid)] of the woman's dog>.
This wordform is obtained in the following way:

*purat>i +**yi** +**yi** ⇒ purat>iyi
woman DAT1 DAT2

Here, the governed DAT1 of the noun ṬUṬU <dog> is imposed by the verb D†RIL (be afraid>, which governs the dative of its object. With the noun PURAT>I <woman>, the DAT1 marks the possession, just as the genitive in other languages (the Dyiwarli dative has this usage); but the agreeing DAT2 'reflects' the dative of its syntactic governor, i.e. of ṬUṬU.
However, two different dative allomorphs can follow each other:

b.

D†uma	+t>i	+Ø	d†iril	+ari	+a
child	PL	NOM	be.afraid	INCHOAT	PRES
tuṭu	**+wu**	*[nana +du+wu+wu ⇔]	nana+du+wu yakan	**+ku+wu**	
dog	DAT1	I **DAT**$_1$ **DAT**$_2$ **DAT**$_3$	wife	**DAT**$_1$**DAT**$_2$	

<The children are afraid [lit. <became afraid>] of my wife's dog [lit. (of wife of me)]>.

For YAKAN <wife>, the DAT1 marks the possession [= (of wife)], and the DAT2 reflects the governed DAT1 of its governor [= T⊓UT⊓U <dog>] (<dog→of wife>). For nANA <I>, the DAT1 also marks the possession [= <of me>], while the DAT2$_a$ and DAT2$_b$ reflect the two DATs of its governor, YAKAN. As we see, from two identical suffixes of the DAT **-wu wu**, one is deleted, but two different suffixes of the DAT following each other, that is, the sequences **-d †u-wu** and **-ku-wu**, remain intact.

Since (15a) involves only identical, or 'repeated,' suffixes, the question arises as to whether it is not a case of haplology-a meaningless morphonological operation, special case of truncation alternation,

[15]For a different description of the Alutor data, see Kibrik 1997[43-54]. Among other things, Kibrik proposes to avoid rule (14b) by means of more complex syntactic rules of verb agreement -namely, having the finite verb to agree in the plural with one actant only (either with the subject or with the object). I am not in a position to make a Principled choice.

dealing with (quasi-)identical phonemic sequences under particular phonological and/or morphological conditions (like in murder+**er** ∼ *murd**er**+**er**+ess ⇒ murderess, adult**er**+**er** ∼ *adult**er**+**er**+ess ⇒ adulteress, while wait+er ∼ waitr+ess, heir ∼ heir+ess, etc.; or else mor**pho**+**pho**nology ⇒ morphonology).[16] The answer is negative: if we judge from the information supplied in the Dench & Evans paper, the deletion in question involves only whole signs and is conditioned only by whole signs. If, for instance, the dative suffix **-ku** could be deleted after the stem-end sequence -ku or after a different suffix having the same signifier /ku/, this phenomenon could be considered as haplology.[17] Yet, as far as I know, this is not the case.

(16) In Turkic languages, the plural suffix of the noun (**-lar** and all its phonological variants) is not compatible with the possessive 3pl suffix (**-lar**I (their) and its phonological variants):

$$[\textbf{suf'}\in\text{PL} + \textbf{suf''}\in\textbf{POSS.3PL}] \Rightarrow \textbf{suf''}$$

Let me illustrate this morphological ellipsis from (Osmanli) Turkish:

a.	at	+lar+larI	+Ø	⇒	*atlar*I <their horses>
	horse	PL	POSS.3PL	NOM	
	ev	+ler+leri	+de	⇒	*evleride* <in their houses>
	horse	PL	POSS.3PL	INESS	

A nominal form of the type atlarI is three-way ambiguous, because along with the above Deep-Morphological Representation, it can have two more:

b.	at	+lar+I	+Ø	⇒		*atlar*I <his/her horses>
	horse	PL	POSS.3SG	NOM		
	at	+Ø	+larI	+Ø	⇒	*atlar*I <their horse>
	horse	SG	POSS.3PL	NOM		

Here, similarly to the preceding case, we cannot see haplology (as is often done): what is involved in (12) are genuine signs; note that the stem-final sequence *-lar* or *-ler* does not trigger the deletion: *dolar+lar* <dollars> ⇒ *dolarlar*, *kiler+ler* <larders> ⇒ *kilerler*, etc.

0.11.3 An Alternative Description of the Same Facts?

Description via ellipsis-by deleting morphs (or morphemes) selected on a deeper level of representation-is of course not the only possible way to account for observed facts. Logically speaking, there is an alternative: we can prevent the respective signs from being selected (in case when the corresponding grammemes are present in Deep-Morphological Representation of the wordform being synthesized). This is BLOCKING approach, and it can be implemented in two ways:

[16] On haplology, see, for instance, Dressler 1987, Stemberger 1981 and Menn and MacWhinney 1984 . A very typical example of haplology is the truncation of the initial segment -/ov/ of the signifier of the Russian productive adjectival suffix **-ovat** <-ish> = (of weak degree) following the stem-final segment -/ov/, which can, but need not, be a (fossilized) suffix (a submorph):

<purple>	/l'ilóv/-	(yj)	⇒	<purplish>	/l'ilov/	+ /ovt/	⇒ /l'ilovát/- (yj)
<rose>	/rózov/-	(yj)	⇒	<rose-ish>	/rozov/	+ /ovt/	⇒ /rozovát/- (yj)
<bad> [slang]	/f'igóv/-	(yj)	⇒	<bad-ish>	/f'igov/	+ /ovt/	⇒ /f'igovt/- (yj)
<stupid> [slang]	/dubóv/-	(yj)	⇒	<stupid-ish>	/dubov/	+ /ovt/	⇒ /dubovt/- (yj)

Cf. cases where the stem does not end in -ov, so that haplology cannot take place:

<grey>	/s'er/-	(yj)	⇒	<greyish>	/s'er/ +	/ovt/	⇒	/s'erovt/-	(yj)
<bad>	/PLox/-	(oj)	⇒	<bad-ish>	/plox/ +	/ovt/	⇒	/PLoxovt/-	(yj)
<stupid>	/glup/-	(yj)	⇒	<stupid-ish>	/glup/ +	/ovt/	⇒	/glupovt/-	(yj)

[17] Cf. in Sanskrit: the singular instrumental suffix **-y,** is optionally deleted after a derivational suffix of abstract nouns having the signifier /j,/: (eloquence, SG.INSTR) *vacas+y,+y,* ⇒ *vacasy,y, /vacasy,*; this is a genuine haplology (Stemberger 1981[799]).

- If no ordering of morphological rules is admitted, we have to write into our rules more complex conditions that will not allow one of the competing signs to be selected if the other one is around. Thus, describing the Georgian subject/object verbal prefixes, we should add the following condition for the selection of the prefix **v-**:“Only if there is no 2nd person object.”
- If (at least) partial ordering of morphological rules is admitted, we have to order our rules in such a way that in the appropriate block, the en-rule is ordered before the **t**-rule, and the latter, before the **s**-rule; each previous rule prevents the following rule from applying. The same method can of course be used for the **v-** and **g-** prefixes: the **g-** rule is ordered before the **v-** rule and thus-if **g-** apears in the wordform under synthesis-prevents **v-** from appearing at any level of morphological representation. (This is the ‘disjunctive ordering,’ proposed by Anderson 1976: 12 and Anderson 1992: 46, 87 and *passim*, and then developed in Carmack 1997.)

The two techniques are perfectly equivalent. The choice must be made based on systemic considerations: one has to prefer the description which guarantees simpler and more elegant model. However, I will not try to solve this situation of non-unicity of linguistic description, because I do not accept the blocking approach to morphic incompatibility altogether, and this for the two following reasons:

1. Under the blocking approach, some (inflectional) meanings are not expressed at all. Indeed, it is the gist of this approach-to prevent particular meanings from being expressed in the presence of some other meanings, whose ‘stronger’ markers compete for the same structural position in the wordform. But I do not like the ‘philosophy’ of morphology that lurks behind the technique of preventing some meanings from being expressed. I believe that all starting meanings have to be expressed in the wordform to which they give rise; if a meaning does not receive an overt marker, this would automatically mean in my parlance that it is expressed by a significant absence, i.e. by a zero. But the zero sign which would have to be postulated in the above examples of morphological ellipses is, however, not allowed in our system-because it is not contrastive. The ellipsis approach avoids this paradox. Meanings that first are expressed by overt signs and later loose their markers are well attested in syntax-these are ellipses of wordforms in particular constructions. So in morphology I prefer to have an obvious analogy to syntax and postulate morphological ellipses.
2. In contrast to blocking, the ellipsis approach ensures an explicit statement of morphic incompatibilities anx1d the relative strength of incompatible morphs. Such a rule as (11c) in Georgian:

   $$[\textbf{pref'}\in \{\text{SUB}\}, \textbf{pref'}\in\{\text{OBJ}\}] \Rightarrow \textbf{pref''},$$

   says directly which morphs cannot be combined within a wordform and which one must be evicted. Under blocking, this information remains implicit. These considerations are sufficiently weighty for me to strongly prefer the ellipsis approach over the blocking one.[18]

0.11.4 Truncation Alternation: A Phenomenon Similar to Morphological ellipsis

Now, in order to add some depth to the notion of morphological ellipsis, I will cite five examples of morphological phenomena that could be easily mistaken for morphological ellipses, while in fact they are not: I am referring to truncation alternations.

Alutor Abridged Verbal Forms

(17) Alutor

[18]The ellipsis approach has its own logical drawback: namely, it admits the creation of intermediate ill-formed representations (Stemberger 1981802-804). However, my linguistic Principles make me prefer the generality, homogeneity and simplicity of the rules over the avoidance of ill-formed intermediate representations.

Some Alutor suffixes, parts thereof or suffixal parts of circumfixes can be optionally dropped at the end of a wordform:

	full form	abridged form
<sit$_{SG}$ down!>	qe+tvagal+γ **i**! ~	qe+tvagal!

[q- . . . -γ **i** is a circumfix of the 2SG subject in the imperative]

<I kissed them$_{PL}$ > t+uvvan+na+**wwi** ~ t+uvvan+na+w ~ t+uvvan+na

-na is a suffix of the 3 person object; **-wwi** is a pluralizer of the object

<Let us$_{PL}$ begin!> **men**+n vu+la+**mek**! ~ men+nvu+la!

men- . . . **-mek** is a circumfix of the 1du subject in the imperative; **-la-** is a pluralizer of the subject

What is omitted in the abridged forms are significative elements, and yet this omission is not a morphological ellipsis:

- The main condition for the omission in question is purely phonological 'at the end of the wordform;' the presence of other signs is irrelevant.
- The omission is impossible if some purely phonological conditions are violated:
  -No omission if the resulting form is not trisyllabic (= does not correspond to the preferred syllabic pattern of Alutor verbal wordforms),
  cf. *qe+n+iv+g i!*
  <Tell me!> ~ *qeniv!
  -No omission if the string to be omitted has triggered obligatory regressive assimilation before itself: *met+uvvat+mek* <we$_{DU}$ kiss> ⇒ *met+uvvan+mek* ~ **met+uvvan*; but the abridged form *met+uvvat*-without assimilation-is correct [met- . . . -mek] is a circumfix of the 1du subject in the indicative].
- The omission of the pluralizer **-wwi** preserves the fleeting **-a** before it: cf. the abridged plural object form t+uvvan+na <I kissed them$_{PL}$> vs. the singular object form t+uvvat+en <I kissed him/ her>, where the final **-a** is impossible. (The same situation is observed in the noun. Thus, nouns of the type MILUT <hare> have a final fleeting **-a** in the stem, which has to fall before the zero suffix of the nominative, but is retained before the pluralizer **-wwi**, even if the latter is omitted: SG.NOM *milut* <*miluta> and pl.NOM *milutawwi*; if this **-wwi** is dropped, the fleeting **-a** still remains: the abridged PL.NOM is *miluta*.
  To sum up: The Alutor abridgment in verbal wordforms is a morphologically controlled phonemic truncation alternation.

Russian Verbal Past-Tense Marker

(18) In Russian, the past tense marker **-l** is not present in the verb forms after a consonant stem before a zero suffix of the masculine gender:

	SG, MASC	SG, FEM	PL
<can, PAST>	mog +Ø	mog +**l**+a	mog +**l**+i
<freeze, PAST>	mërz+Ø	mërz+**l**+a	mërz+**l**+i
<row, PAST>	grëb +Ø	greb +**l**+a	greb +**l**+i

In all the verbs whose stem does not end in a consonant or has a thematic vowel this **-l** appears without exception:

<dance, PAST>	pljasa+**l**+Ø	pljasa+**l**+a	pljasa+**l**+i
<grind, PAST>	molo +**l**+Ø	molo +**l**+a	molo +**l**+i
<love, PAST>	ljubi +**l**+Ø	ljubi +**l**+a	lubi +**l**+i

How can we describe the absence of -l in the verb forms of the type *mog*?

- The context in which this omission happens is strictly phonological-after a consonant at the word end; therefore, the omission of **-l** is not a morphological ellipsis.

•Logically, one can introduce here a zero suffix -$Ø_{PAST}$, allomorph of the same morpheme as **-l**; it satisfies all the requirements imposed on zero signs: it is informative, exclusive, and contrastive. But the conditions of its appearance are 'too' phonological, so that I prefer the third solution left: The omission of the past tense suffix **-l** in the cases of the above-mentioned type is a phonemic truncation alternation that is morphologically controlled (it applies only to /l/ of the suffix of the past; within radicals, the endings of the type *krugl* <[it is] round>, *smysl* <meaning> or *čezl* <rod, staff> are possible):

$$/l/ \Leftrightarrow \Lambda \parallel /C/+ \#;/l/\in\{PAST\}$$

English Derivation

(19) In English derivation, numerous omissions of phonemic material before the derivational suffix are found:

Stalinism	∼ *Stalin**ism**ist	⇔	Stalinist	nominate	∼ *nomin**at**+ee	⇔	nominee
atavism	∼ *atav**ism**istic	⇔	atavistic	tolerate	∼ *toler**at**+able	⇔	tolerable
ambiguous	∼ *ambigu**ous**ity	⇔	ambiguity	translate	∼ *transl**at**ation	⇔	translation

Following Aronoff 1976, all such cases are considered as truncation alternation; is this correct? I think so. True, in many derivational patterns of the types shown in (19) we deal with the deletion of signs under the impact of other signs: thus, in highly productive series Stalinism ∼ Stalinist, Marxism ∼ Marxist, Darwinism ∼ Darwinist, etc. we see the incompatibility of two genuine signs, the suffixes **-ism** and **-ist**; here we could speak of morphological ellipsis. However:

- In extremely numerous cases the element affected by truncation is not the signifier of a sign: **-ate** in nominate, translate, etc.
- Moreover, such strings as -ism or -ous, which are signifiers of the corresponding suffixes, can very often be non-significative elements: cf. **-ism** in *atavism* and **-ous** in *ambiguous* (these are morphoids et submorphs: see Mel'čuk 1997[245-251]).
- Even in cases where the truncated element is the signifier of a sign, this is not relevant for the operation: Aronoff's truncation does not pay attention to the sign/non-sign character of the material to be deleted. Therefore, to ensure a homogeneous description of all cases, I prefer to treat the phenomenon indicated in (19), pace Aronoff and his followers, as a phonemic truncation alternation.

English Possessive Marker

(20) In English, the marker of possessivity (= 'Saxon Genitive') is incompatible with an s marker of the regular plural and is evicted by the latter (in writing, the possessive is indicated by an apostrophe). For instance: **crook+s/s/+'s ⇒ crook+s'; *king+s/z/+'s ⇒ king+s'; *fox+es/Iz/+'s ⇒ fox+es'* The traditional description is via a zero allomorph of the morpheme POSSESSIVE, see, e.g., Quirk et al. 1991: 320 (the 'zero genitive'). But I cannot accept this: such a form as **crooks** or **kings** cannot be taken to have a zero suffix *-ØPOSS, because this zero would not be contrastive-it cannot be opposed to an overt possessive suffix.

Is this a morphological ellipsis? We could say so for the examples of the above type; but then the possessivity marker is also deleted

- in proper names in -/z/ (optionally): *Jones'/Jones's, Grimes'/Grimes's, Dickens'/Dickens's*, etc.;
- in Greek proper names that end in /s/ or /z/: *Euripides', Socrates'*, . . . ;

- in some Latinate nouns that end in /s/ or /z/: *series', rabies'*, ... ;
- in phraseological expressions of the type for goodness'/conscience'/politics' sake.

In all these cases, the possessive suffix is not evicted by another sign, but deleted as a function of the phonemic context. Therefore, in order to have a homogeneous description of the deletion of the possessive suffix in all cases, I prefer to say that the deletion of the possessive -s is-everywhere-a phonemic truncation alternation, which is controlled morphologically (in native English words a stem-final /s/ or /z/ which is not the signifier of a plural suffix does not trigger the truncation: *niece's* /n?'sIz/, *cheese's* /<?'zIz/ or *rose's* /ro^{u}zIz/; on the other hand, non-sibilant plural suffixes do not trigger this truncation, either: *virtuosi's, alumni's, phenomena's, children's, oxen's).* Stemberger 1981[792-795], who an alyzes the deletion of the possessive **-s** in detail explicitly qualifies it as 'morphological haplology.'

Russian Shortened Vocative Forms

(21) The familiar (= shortened) form of a Russian first name (of the 1st declension) in the vocative singular can either be identical to the nominative singular, marked by the suffix **-a**, or it can lack this **-a:**

feminine			masculine		
Svet +a!	∼	*Svet!*	Vas+ja!	∼	*Vas!*
Maä +a!	∼	*Maä!*	Pet +ja!	∼	*Pet!*
Ver +a!	∼	*Ver!*	Bor+ja!	∼	*Bor!*

The deletion of the final-a in Russian familiar first names is meaningful: it expresses a higher degree of intimacy, so that *Lid!* or *S!* shows more 'closeness' between interlocutors than *Lida!* or *Saäa!* Therefore, this deletion is impossible with basic (= unshortened) forms of first names (**Olg!, *Svetlan!, *Ljudmil!*), which are not intimate enough, while it is readily applicable to ultrafamiliar forms of the type *Vitk!, Saäk!, Valk!, Genk!*, etc. These shortened forms are interesting in the following respect: in Russian, the final consonant in a wordform cannot be voiced (it undergoes automatic devoicing); yet these forms retain their final voiced consonant: *Serëĕ*/s'ir'**ĕ**/, *Nad'* /na**d'**/,*Roz* /ro**z**/, *Vov* /vo**v**/, etc. are normal pronunciations. This is especially well seen in the vocative construction Nad', a Nad'!, where the impossibility of devoicing is absolutely obvious; cf. Reformatskij 1979[50-51]. Note that the same forms taken to be the genitive plural-with a zero suffix -Ø$_{PL.GEN}$-cannot end in a voiced consonant: *iz-za vsex ètix Serëč* /s'ir'óbf ä/, *Nad* /na**t'**/, *Roz* /ro**s**/, *Vov* /vo**f**/ <because of all these ... >. How can we describe these facts?

- We cannot say that in the vocative of the nouns in question we have ellipsis of the nominative suffix: this omission is meaningful, and ellipses are meanigless by definition.
- We cannot say, either, that the vocative is expressed by a zero suffix: before a zero suffix the devoicing of the stem-final consonant is obligatory.
- What happens in these forms is a meaningful phonemic truncation alternation (= apophony). It has to apply in the vocative forms after all phonemic alternations, including /$C_{[+voiced]}$/ ⇔ /$C_{[-voiced]}$/, have applied: /s'ir'ó**ĕ**a/ ⇔ /s'ir'ó**ĕ**/; or it has to leave a 'trace'-in order to prevent the devoicing alternation from applying.

0.12 Derivational Zero Signs? No way!

For morphological grammatical zero signs, the IZLS Principle needs an additional condition:

Additional condition for the introduction of morphological grammatical zero signs

4. Obligatoriness: If a zero sign X is a morphological grammatical sign, then the meaning $< X >$ (the signified of X) is inflectional, i.e. it is a grammeme or a combination of grammemes.
In other words, $< X >$ (or each of its components) has to belong to an obligatory morphological category: a meaning of this category MUST be expressed in the given position. $< X >$ cannot be a derivateme: derivational zero signs should not be allowed. Being non-obligatory, derivatemes are unable to exert enough pressure on the morphological system of the language in order to give rise to zero affixes. Consider, for example, the pairs of the following type in English:

(22) *[to] cook* ∼ *[a] cook or [to] gossip* ∼ *[a] gossip* [19]

In the nouns of these pairs, no agent zero suffix that would be parallel to -er can be postulated, because the meaning <[person] that X-es>, which such a suffix would express, is not inflectional in English. If we admit here an agent zero suffix -$Ø^{AGENT}$, it would violate Condition 3 of the IZLS Principle: this presumed zero suffix would not be contrastive, since no other derivational suffix appears in this position to mark the underlying radical as 'non-derived.' Such is the case of all derivational affixes: a derivational affix is never obligatory (by definition), and an absence in a non-obligatory position cannot be significative. The 'Overt Analogue Criterion' (Sanders 1988[156]) that is, the existence of a non-zero derivational affix that expresses the same meaning which we are about to ascribe to the presumed zero affx-is not sufficient.

Moreover, this derivational zero suffix would violate as well Condition 2 of the IZLS Principle: there is an overt linguistic means to which the meaning in question can be ascribed. Namely, the linguistic means used to derive *[a] cook and [a] gossip* from, respectively, *[to] cook* and *[to] gossip* is conversion: a regular technique consisting in modification of the syntactics of the initial radical, in this case the substitution Verb ⇒ Noun (on morphological conversion, see Mel'<uk 1982: 102-104 and 1993-2000, vol. 4: 309ff [20]).

The impossibility of derivational zeroes follows from the fact that derivational oppositions are PRIVATIVE (Plungjan): a derived unit *X+a* <X+a> is semantically always more complex than the underlying unit *X* <X>, which does not include any meaning opposed to <a> (thus, Russian diminutives of the type *äar+ik* <[a] small ball> express the meaning <small>; but the underlying radicals do not express the meaning <big> or <not small>; *äar* can denote a very big and a very small ball-cf. *kroxotnyj äar* <[a] tiny ball>. On the contrary, inflectional oppositions are necessarily equipollent: as a rule, one inflectional form *X+b* <X+b> contrasts with another inflectional form *X+c* <X+c>, so that both forms are of equal semantic complexity.

[19] There is another lexeme with the same meaning: GOSSIPER; it is derived from *[to] gossip* by the regular agent suffix **-er**. Its existence does not in any way affect my reasoning or this example.

[20] On the opposition 'conversion ∼ zero-affixation,' see Lieber 1981[119*ff*], "Against Zero-Affixation." Lieber's main argument against derivational zero suffixes in the cases like Germ. ruf(-en) <[to] call> ∼ der Ruf <[a] call> (PL. Rufe) or bind(-en) <[to] bind, tie> ∼ das Band <[a] tie> (PL. Bänder) is that one zero suffix would not be sufficient, since the derived nouns are of different genders and different declension types. In point of fact, on the same grounds, Lieber objects against conversion as a directed derivational means: she proposes to consider conversion as a static symmetrical relation between two stems in the lexicon. As for the cases of the type of Latin supines, formed from a past participle: *perd+it-[us]* ∼ *perd+it+um* (in order to lose), *ses+s-[us]* ∼ *ses+s+um* (in order to sit (down>>, or English deverbal adjectives, also formed from a past participle: annoyed, inhabited, etc., Lieber 1981[144-148]) admits a derivational zero suffix here, given the absolute uniformity of the derived elements. However, for me, a derivational zero suffix in these forms is inadmissible because it is not contrastive: in the supine stem *perd+it*+ØSUPINE , it would be opposed only to the absence of any derivational suffix in the participle stem *perd+it-*, and this is not allowed by the IZLS Principle. The same argument applies to Lieber's proposal (1992: 66-67) to see a derivational zero suffix in French compounds of the type *essuie-glace* <windshield wiper> or *tire-bouchon* <corkscrew>: all of them have similar meanings ((instrument for ...)> and are of the masculine gender. Yet I believe that this is a pure linguist's zero, postulated in order to make our description more elegant; it is not contrastive and thus cannot be deemed a language zero, the only variety I work with.

0.13 Most Current Zero Signs

It seems useful to give here a list of the zero signs known to me that are most currently mentioned in the literature.

0.14 Morphological Zeroes: Zero Morphs and Morphemes

0.14.1 Nouns

SG	Eng.	*table*+$\emptyset_{SG}$
		vs.*table*+s
	Sp.	*mesa*+$\emptyset_{SG}$ <table>
		vs.*mesa*+s
	Hung.	*asztal*+$\emptyset_{SG}$-
		vs.*asztal+ak*- <tables>
NOM	Hung.	*asztal*+$\emptyset_{SG}$ + $\emptyset_{NOM}$ <table, NOM>
		vs.*asztal*+$\emptyset_{SG}$+*at* <table, ACC>,
		asztal+$\emptyset_{SG}$+*ban* <table, INESS>, ...
SG.NOM	Rus.	*stol*+$\emptyset_{SG.NOM}$ <table, SG.NOM>
PL.GEN	Rus.	*ruk*+$\emptyset_{PL.GEN}$ <hand/arm, PL.GEN>
DEF(inite)		
	Basque	mendid+$\emptyset_{DEF}$+*tik* <the mountain, ABL>
	vs.	*mendi+ta +tik* <mountain, ABL>
INDEF(inite)		
	Basque	*mendi*+$\emptyset_{INDEF}$ + $\emptyset_{NOM}$ <mountain, NOM>
	vs.	*mendi+a* +$\emptyset_{NOM}$ <the mountain, NOM>
		(The distribution of the markers of definiteness on a Basque noun depends on its case.>

0.14.2 Adjectives

MASC	Sp.	*inglés*+$\emptyset_{MASC-}$	<English, MASC>
		vs. *ingles+***a-** <English, FEM>	
SG	Sp.	*inglés*+$\emptyset_{MASC}$+$\emptyset_{SG}$	<English, MASC, SG>
		vs. *ingles*+$\emptyset_{MASC}$+es <English, MASC, PL>	
MASC	Rus.	*xoroä*+$\emptyset_{MASC}$	<[is] good, MASC>
		vs. *xoroäa* <[is] good, FEM>	
		xoroä+i <[are] good, PL>	

0.14.3 Zero Radicals

<this>	Kirundi -$\emptyset^{THIS}$: *uwu*+$\emptyset^{THIS}$, *aba*+$\emptyset^{THIS}$, ... vs. *uw +o, ab +o*, ...
<he>	Serbo-Cr. $\emptyset^{HE-}$: $\emptyset^{HE}$+*ga*, $\emptyset^{HE}$+*mu* vs. *nj +ega*, *nj+emucheap'*

0.14.4 Verbs

PRES.IND	Sp.	*habla*+$\emptyset_{PRES.IND}$- <[to] speak, PRES.IND>
		vs. *habla*+ba- [IMPF], *habla*+r- [FUT],*habla+se*- [IMPF.SUBJ]
IMPER	Sp.	*habla*+$\emptyset_{IMPER}$ <[to] speak, IMPER>
AORIST	Alutor	*t+uvvat*+$\emptyset_{AOR}$+nin <[to] kiss, AOR, $1_{SG}.sub - 3_{SG}$.obj> [⇔ tuvvannin]
		vs.*t+uvvat+tk*[PRES]+nin
1_{SG}	Sp.	*habla+ba*+$\emptyset 1_{SG}$ <I spoke> ∼ *habla+se*+$\emptyset 1_{SG}$ <[that I spoke> ∼
		hable+$\emptyset_{PRES} + \emptyset 1_{SG}$ <[that] I speak>
		vs. *habla+ba+s* <you$_{SG}$ spoke>, *habla+se+s* <[that] you$_{SG}$ speak>, etc.
2_{SG}		Georg. $\emptyset 2_{SG}$+*ici* <[you$_{SG}$] know>
[*Indic.*]		vs. *v+ici* <[I] know>, *ici +s* <[he] knows>
2_{SG}	Sp.	*habla*+$\emptyset_{IMPER} + \emptyset 2_{SG}$ <speak$_{SG}$!>
[*Imper.*]		vs. habla+$\emptyset_{IMPER}$+d <speakPL!>
	Czech	*čeen*+ $\emptyset_{IMPER} + \emptyset 2_{SG}$ < *drive*$_{SG}$!>
		vs. *čeen*+$\emptyset_{IMPER}$+me <let's drive>,*čeen*+$\emptyset_{IMPER}$+te <drive$_{PL}$!>
3SG	Sp.	habla+$\emptyset_{PRES.IND} + \emptyset_{3SG}$ <s/he speaks> ∼ *habla+ba*+$\emptyset_{3SG}$ <he spoke> ∼
		habla+se+$\emptyset 3_{SG}$ <[if] he spoke> ∼ *hable*+$\emptyset_{PRES} + \emptyset_{3SG}$ <[that] he speak>
SG	Georg.	*v+ici* +$\emptyset_{SG}$ <[I] know>
		vs. *v+ici +t* <[we] know>
MASC	Rus.	*spal*+$\emptyset_{MASC}$ <[I$_{MASC}$ <you$_{MAS}$C, he>] slept>
		vs. *spal+a* <[IFEM <you$_{FEM}$, she>]
		slept>, *spal+i* <[we <you$_{PL}$, they>] slept>
ACTIVE	Lat.	*orn +o*+$\emptyset_{ACT}$ <[I] adorn>
		vs. *orn +o+r* <[I] am being adorned>, *orna+t*+$\emptyset_{ACT}$ <[S/he] adorns>
		vs. *orna+t+ur* <[He] is being adorned>

0.15 Syntactic Zeroes: Zero Wordforms (= Lexes) and Lexemes

Copula	
Rus. Ø:	*Leo bogatyj* <Leo is rich> <a zero lex of a non-zero lexeme>
Impersonal pronoun	
Sp. Ø:	*Nieve* <It snows>; *Es difícil de dormir* <It is difficult to sleep>.
Pashto Ø:	*Sari> xand+l* <[The] man [OBL] laughs [PL]>,
	lit. <Things [PL] laugh man [= DirO]>.
People Rus.Ø:	*Ivana ubili* <Ivan was killed [by people]>,
	lit. <They killed Ivan [= DirO]>.
Massai Ø:	*ríkíenkI;te;n* <The cow was led>,
	lit. <They were led the cow [= DirO]> <Mel'čuk 1997>.
Elements Rus. Ø:	Ivana ubilo <Ivan was killed [by something/by mysterious forces]>,
	lit. <It killed Ivan>.

0.16 Zero Signs vs. Non-Significative Zeroes

One of the greatest merits of Haas 1968bis, I think, having established the two-way distinction between typical uses of 'zero' terminology in modern linguistics: zeroes belonging to language-something that we can call 'linguistic elements/units,' whatever the meaning of the latter expression-and zeroes belonging to linguistics, that is, different devices or *facons de parler* that help linguists to formulate their descriptions. Such zeroes are non-significative: they are not used to convey information; they are, so to speak, the linguist's zeroes. The linguist's zeroes can be further subdivided into two types:

- Vague or metaphorical uses of the term zero-such that it does not intend any reference to linguistic elements, but can designate "any linguistic notion, whenever for one reason or another such notion is found to be inapplicable ... ‘Zero,’ as thus used, is merely the negative particle transposed into the category of nouns" (Haas 1968b[34], fnt. 1): *zero-expression, zero word order, zero function, zero contrastg, zero style, zero case* (= the nominative or the absolutive), *zero tense* (= the present), *zero degree verb* (= the infinitive), *zero derivation*, ...
- Uses that are justified only by the requirements of general statements (Haas 1968b[50]): for instance, in cases like sheep one could say-by analogy with tens of thousands of English nouns-that plurality is expressed by a zero suffix (these uses go back to Bloomfield and further to P,n>nini). To the extent that we are interested in the ‘real’ zeroes of language, all such uses should be carefully avoided in order not to create confusion. (But cf., however, Kibrik’s considerations on ‘system-justified’ zeroes: Kibrik 1997[55-56]) language zeroes are necessarily either linguistic signs or -emic sets of signs. More precisely, there are
    - either morphological zeroes: morphs/morphemes, reduplications/reduplicationemes, apophonies/ apophonemes and conversions/conversionemes;
    - syntactic zeroes: wordforms (= lexes) and lexemes. These zeroes, as I have already said, can also be called lexical: they are lexical units used in syntactic structures.

There are no other zeroes in language. Among other things, no phonemic zeroes (such as, e.g., zero juncture) are possible. A phoneme is not a sign; and a phonemic zero element would not be admitted by our Principle for the introduction of zero signs. Interestingly, even researchers who like to speak of ‘phonemic’ or ‘phonetic’ zeroes do not propose to actually write them in the transcription, while morphological and syntactic zeroes (= zero signs and zero -emic sets of signs) are always written in the corresponding representation.[21]

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A Dynamic Modal Arrow Logic for the Analysis of Aspectual Phenomena in Natural Language

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Abstract

In Naumann (1998b,1999a) and Naumann/Mori (1998) a *Dynamic Event Semantics* (DES) for the analysis of aspectual phenomena was developed that is basically a many-sorted type theory in the sense of Gallin. DES is based on the intuition that non-stative verbs express changes. A change can be conceived of either as an object (event, action) or as a transformation of state. The first perspective is captured in Event Semantics and in Arrow Logic, whereas the second perspective underlies Dynamic Logic. In models for DES the first perspective of changes as objects is accounted for by an eventuality (sub-)structure **E** whereas the second perspective is captured by a transition (sub-)structure **S**.

As noted in the above mentioned articles, the aspectual properties of expressions only depend on the properties the execution-sequences of events have with respect to the result they bring about. This raises the question of whether it is possible to express the aspectual properties of linguistic expressions in weaker logics. In this paper two logics L and L^h are defined. L combines (fragments of) Arrow Logic and Dynamic Modal Logic. It is two-sorted: besides s-formulas there are e-procedures. The latter are primary because verbs and their projections are translated as e-procedures. In L it is not possible to express the dependency of results that are brought about by an event e on particular objects that participate in e. This is possible in the hybrid extension L^h of L. The resulting theory is applied to modification of VPs with directional PPs and the temporal conjunction ‘until’, the interpretations of which are sensitive to the aspectual properties of verbs.

1 Data and Evidence

Modifiability with *in*-, *for*- and *at*-adverbials of an expression depends on the underlying verb, witness the data in (1).

(1) a. John ate an apple in ten minutes/*for ten minutes/*at three.
b. Mary pushed the cart *in ten minutes/for ten minutes/*at three.
c. Peter was ill *in ten minutes/for two weeks/at three.
d. Susan reached the station in ten minutes/*for ten minutes/at three.

These differences are the basis for the Vendler-classification of verbs: Accomplishments (‘eat’), Activities (‘push’), stative verbs (‘be ill’) and Achievements (‘reach’). This classification is not exhaustive as the examples in (2) show.

(2) a. Bill knocked at the door *in ten minutes/*for ten minutes/at three.
b. Mary painted the wall in ten minutes/for ten minutes/*at three.
c. Mary gave Bill the book *in ten minutes/*for ten minutes/at three.

Whereas a verb like ‘knock’ neither admits of modification with ‘in’ nor of that with ‘for’-adverbials (the latter yields an iterative interpretation), for a verb like ‘paint’ both adverbials are admissible. In Naumann (1999a,c) ‘knock’ and ‘paint’ are classified as Point- and Proc-Acco-verbs, respectively. In contrast to Achievement-verbs, a resultative reading of the (present) perfect, a so-called Perfect of Result, is not possible for Point-verbs: ‘John has knocked at the door’ can only be interpreted as an experiential perfect, i.e. in the sense of John knocked at the door at least once in the past.[1] Transfer-verbs differ from Accomplishment-verbs with respect to the progressive. Whereas the inference from ‘John was eating an apple’ to ‘John ate part of the apple’ (= there was an event of eating) is valid, this does not hold for ‘give’: Mary was giving Bill the book’ does not imply that there was an event of giving because from this it would already follow that Bill got the book from Mary, i.e. the inference from ‘Mary was giving Bill the book’ to ‘Mary gave Bill the book’ would be valid, contrary to the evidence. Contrary to Achievement-verbs, modification with an ‘in’-adverbial of expressions containing an Accomplishment-verb is not (quasi)-synonymous with a corresponding sentence using ‘after’ instead of ‘in’: Susan reached the station in ten minutes = Susan reached the station after ten minutes; John ate an apple in ten minutes ≠ John ate an apple after ten minutes.

The behaviour with respect to temporal adverbials can be changed by various types of modifying expressions like directional PPs, (3a), secondary predication, (3b), and ‘for’-adverbials, (3c), (4)-(5).

(3) a. Mary pushed the cart to the station in ten minutes/*for ten minutes/*at three.
b. Mary painted the wall blue in ten minutes/*for ten minutes/*at three.
c. John pushed a cart for twenty minutes.

(4) a. Mary was pushing the cart $\Rightarrow$ Mary pushed the cart
b. Mary was pushing the cart for twenty minutes $\not\Rightarrow$ Mary pushed the cart for twenty minutes.

(5) a. Mary almost pushed the cart $\Rightarrow$ Mary did not push the cart (but, say, had the intention to do it)
b. Mary almost pushed the cart for twenty minutes
$\Rightarrow$ Mary did not push the cart (but, say, had the intention to do it)
$\Rightarrow$ Mary pushed the cart, say, for eighteen minutes

[1]For an analysis of the Present Perfect in English see Naumann (1999b).

Whereas (1b) is an Activity-expression, (3a) is an Accomplishment-expression. A similar argument applies to (2b) and (3b): depictive adjectives change the Proc-Acco-expression (2b) into an Accomplishment-expression, (3b). The examples in (3c-5) show that modification with a 'for'-adverbial changes the aspectual behaviour too. For the modified expression one gets both a so-called imperfective paradox, (4), and two readings for modification with 'almost', (5). Examples like these show that the aspectual behaviour of an expression is only partly determined by the underlying verb. Another determining factor are modifying expressions. Furthermore, certain expressions impose aspectual restrictions on their use. An example is the temporal conjunction 'until'. It imposes an aspectual restriction on the expression in the main clause: only stative- and Activity-expressions are admitted, (6a). Both Accomplishment- and Achievement-expressions are excluded, (6b).

(6) a. John was ill/worked on the article until Mary arrived.

b. *Bill wrote the article/reached the station until Mary arrived.

The task, then, consists in finding interpretations of verbs in the lexicon on the basis of which (i) the differences in aspectual behaviour at the lexical level and (ii) the process of aspectual composition (changes in aspectual behaviour triggered by modifying expressions) can be explained.

2 Dynamic Event Semantics

Changes as Objects and Changes as Transformations of States Dynamic Event Semantics (DES), Naumann (1998,1999) and Naumann/Mori (1998), is based on the intuition that non-stative verbs like 'eat' express changes. The intuitive notion of a change comprises at least two aspects that are complementary to each other: (i) something (an object: action, event) which brings about the change; (ii) something (a result) which is brought about by the change and which did not hold before the change occurred. In (i) 'change' is understood as the result that is brought about, i.e. in the sense that is captured by (ii), whereas in (ii) 'change' is meant as the object that brings about the result. The second aspect can be described as a transformation of state (TS). Before the change occurred, the world was in a particular state, say s, at which some result Q did not hold, whereas after the change has occurred, the world is in a state s' at which Q does hold. E.g., the eating of an apple is an event of type eating if conceived of as an object. On the perspective of a change as a TS one gets: a state s at which there is a complete apple is transformed into a state s' where the apple no longer exists ($Q =$ the apple does not exist). For the pushing of a cart the change as an object is an event of type pushing whereas the transformation can be described as 'a state s is transformed into a state s' such that relative to s the cart traversed a non-empty path' ($= Q$). The first perspective, changes as objects, is captured in Event Semantics as well as in Arrow Logic, whereas the second perspective, changes as TS, is captured in Dynamic Logic (or Temporal Logic).

Verb Classification in Dynamic Event Semantics. The double perspective on the intuitive notion of a change either as an object or as a transformation of state that brings about a result is modeled in DES by having both an eventuality-structure **E** with an underlying domain E of events and a transition-structure **S** with an underlying domain S of states. The aspect of a change as an object is captured at the level of E whereas the aspect as a transformation of state is captured at the level of S. The elements of S are basic objects without structure that are ordered by a strict, linear ordering $<_S$. The domain E is related to the domain S by two functions $\alpha : E \rightarrow S$ and $\omega : E \rightarrow S$ that assign to each $e \in E$ its beginning point $\alpha(e)$ and end point $\omega(e)$, respectively. Together, α and ω determine the execution sequence $\tau(e)$ of e : $\tau(e) = \{s \mid \alpha(e) \leq s \leq \omega(e)\} = (\alpha(e), \omega(e))$. The domain E is structured by a part-of relation $\leq_E$ in terms of which a composition relation C is defined that is required to be associative.

(7) $C = \{(e, e_1, e_2) \mid e_1 \leq_E e \wedge e_2 \leq_E e \wedge \omega(e_1) = \alpha(e_2) \wedge \alpha(e) = \alpha(e_1) \wedge \omega(e) = \omega(e_2)\}$

C corresponds to the operation of (sequential) composition in Dynamic Logic (DL). In terms of C the following relations can be defined.

(8) a. $R_B = \{(e, e') \mid \exists e'' : Ce', ee''\}$
b. $R_{B^*} = \{(e, e') \mid R_B ee' \wedge e \neq e'\}$
c. $R_{E'} = \{(e, e') \mid \exists e'' : Ce, e''e'\}$

The *interior* $\tau^*(e)$ *of* $\tau(e)$ is defined as $\tau^*(e) = \tau(e) - \{\alpha(e), \omega(e)\}$. The set of point-events is defined as $I = \{e \mid \alpha(e) = \omega(e)\}$. The domain E is sorted by the label set $VERB$: for $v \in VERB$, $P_v \subseteq E$ is the set of all events of type v, e.g. the set of all eating events if $v = eat$. The elements of the set $\{P_v \mid v \in VERB\}$ are called *basic* event-types. A verb v in the lexicon is always interpreted with respect to P_v, i.e., P_v is the event-type *corresponding* to v.[2] Results Q are subsets of S. A TS is a pair $< Q, < s, s' >>$ with (s, s') a finite sequence of states and Q a result such that $s \notin Q$ and $s' \in Q$. An event-type P_v determines for each of its elements e a set $Res(P_v, e)$ of results that e can possibly bring about. The relationship between an event e and TSs is therefore in general not one-one but one-many: e can bring about more than one result. Different types of results can be distinguished by the way they are evaluated on the execution sequence of events $e \in P_v$. Basically, three different types of results are distinguished: (i) a result Q is *s-minimal* just in case it holds at all states of the execution sequence of e in between e's beginning point and a state s of the execution sequence at which Q holds; (ii) a result Q is *w-minimal* if it holds at all states of the execution sequence of e that are end points of initial stages e' of e that are of type P_v and (iii) a result Q is *maximal* if it only holds at the end point of e (if it holds at all on e's execution sequence). For an event of type 'John eat a fish' examples for the three types of results are: s-minimal: the results brought about by the initial actions by John (e.g. John's opening and closing his mouth, putting a piece of the fish into his mouth)[3]; w-minimal: the results brought about by swallowing part of the fish: partial decrease in the

[2]Note that v is used ambigously. On the one hand, it is an element of the label set $VERB$, the interpretation of which is a subset of the basic domain E of events. On the other hand, v is an element of the lexicon of a particular language, say English, which is interpreted with respect to $v \in VERB$. The translation of v in this second sense in DMAL is a complex e-procedure, see section (3.5) for details.

[3]Note that these actions do not necessarily involve the fish.

mass of the fish due to the swallowing; maximal: total decrease of the mass of the fish: its mass is zero. The set $Res(P_v, e)$ is temporally ordered: the s-minimal results are brought about first, followed by the w-minimal results and the maximal results are brought about last. It is therefore possible to define an ordering $\leq_v$ on $Res(P_v, e)$ based on the relation 'not before': $Q \leq_v Q'$ if Q' is brought about not before Q on the execution sequence of e. Intuitively, Q' is brought about not before Q (relative to an event-type P_v) just in case whenever Q' holds at some point s of the execution sequence $\tau(e)$ of an event $e \in P_v$, then there is a point $s' \in \tau(e)$ with $s' \leq s$ at which Q holds, Latrouite/Naumann 1999a,b.

Verbs v are classified on the basis of (i) the *types* of results that are determined by the corresponding event-types P_v, (ii) the *sort* to which the event-types P_v belong and (iii) the *sort* of the results that are determined and which are maximal elements of $Res(P_v, e)$ with respect to the ordering $\leq_v$ for $e \in P_v$. E.g., Accomplishment-verbs like 'eat' determine all three types of results whereas Activity-verbs like 'push' do not determine a maximal result but only an s- and a w-minimal one. Transfer-verbs determine two maximal results, whereas Point- and Achievement-verbs determine only one. Three different sorts of event-types are distinguished: an event-type P_v is *P-atomic* if no proper initial stage (prefix) e' of an event e belonging to P_v is of this type too, (9a); an event-type P_v is *instantaneous* if each of its elements has an execution sequence that consists of a single state (i.e., the beginning point is identical to the end point), (9b), Naumann 1999b. Instantaneous and P-atomic event-types together form the *atomic* event-types. Finally, an event-type P_v is non-atomic if it is not P-atomic and if the execution sequence of each of its elements is not a singleton. Event-types of sort Accomplishment and Activity are non-atomic, (9c).

(9) a. $\forall P[P - Atomic(P) \Leftrightarrow \forall e[e \in P \rightarrow \neg\exists e'[prefix(e', e) \wedge e' \in P]]]$

b. $\forall P[Instant(P) \Leftrightarrow \forall e[e \in P \rightarrow \alpha(e) = \omega(e)]]$

c. $\forall P[Non - Atomic(P) \Leftrightarrow \neg P - Atomic(P) \wedge \forall e[e \in P \rightarrow \alpha(e) <_S \omega(e)]]$

Examples for P-atomic event-types are those corresponding to Transfer-verbs like 'give' and 'buy'. Point- and Achievement-verbs like 'hit' and 'reach', respectively, correspond to instantaneous event-types. Events belonging to an instantaneous event-type presuppose other events of which they are right boundaries (for details see Naumann (1999a)).[4]

Results that are of the same type can differ with respect to the *sort* to which they belong. Consider events of type 'John eat an apple' and 'Mary sing a song', respectively. Both expressions are of type Accomplishment and therefore define a maximal result which can be paraphrased as: 'the mass of the apple is zero' and 'relative to the beginning-point of the event the partition of the song was went through completely'. The difference between the two results is that the former but not the latter continues to hold after the event terminated. The former result is therefore *state-related* whereas the latter is *event-related*. A result is state-related if it continues to hold after the end of an event until it is undone (by some other (non-stative) event). A result is event-related if it is true

[4]Furthermore, for events belonging to an instantaneous event-type as well as for events that are denoted by unaccusative forms like 'The bottle emptied' the results are evaluated with respect to a presupposed event.

only during the the executions of (non-stative) events (for details see Naumann (1999a)). The classification based on the three criteria is given in Table 1 ((i) = type of maximal elements of $Res(P_v, e)$ relative to $\leq_v$, (ii) = sort of event-type and (iii) = sort of (i)).

	(i)	(ii)	(iii)
Acco.	maximal	non-atomic	state- or event-related
Act.	w-minimal	non-atomic	event-related
Transfer	maximal	P-atomic	state-related
Point	maximal	instantaneous	event-related
Achievement	maximal	instantaneous	state-related
stative verbs	s-minimal	non-atomic	state-related

Table 1

2.1 The Relationship between Changes as Objects and Changes as Transformations of States

Recall that $Res(P_v, e)$ is the set of results assigned to e by P_v. In general, e is not required to bring about each element from this set. The question of which results must be realized by e must be split into the following two.

(i) What (types of) results must be brought about by an event e in order to be of type P_v, i.e., what requirements on the results are imposed by P_v?

(ii) What (types of) results must be brought about by an event $e \in P_v$ that is denoted by the verb v in the lexicon, i.e., what requirements on the results are imposed by (the interpretation of) v in the lexicon?

Consider an event of type eating with John as Actor and an apple as stuff that is eaten. If only the s-minimal result is brought about, e.g., John opened his mouth, the corresponding event is not of type 'eat', i.e., it does not belong to P_{eat}. What is minimally required is that John swallow at least part of the apple. This result corresponds to the w-minimal one, i.e. to a partial decrease in the mass of the apple. P_{eat} does not impose any further requirements, in particular it is not required that the maximal result (the mass of the fish is zero) be brought about. From this it follows that an event $e \in P_{eat}$ need not bring about all results from $Res(P_v, e)$. On the other hand, the w-minimal result is not sufficient for e to be an element of the set of events of type eating denoted by 'eat' in the lexicon. 'John ate an apple' is true only if John ate the apple completely and not only an arbitrary part of it. Thus, for 'eat' the requirement imposed on the results that must be brought about is stronger than the one imposed by P_{eat}: P_{eat} : w-minimal result; (interpretation of) 'eat': maximal result.

For an event of type push with Mary as Actor and a cart as object that is pushed, the minimal requirement imposed by P_{push} is the same as that imposed by P_{eat}: the w-minimal result must be brought about (the cart traversed a non-empty path). The s-minimal results corresponding to the initial actions by John only count as attempts at pushing the cart but do not in themselves constitute an event of this type. In this case the interpretation of 'push' in the lexicon cannot strengthen the condition imposed by P_{push} because this event-type does not determine a maximal result, i.e., the maximum of $Res(P_{push}, e)$

with respect to $\leq_{push}$ is the w-minimal result. Consequently, for Activity-verbs v like 'push' the requirement imposed by P_v is identical to that imposed by the interpretation of v.

What is common to the requirement imposed in English by Accomplishment- and Activity-verbs in the lexicon is that the maximum of $Res(P_v, e)$ with respect to the ordering $\leq_v$ must be brought about. This condition is equivalent to the requirement that each element from $Res(P_v, e)$ must be brought about. A similar argument applies to the other classes of verbs like Transfer- or Point-verbs discussed above. In each case the interpretation of a verb v belonging to one of these classes requires an event $e \in P_v$ to bring about the maximum of $Res(P_v, e)$ with respect to $\leq_v$ and therefore each element from $Res(P_v, e)$. The type of the maximal elements of $Res(P_v, e)$ depends on the aspectual class to which v belongs. For verbs of type Accomplishment, Transfer, Point and Achievement this is the type of maximal results whereas for Activity-verbs and stative verbs the maximum is w-minimal and s-maximal, respectively.

3 The Analysis

3.1 Dynamic Nucleus Structures and Dynamic Modes

3.1.1 Dynamic Modes

In section (2) it was shown how various aspectual classes can be distinguished in terms of four criteria that are all related to the dynamic-temporal structure of events of the types belonging to one of these classes. A first criterion are the types of results that are determined by P_v. Important for the classification is the type of the maximal elements of $Res(P_v, e)$ with respect to the ordering $\leq_v$. Types of results differ with respect to the way they are evaluated on the execution-sequences of events. E.g., maximal results only hold at the end-point (if at all) whereas w-minimal results hold at the end-points of all initial stages e' of e that are of type P_v. As maximal elements of $Res(P_v, e)$ are required to hold at $\omega(e)$ by the interpretation of v in the lexicon, it follows that they are evaluated on $\tau(e)$ in the way determined by their type if they belong to the subset of P_v denoted by v. This way corresponds to a dynamic mode from Dynamic Modal Logic (DML), de Rijke (1993). DML is a two-sorted logic that is based on a two-level architecture of formulas and procedures (programs), together with two types of operations modeling the interaction between the two sorts of expressions: dynamic modes (DM) map formulas to procedures and static projections map procedures to formulas. A DM is interpreted as an operation from $\wp(S)$, the power set of the underlying domain of states, to $Re(S)$, the set of binary relations on S: $\lambda Q \lambda s s' [\![DM]\!](Q)(s)(s')$. Two examples for dynamic modes are given in (10).

(10) a. $R_{Min-BEC}(Q) = \{(s, s') \in S \times S \mid s <_S s' \wedge \neg Q(s) \wedge Q(s') \wedge \forall s''[s <_S s'' <_S s' \rightarrow \neg Q(s'')]\}$

b. $R_{Con-BEC}(Q) = \{(s, s') \in S \times S \mid s <_S s' \wedge \neg Q(s) \wedge Q(s') \wedge \forall s''[s <_S s'' <_S s' \rightarrow Q(s'')]\}$

The crucial observation is that dynamic modes exactly fit the description given above in section (2) of how changes as transformations of states (TS) are brought about. There a TS was defined as a pair $< Q, < s, s' >>$ consisting of a result $Q \subseteq S$ and an element from $S \times S$ such that Q is false at s and true at s'. For a given dynamic mode DM, $< Q, < s, s' >>$ is either an element of its denotation or not. If Q is a result that must be brought about by an event $e \in P_v$, in particular if $Q \in Res_{\max}(P_v, e) = \{Q \mid Q \in Res(P_v, e) \wedge \forall Q'[Q' \in Res(P_v, e) \rightarrow Q' \leq_v Q]\}$, and if Q is of type i, for i one of the three basic types, Q is evaluated on $\tau(e)$ in the way determined by i. If this way is expressed by the dynamic mode DM and if $(s, s') = \tau(e)$, one gets: $< Q, < s, s' >> \in [\![DM]\!]$. Thus, Q corresponds to the result brought about, whereas $[\![DM]\!]$ (or $[\![DM]\!](Q)$) corresponds to the way the result is brought about. E.g., for the type of change expressed by an Accomplishment-verb v, a maximal result Q assigned to an $e \in P_v$ is mapped to a binary relation such that Q only holds at the output-state s' and at no other state of the execution-sequence $\tau(e)$.

From the present perspective, the disadvantage of DML consists in the lack of a separate domain E of events (actions). DML-models are one sorted: there is only one domain S of states. As a consequence, transitions are not basic objects but are interpreted as binary relations on S such that the aspect of a change as an object cannot be modeled. Models for Arrow Logic, on the other hand, are two sorted: transitions are not interpreted as (elements of) binary relations but as elements of a separate domain of arrows. In models for DES defined in section (2) above both perspectives are combined because there is both a domain E of events and a domain S of states that are systematically related to each other.

The relationship between dynamic modes and the domain E of events can be defined as follows. Recall that each event $e \in E$ belongs to an event-type $P_v \subseteq E$ that is an element of the basic set $\{P_v \mid v \in VERB\}$ for $VERB$ a subset of the verbs in English. Each P_v induces the binary relation R_v on $S \times S$.

(11) $R_v = \{(s, s') \in S \times S \mid \exists e[e \in P_v \wedge \tau(e) = (s, s')]\}$

Compared to DL, R_v can be interpreted as corresponding to the binary relation $R\pi$ denoted by a (basic) program letter π in the following sense: each $e \in P_v$ corresponds to an element $(s, s') \in R_v$ and vice versa. On this perspective, e is the object (change) that brings about the transition from s to s'. Each P_v can therefore be seen as a kind of accessibility-relation corresponding to R_v.

If $(s, s') \in R_v$, the minimal requirement on the result imposed by P_v is satisfied. As was shown above, this requirement can be weaker than the one imposed by the interpretation of v. The relation corresponding to this interpretation is R_v^*.

(12) $R_v^* = \{(s, s') \in S \times S \mid \exists e \exists Q[e \in P_v \wedge \tau(e) = (s, s') \wedge Q \in Res_{\max}(P_v, e) \wedge [\![DM]\!](Q)(s)(s')]\}$

An element of R_v belongs to R_v^* if an element Q of the set of maximal elements of $Res(P_v, e)$ with respect to $\leq_v$ is brought about, i.e. holds at $\omega(e)$. DM is the way Q is evaluated on $\tau(e)$.

As the elements (s, s') of $[\![DM]\!](Q)$ one is interested in are the execution sequences of events, a dynamic mode can be interpreted as an operation from $\wp(S)$ to $\wp(E)$. Yet, this is in general too simple because a type of result can impose restrictions that depend on a basic event-type P_v. This is the case for w-minimal results. In DES dynamic modes are therefore interpreted as operations from $\wp(E) \times \wp(S)$ to $\wp(E)$. Three examples are given in (13).

(13) a. $R_{Min-BEC_{<_S}} = \lambda P \lambda Q \lambda e[e \in P \wedge \alpha(e) \notin Q \wedge \omega(e) \in Q \wedge \forall s[\alpha(e) <_S s <_S \omega(e) \rightarrow s \notin Q]]$

b. $R_{Con-BEC_{<_S}} = \lambda P \lambda Q \lambda e[e \in P \wedge \alpha(e) \notin Q \wedge \omega(e) \in Q \wedge \forall s[\alpha(e) <_S s <_S \omega(e) \rightarrow s \in Q]]$

c. $R_{Con-BEC_v} = \lambda P \lambda Q \lambda e[e \in P \wedge \alpha(e) \notin Q \wedge \omega(e) \in Q \wedge \forall e' \forall s[\alpha(e) <_S s <_S \omega(e) \wedge prefix(e', e) \wedge \omega(e') = s \wedge e' \in P \rightarrow s \notin Q]]$

Note that all three modes require that Q be evaluated differently at the beginning- and the end-point of e, in accordance with the definition of TS given above. From this it follows that they cannot be used for the interpretation of verbs v whose corresponding event-types P_v are instantaneous because the execution sequences of events $e \in P_v$ are singletons such that no TS corresponds to $\tau(e)$. This possibility is related to the fact that the three basic types of results do not exclude each other. What types are indistinguishable for an event-type P_v depends on the sort to which it belongs. For instantaneous event-types, all three types coincide. For P-atomic event-types, the distinction between w-minimal and maximal results collapses: each w-minimal result is maximal and vice-versa. This property distinguishes the (non s-minimal) results assigned to verbs of this class from those assigned to verbs belonging to the class of Accomplishments or Activities where neither w-minimal results are maximal nor maximal results w-minimal. Maximal results that are not w-minimal are called *strongly maximal* (*s-maximal*) whereas maximal results that are also w-minimal are called *weakly maximal* (*w-maximal*). Results that are w-minimal but not maximal are called *w*-minimal*, Latrouite/Naumann (1999a,b).

The problem of defining dynamic modes for instantaneous event-types can be solved by distinguishing two subtypes of s-minimal results: those that are false at $\alpha(e)$ and those which are true at $\alpha(e)$. The mode corresponding to the former, s-minimal$_1$, is $R_{Con-BEC_{<_S}}$, whereas the mode corresponding to the latter, s-minimal$_2$, is $R_{HOLD_{<_S}}$, defined in (14).

(14) $R_{HOLD_{<_S}} = \lambda P \lambda Q \lambda e[e \in P \wedge \forall s[\alpha(e) \leq_S s \leq_S \omega(e) \rightarrow s \in Q]]$

Results that are s-minimal$_2$ characterize both Point-, Achievement- and stative verbs. For the two former classes these results are, in addition, s-minimal, w-minimal and maximal, whereas for the latter class these results are s-minimal and w-minimal but not maximal.

The correspondence between types of results and dynamic modes is given in Table 2.

s-minimal$_1$	$R_{Con-BEC_{<_S}}$
s-minimal$_2$	$R_{HOLD_{<_S}}$
w*-minimal	$R_{Con-BEC_v}$
w-maximal	$R_{Min-BEC_{<_S}}$
s-maximal	$R_{Min-BEC_{<_S}}$

Table 2

From this correspondence between types of results and dynamic modes the following partial characterization of the aspectual classes can be derived.

Accomplishment	$R_{Min-BEC_{<_S}}$
Activity	$R_{Con-BEC_v}$
Point	$R_{HOLD_{<_S}}$
Achievement	$R_{HOLD_{<_S}}$
Transfer	$R_{Min-BEC_{<_S}}$
stative verbs	$R_{HOLD_{<_S}}$

Table 3

There is no distinction between (i) Accomplishments- and Transfer-verbs and (ii) between stative verbs, Points and Achievements at this level. These classes can be distinguished if in addition the following two further criteria discussed in section (2) are used: (a) sort of event-type and (b) sort of result. Although Accomplishment- and Transfer-verbs do not differ with respect to the way a $Q \in Res_{\max}(P_v, e)$ is evaluated on the execution sequences of events $e \in P_v$, they differ with respect to the property of being strongly non-closed under (proper) prefixes of the same event-type. For event-types P_v, v of type Transfer, (15a) holds, whereas it fails to hold for v of type Accomplishment.

(15) a. $P_v \cap cl(P_v) = \emptyset$

b. $cl(P_v) = \{e \mid \exists e'[e' \in P_v \wedge prefix(e', e) \wedge e \in P_v]\}$

(15a) is captured by the dynamic mode $R_{Min-BEC^*_{<_S}}$.

(16) $R_{Min-BEC^*_{<_S}} = \lambda P \lambda Q \lambda e[R_{Min-BEC_{<_S}}(P)(Q)(e) \wedge \forall e'[prefix(e', e) \rightarrow e' \notin P]]$

The execution sequences of events belonging to an event-type P_v with v of type stative verb are non-singletons whereas those of events $e \in P_v$ with v either of type Point or of type Achievement are singletons. This difference can be captured by distinguishing the two $HOLD$-modes R_{HOLD^1} and R_{HOLD^2}.

(17) a. $R_{HOLD^1} = \lambda P \lambda Q \lambda e[R_{HOLD_{<_S}}(P)(Q)(e) \wedge \alpha(e) = \omega(e)]$

b. $R_{HOLD^2} = \lambda P \lambda Q \lambda e[R_{HOLD_{<_S}}(P)(Q)(e) \wedge \alpha(e) <_S \omega(e)]$

As was shown in sections (1) and (2), Achievement-verbs differ from Point-verbs in the sort of the maximal element from $Res(P_v, e)$ that is defined. For Achievement-verbs this result is state-related, i.e., it continues to hold after the end of the event until it is undone. For Point-verbs, on the other hand, this result is event-related, i.e., it is false after the end of the event. This difference shows up at the linguistic level in the possibility of a resultative reading for the Perfect: 'John has reached the station' has a resultative reading, implying that he is still at the station at speech-time. For 'John has knocked at the door' there is no such reading because the knocking does not bring about any result that holds beyond the end of the event. This distinction can be captured by defining variants of dynamic modes which require the result Q to hold an indefinite time after the end of the event. The general scheme of this variant is defined in (18).

(18) $DM^h = \lambda P \lambda Q \lambda e \exists e' \exists e''[DM(P)(Q)(e) \wedge HOLD(E)(Q)(e') \wedge Ce'', ee']$

The aspectual classes from section (1) can now be characterized in terms of dynamic modes as given in Table 4.

Accomplishment	$R_{Min-BEC_{<S}}$
Activity	$R_{Con-BEC_v}$
Point	R_{OCCUR}
Achievement	R_{END}
Transfer	$(R_{Min-BEC^*_{<S}})^h$
stative verbs	R_{HOLD^2}

Table 4

The dynamic modes R_{OCCUR} and R_{END} are defined in (19). They make explicit that an event $e \in P_v$ for v either of type Point or of type Achievement is the right bounary of another (presupposed) event e'. This is defined by the mode $HOLD^{1+}$.

(19) a. $R_{OCCUR} = R_{HOLD^{1+}}$

b. $R_{END} = (R_{HOLD^{1+}})^h$

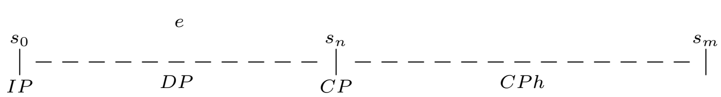
c. $R_{HOLD^{1+}} = \lambda P \lambda Q \lambda e \exists e' \exists e''$

$$[R_{HOLD^1}(P)(Q)(e) \wedge Ce', e''e \wedge R_{Min-BEC^*_{<S}}(E)(Q)(e')]$$

3.1.2 Dynamic Nucleus-Structures

In the last section aspectual classes were defined in terms of the dynamic-temporal structure of events. The dynamic-temporal structure of an event can be depicted in terms of a *Dynamic Nucleus Structure* (DNS), Moens/Steedman 1988. A DNS consists of four parts: IP (Inception-Point), DP (Development-Portion), CP (Culmination-Point) and CPh (Consequent-Phase).

Dynamic Nucleus Structure (Moens/Steedman 1988)



The DNS characterizing an aspectual class AC corresponds to the execution of an event $e \in P_v$, with $v \in AC$, that brings about all results from $Res(P_v, e)$. The IP and the CP can therefore be identified with $\alpha(e)$ and $\omega(e)$, respectively. The DNS of an aspectual class is determined by (i) the way a result $Q \in Res(P_v, e)$ is evaluated on the four parts and (ii) the way proper prefixes e' of e are related to P_v, i.e., whether they can be of type P_v or not. All DNSs agree on the way Q is evaluated at the CP: it is true at this point. For the other parts one gets:

(i) if Q is true at the IP, Q is strongly s-minimal (Point, Achievement, stative verb)

a. if Q is false on the DP, it is maximal (Accomplishment, Transfer)

b. if Q is true and false on the DP, the DP is empty (Point, Achievement)

(ii) if Q is true on the CPh, Q is state-related (Transfer, Achievement)

(iii) if Q is false on the CPh, Q is event-related (Activity, Point, stative verbs)

If P_v is P-atomic and therefore strongly non-closed under prefixes of this type, the answer to (ii) is *no*, otherwise *yes*.

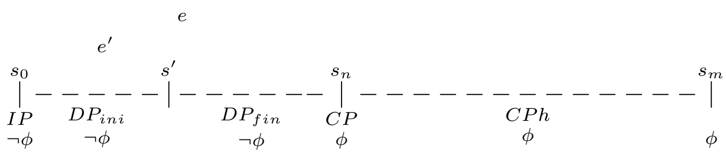
Table 5 summarizes the results.

	IP	DP	CPh	$e' \in P_v$
Acco	false	false	false/true	yes
Act	false	true	false	yes
Transfer	false	false	true	no
Point	true	false∧true	false	yes
Ach	true	false∧true	true	yes
stative verb	true	true	false	yes

Table 5

Below the DNS of the aspectual classes discussed in this paper are given.

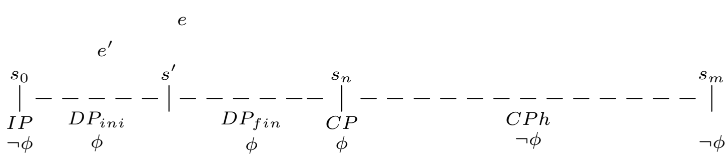
Dynamic Nucleus Structure of Accomplishment-verbs ('eat')



⇒ ϕ expresses the maximal result (the mass of the object denoted by the internal argument is zero)

⇒ $e' \in P_v$, $\omega(e')$ $(= s') \notin Q\phi$

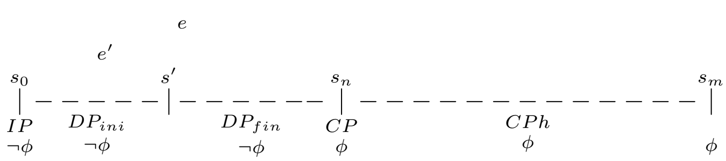
Dynamic Nucleus Structure of Activity-verbs ('push')



⇒ ϕ expresses the w^*-minimal result (the mass of the object denoted by the internal argument is zero)

⇒ $e' \in P_v$, $\omega(e')$ $(= s') \in Q\phi$

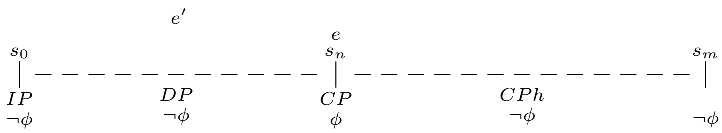
Dynamic Nucleus Structure of Transfer-verbs ('give')



⇒ ϕ expresses the maximal result (the object denoted by the oblique argument has the object denoted by the internal argument)

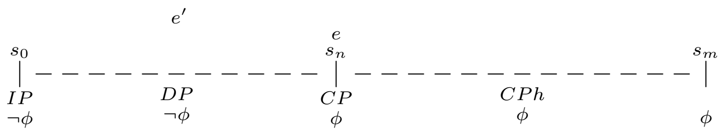
⇒ $e' \notin P_v$, $\omega(e')$ $(= s') \notin Q\phi$

Dynamic Structure of Point-verbs ('knock')



$\Rightarrow e \in P_v$; e' = presupposed event

Dynamic Structure of Achievement-verbs ('reach')



$\Rightarrow e \in P_v$; e' = presupposed event

3.2 Dynamic Modal Arrow Logic

In this section a logic L, Dynamic Modal Arrow Logic (DMAL), is defined in which it is possible to express the characterizing properties of aspectual classes from the last section. L combines elements from Dynamic Modal Logic (DML), de Rijke (1993), and Arrow Logic (AL), van Benthem (1996). It is two-sorted: there are both s-formulas that are evaluated at elements from S and e-procedures that are evaluated relative to elements from the domain E of events. s-formulas are used to make assertions about what is true or false at particular points of the execution sequence $\tau(e)$ of an event. They are therefore related to the level of changes as transformation of states; e-procedures, on the other hand, are related to the level of changes as objects. They admit to make assertions about properties of events. The link between e-procedures and s-formulas is established by basic e-procedures of the form $L\phi$, $R\phi$ and $D\phi$, which make assertions about the beginning-point $\alpha(e)$, the end-point $\omega(e)$ and an intermediate point of the execution sequence of an event e, respectively. The interpretation of s-formulas is standard. The interpretation of e-formulas is given in (20) (Recall that C is the (de)composition relation defined in (7) above; $\alpha(e)$ and $\omega(e)$ are the beginning and end point of e, respectively).

(20)
a. $M, e \models \sqsubseteq$ iff $e \in E$
b. $M, e \models \sqsubseteq_v$ iff $e \in P_v$
c. $M, e \models \pi \cap \pi'$ iff $M, e \models \pi \wedge M, e \models \pi'$
d. $M, e \models \pi \cup \pi'$ iff $M, e \models \pi \vee M, e \models \pi'$
e. $M, e \models \pi \bullet \pi'$ iff $\exists e_1, e_2[Ce, e_1e_2 \wedge M, e_1 \models \pi \wedge M, e_2 \models \pi']$
f. $M, e \models \sim \pi$ iff *not* $M, e \models \pi$
g. $M, e \models \delta$ iff $\alpha(e) = \omega(e)$
h. $M, e \models R\phi$ iff $M, \omega(e) \models \phi$
i. $M, e \models L\phi$ iff $M, \alpha(e) \models \phi$
j. $M, e \models D\phi$ iff $\exists s[DP(e, s) \wedge M, s \models \phi]$
$DP = \{(e, s) \mid s \in \tau(e) - \{\alpha(e), \omega(e)\}\}$
k. $M, e \models [X]\pi$ iff $\forall e_1[Xe_1, e \rightarrow M, e_1 \models \pi]$ $X \in \{B, B^*, E'\}$

Semantically, the function of $\sqsubseteq_v$ is similar to that of the program letters π in DL. They admit to impose a sortal restriction on an event e. In DMAL dynamic modes are interpreted as operations from $\wp(E) \times \wp(S)$ to $\wp(E)$. As said in the previous section, this change of interpretation relative to DML reflects, first, that transitions qua events are basic objects in DMAL and, second, that a transformation of state is relativized to an event-type P_v. The e-procedures of the form $DM(\pi, \phi)$ used in the translation of the aspectual classes are defined in (21) ($\langle X \rangle \, \phi =_{def.} \neg \, [X] \, \neg \phi$).

(21) a. $Min\text{-}BEC_v(\sqsubseteq_v, \phi) =_{def.} L\neg\phi \cap \sqsubseteq_v \cap R\phi \cap \sim [(\sqsubseteq_v \cap R\phi) \bullet \sqsubseteq_v]$
b. $Min\text{-}BEC_{<_S}(\sqsubseteq_v, \phi) =_{def.} L\neg\phi \cap \sqsubseteq_v \cap R\phi \cap \sim D\phi$
c. $Min\text{-}BEC^*_{<_S}(\sqsubseteq_v, \phi) =_{def.} Min\text{-}BEC_{<_S}(\sqsubseteq_v, \phi) \cap \sim [\sqsubseteq_v \bullet \sqsubseteq_v]$
d. $Con\text{-}BEC_v(\sqsubseteq_v, \phi) =_{def.} L\neg\phi \cap \sqsubseteq_v \cap R\phi \cap \sim [(\sqsubseteq_v \cap R\neg\phi) \bullet \sqsubseteq_v]$
e. $Con\text{-}BEC_{<_S}(\sqsubseteq_v, \phi) =_{def.} L\neg\phi \cap \sqsubseteq_v \cap R\phi \cap \sim D\neg\phi$
f. $BEC^*(\sqsubseteq_v, \phi) =_{def.} \sqsubseteq_v \cap R\phi$
g. $HOLD_{<_S}(\sqsubseteq_v, \phi) =_{def.} L\phi \cap \sqsubseteq_v \cap R\phi \cap \sim D\neg\phi$
h. $HOLD^1(\sqsubseteq_v, \phi) =_{def.} HOLD_{<_S}(\sqsubseteq_v, \phi) \cap \delta$
i. $HOLD^2(\sqsubseteq_v, \phi) =_{def.} HOLD_{<_S}(\sqsubseteq_v, \phi) \cap \sim \delta$
j. $END(\sqsubseteq_v, \phi) =_{def.} HOLD^1(\sqsubseteq_v, \phi) \cap \langle E' \rangle \, Min\text{-}BEC_{<_S}(\sqsubseteq, \phi) \cap \langle B \rangle \, HOLD_{<_S}(\sqsubseteq, \phi)$
k. $OCCUR(\sqsubseteq_v, \phi) =_{def.} HOLD^1(\sqsubseteq_v, \phi) \cap \langle E' \rangle \, Min\text{-}BEC_{<_S}(\sqsubseteq, \phi)$

Analyzing Accomplishment- and Activity verbs in the way suggested above solves two problems that event semantics faces. First, the $Con\text{-}BEC_v$ mode can be taken to express a weakened form of the property of divisivity which requires an event-predicate to be true of all subevents e' of an event e that satisfies the predicate. As a partial characterization of Activity-expressions this property is too strong as the example of waltzing shows. As it takes three steps to waltz, there are initial-stages e' of an event of waltzing that are not of this type. The $Con\text{-}BEC_v$ mode, on the other hand, requires the condition expressed by its second argument to only hold at those subevents e' that are of type P_v, thereby restricting the relevant set of subevents to those satisfying a particular condition (see Naumann 1999a for details). Second, the $Min\text{-}BEC$ mode expresses a weakened form of the property of being quantized that requires an event predicate to be false of each subevent e' of an event e satisfying the predicate. This property cannot characterize Accomplishment-expressions as the example of 'walk to the station' shows. Each (non-minimal) final stage e' of an event e of this type belongs to this type too such that the predicate is not quantized. The $Min\text{-}BEC_v$ mode only requires that its second argument ϕ is false at the end-point of all proper initial stages of e. In the case of Accomplishment-verbs (as well as Transfer-verbs) even something stronger holds: ϕ is false for all states of e's execution sequence, except at $\omega(e)$. The corresponding dynamic mode is $Min\text{-}BEC_{<_S}$, (21b). The translation of a verb v (or a sentence containing that verb) is an e-procedure of the form $DM(\sqsubseteq_v, \phi)$ such that DM is the dynamic mode defining the aspectual class to which v belongs. The second argument ϕ is not uniquely determined because it depends on an object that undergoes a change (e.g. an apple that is eaten) such that the translation is a translation-scheme. For different objects one gets different ϕ (see section (3.5) for details). In (22a) the general scheme and in (22b,c) two examples are given.

(22) a. $v \rightsquigarrow DM(\sqsubseteq_v, \phi)$

b. eat $\rightsquigarrow$ $Min\text{-}BEC_{<_S}(\sqsubseteq_{eat}, \phi)$

c. push $\rightsquigarrow Con\text{-}BEC_v(\sqsubseteq_{push}, \psi)$

3.3 The Interpretation of Modifying Expressions like directional PPs

In section (2) it was shown (i) that each event-type P_v imposes a minimal requirement on the result that must be brought about and (ii) that this requirement can be strengthened by the interpretation of v in the lexicon. E.g., whereas P_{eat} requires only that the w^*-minimal result be brought about, the interpretation of 'eat' imposes the stronger condition that the s-maximal result be realized. The semantic function of modifying expressions like directional PPs and secondary predications is analyzed in an analogous way. They impose a further condition that an event e must satisfy. For instance, the PP 'to the station' requires the object with respect to which e brings about the change to be at (or in) the station at the end-point of e. Expressed in terms of the path that is traversed, the condition amounts to the requirement that the end of the path is at (or inside) the station. Consequently, both the verb, e.g. 'run', and the PP impose a condition on the path that is traversed by the object undergoing the change. The condition imposed by the former is a strengthening of that imposed by the latter such that the change of aspectual properties is monotone. Furthermore, the condition that the object undergoing the change be at the station at the end-point of the event e must be satisfied only at that point and at no other point of the execution sequence of e such that it is s-maximal. This second requirement imposes a condition on the *way* the result determined by the PP must be brought about. Thus, a directional PP imposes both a result and a condition on how this result is brought about. It can therefore be translated as an e-procedure of the form $DM(\pi, \phi)$. The e-procedure π will express some accessibility-relation. As a directional PP like 'to the station' can be combined with expressions belonging to different event-types, π must be $\sqsubseteq$, which expresses the most general accessibility-relation. The translation of 'to the station' is (23).

(23) to the station $\rightsquigarrow Min\text{-}BEC_{<_S}(\sqsubseteq, \psi_{at\_the\_station})$

(23) is a translation-scheme because the second argument of $Min\text{-}BEC_{<_S}$ depends on a particular object that undergoes a change such that for different objects one gets different ψ. The translation of 'run to the station' is (24).

(24) run to the station $\rightsquigarrow Con\text{-}BEC_v(\sqsubseteq_{run}, \phi) \cap Min\text{-}BEC_{<_S}(\sqsubseteq, \psi_{at\_the\_station})$

(24) is an instance of (25).

(25) $DM(\pi, \phi) \cap DM'(\pi', \phi')$

(25) represents a general scheme of how changes (or, more generally, additions) of aspectual properties, triggered e.g. by modifying expressions, are accounted for in DMAL. The properties the execution sequence $\tau(e)$ of an event e satisfying $DM(\pi, \phi)$ has with respect to the property expressed by ϕ will in

general be different from those $\tau(e)$ has relative to the property expressed by ϕ and ϕ'. These properties can be calculated from implications like (26).[5]

(26) $Min\text{-}BEC_{<_S}(\pi, \phi) \cap Con\text{-}BEC_v(\pi, \psi) \rightarrow Min\text{-}BEC_{<_S}(\pi, \phi \wedge \psi)$

3.4 The Interpretation of 'Until'

'Until' requires an initial stage e_1 of the event e denoted by the main clause MC to go on until a particular point of the event e' denoted by the SC. If the procedure π' in the subordinate clause SC is of the form $Min\text{-}BEC_{<_S}(\sqsubseteq_v, \phi)$, i.e., if the event e is strongly non-closed under initial stages with respect to this procedure, the SC determines two points: either the beginning-point $\alpha(e')$ of e' or its end-point $\omega(e')$ as possible values for $\omega(e_1)$. If π' is of the form $Con\text{-}BEC_v(\sqsubseteq_v, \phi))$ or if the procedure requires e' to be an instantaneous event, it is required that $\omega(e_1) = \alpha(e')$. For instantaneous events e', this is equivalent to $\omega(e_1) = \omega(e')$ because $\alpha(e') = \omega(e')$. This difference with respect to the number of possible end points for e_1 is a consequence of a difference at the level of the dynamic structure. For an event-type that is characterized by the $Min\text{-}BEC_{<_S}$ mode it is possible that after the execution of a finite number of events of this type the result determined in the lexicon does not hold. For events belonging to an event-type that is characterized by another mode this is not possible (see Naumann (1998a,b), Naumann/Osswald (1999) for details). The aspectual restriction follows from the fact that the run-time $\tau(e_1)$ of e_1 can be arbitrarily restricted by the event e' denoted by the SC. What must be guaranteed is that for any choice of $\alpha(e')$ or $\omega(e')$ e_1 satisfies π, i.e. the procedure in the MC. This only holds if the interpretation of π $(= DM(\sqsubseteq_v, \phi))$ is closed under initial-stages modulo P_v and events satisfying π are not required to be instantaneous, i.e., $\alpha(e) = \omega(e)$ must not hold. The interpretation of 'Until' is given in (27).

(27) $M, e \models U(\pi, \pi')$ iff there are e_1, e' s.t.

- (i) $Be_1, e \wedge M, e_1 \models \pi$
- (ii) if $\forall e[M, e \models \pi \Rightarrow M, e \models [B^*] \sim \pi]$ then $\omega(e_1) = \omega(e') \vee \omega(e_1) = \alpha(e')$, else $\omega(e_1) = \alpha(e')$
- (iii) $M, e' \models \pi'$

3.5 The Hybrid Language L^h

The analysis developed so far faces the following two, related problems. First, in $DM(\pi, \phi)$ ϕ can be an arbitrary s-formula. It need not be a result that e can bring about, in particular it need not be a maximal element of $Res(P_v, e)$ with respect to $\leq_v$. Second, ϕ is brought about with respect to a particular participant d of e. This aspect is not captured either. This leads to problems in the case of modification. E.g., for an event of pushing a cart to the station, it is the cart which must be at the station at the end of the event and not some arbitrary object. These problems can be solved as follows. Recall that each basic event type P_v determines for each of its elements e a set $Res(P_v, e)$ of results that e can possibly bring about. Each element of $Res(e, P_v)$ is brought

[5] An interpretation of secondary predication in a slightly different framework is presented in Naumann (1999c).

about with respect to at least one object that participates in e. E.g., if e is of type 'John eat a fish', the results are brought about with respect to John and the fish. John is assigned both the s-minimal result (e.g. his mouth is open) and a w^*-minimal one (part of the fish is in his stomach) whereas the fish is assigned a w^*-minimal result (its mass partly decreased) and the s-maximal one (its mass is zero). In the case of an event e of type 'Bill push the cart' Bill is assigned the s-minimal result (his actions towards the cart) and possibly a w^*-minimal one (Bill traverses a non-empty path) whereas the cart is assigned only a w^*-minimal result (the cart traverses a non-empty path). The relationship between an event $e \in P_v$, an object d participating in e and a result $Q \in Res(P_v, e)$ that e can possibly bring about with respect to d is captured by a relation Δ_v on $E \times O \times \wp(S)$; see Latrouite/Naumann 1999b for details.

The result that is determined by the interpretation of verbs as well as of modifying expressions is brought about with respect to a particular participant d of e that can be defined in terms of the results that are assigned to it, together with a temporal maximality condition: (i) d is assigned all results that are maximal with respect to $\leq_v$ and (ii) d is the participant of e that is involved last with respect to the objects that satisfy condition (i), for details see Latrouite/Naumann (1999b). This relationship is captured by a (functional) relation Ω_v: $\Omega_v(e) = d$ just in case d is the object satisfying (i) and (ii). In terms of Δ_v the sets $\Delta^*_v(e)(d) = \{Q \mid \Delta_v(e)(d)(Q) \wedge \forall Q'[\Delta_v(e)(d)(Q') \rightarrow Q' \leq_v Q]\}$ and $\Delta^*(e)(d) = \cup_{v \in VERB} \Delta^*_v(e)(d)$ are defined.

At the formal level L is extended in two ways to a hybrid language L^h. First, for each $v \in VERB$ two operators Σ and Σ^* are added to L that bind variables which take their values in $E \cup O$. In L^h e-procedures are evaluated at elements from E relative to a variable assignment g. Second, besides e-procedures and s-formulas there are e,d-formulas that are evaluated at elements from $E \times O$. At the syntactic level, the following clauses are added: if ϕ is an atomic s-formula, $\Sigma_{v_x}\phi$ and $\Sigma^*_{v_x}\phi$ are e-procedures. Each s-formula is an e,d-formula (and nothing else is an e,d-formula). The relevant semantic clauses are given in (28) (note that the clauses in (20) above must be made dependent on a variable assignment too).

(28) a. $M, e \models_g \Sigma^*_{v_x}\phi$ iff $V(\phi) \in \Delta_v(e)(g(x))$

b. $M, e \models_g \Sigma_{v_x}\phi$ iff $\Omega_v(e)(g(x))$ and $M, e, g(x) \models \phi$

c. $M, e, d \models p$ iff $V(p) \in \Delta^*(e)(d)$ for p an atomic e,d-formula

d. $M, e, d \models \neg\phi$ iff not $M, e, d \models \phi$

e. $M, e, d \models \phi \wedge \psi$ iff $M, e, d \models \phi$ and $M, e, d \models \psi$

$\Sigma_{v_x}\phi$ is true relative to some $e \in E$ (and relative to g) only if (i) $e \in P_v$, (ii) $g(x)$ is the object that is assigned all results that are maximal with respect to $Res(P_v, e)$ relative to $\leq_v$ and (iii) $Q\phi$ is a maximal element of $Res(P_{v'}, e)$ (relative to $\leq_{v'}$) for some $P_{v'}$ to which e belongs.

The intuition behind letting each s-formula be an e,d-formula and vice versa is the following: s-formulas are used to make assertions about what holds at particular points of the execution sequence $\tau(e)$ of an event e (or, more generally, about the corresponding Dynamic Nucleus-Structure). Yet one is only interested in the truth or falsity of these s-formulas that express properties which are directly related to the particular event e and not on those that express properties which happen to hold or fail to hold during the execution of e due to the execution of other events. This second aspect, which is not accounted for by s-formulas, is captured by the e, d-formula that corresponds to a given s-formula.

The dependency of the second argument of $DM(\sqsubseteq_v, \phi)$ on an object $d \in O$ in the translation of the verb v can be made explicit by adding the clause $\Sigma^*_{v_x}\phi$, yielding (29a). Like (22a), (29a) is a translation-scheme. In contrast to (22a), (29a) is false if ϕ expresses a property that is evaluated according to DM on $\tau(e)$ but which is not brought about by e (with respect to $g(x)$). In (29b) the translation of 'eat' is given. Similarly, to the translation of a modifying expression the clause $\Sigma_{v_x}\psi$ is added. This yields (29c). In (29d) the translation of 'to the station' is given and in (29e) that of 'run to the station'.

(29) a. $v \rightsquigarrow DM(\sqsubseteq_v, \phi) \cap \Sigma^*_{v_x}\phi$

b. eat $\rightsquigarrow Min - BEC_{<_S}(\sqsubseteq_{eat}, \phi) \cap \Sigma^*_{eat_x}\phi$

c. $MOD(EXP\psi) \rightsquigarrow TR(EXP\psi) \cap Min - BEC_{<_S}(\sqsubseteq, \psi) \cap \Sigma_{v_x}\psi$

d. to the station $\rightsquigarrow Min - BEC_{<_S}(\sqsubseteq, \psi_{at\_the\_station}) \cap \Sigma_{v_x}\psi_{at\_the\_station}$

e. run to the station $\rightsquigarrow Con - BEC_v(\sqsubseteq_{run}, \phi) \cap \Sigma^*_{run_x}\phi \cap Min - BEC_{<_S}(\sqsubseteq, \psi_{at\_the\_station}) \cap \Sigma_{v_x}\psi_{at\_the\_station}$

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A Minimal Logic for Minimalism

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1 Chomsky's Minimalist Program

1.1 The Framework

According to Chomsky [1], language must be studied by taking his 'legibility conditions' as a starting point, that means that we have to explain how it is constrained by our mental architecture and our sensorimotor apparatus. In an 'ideal' situation, words (or more precisely, lexical items) would have no other features than those interpreted at the interfaces: properties of sound and meaning, and the 'computational system' attached to the language would use these features and only these ones, no other element would be introduced. But, as Chomsky says, we know that this is not the current situation and that there are some 'imperfections' with regards to this ideal picture: *commonly, phrases are interpreted in positions other than those where they are heard, though in analogous expressions these positions are occupied, and interpreted under natural conditions of locality* (**dislocation property**), and there are *uninterpretable features* like *Structural Case*. Of course, the situation would be nicer if we could reduce these two imperfections into only one: dislocation would be caused by the need for deleting uninterpretable features. Chomsky expresses this idea in terms of *Suicidal Greed*: features are attracted by *attractors* and attractors are deleted when they have attracted their matching feature.

1.2 Minimalist Grammars

E. Stabler ([9], [10]) gives a precise formulation of the Minimalist Principles in his *Minimalist Grammars*. In these grammars, he defines *Merge* and *Move* as operations between trees (*Merge*) or on trees (*Move*) the leaves of which are labelled by lists of features. The goal of a derivation in this system is to delete all the uninterpretable features. The whole derivation yields a tree where some leaves are simply labelled by phonetic features and others by logical ones.
In these grammars, each item of the lexicon consists in a sequence of features, which are divided as follows:

- Phonetic features for example */speaks/,/linguist/,/some/,...*
- Semantic features for example *(speaks),(linguist),(some),...*
- Syntactic or formal features:
  - categorial features (categories) involved in *merge*:
    $\textsc{base} = \{\mathtt{c}, \mathtt{t}, \mathtt{v}, \mathtt{d}, \mathtt{n}, \ldots\}$
  - functional features involved in *move*:
    $\textsc{fun} = \{\overline{\mathtt{k}}, \overline{\mathtt{K}}, \overline{\mathtt{wh}}, \ldots\}$

These sequences are described by the following regular expression:
$\textsc{label} = \textsc{select}^* \ (\textsc{licensors}) \ \textsc{select}^* \ \textsc{base} \ \textsc{licensees} \ P^* \ I^*$

- P phonetic features

- I semantic features
- SELECT $= \{=\mathtt{b}, =\mathtt{B}, \mathtt{B}= | \mathtt{b} \in$ BASE$\}$ select a category
- LICENSEES $= \{-\mathtt{x} | \mathtt{x} \in$ FUN$\}$ needs a move feature
- LICENSORS $= \{+\mathtt{x}, +\mathtt{X} | \mathtt{x} \in$ FUN$\}$ provides a move feature

Merge is defined between two T-markers u and t the head of u starting with $=\mathtt{x}$ and the head t starting with x with $x \in$ BASE. Let u' (resp. t') denote u (resp. t) in which the $=\mathtt{x}$ (resp. x) feature starting the head is cancelled,

- if u is a lexical item then the resulting tree is $u' < t'$ (so u' is the head and is on the left)
- otherwise the resulting tree is $t' > u'$ (so u' is the head in this case as well, but it is on the right)

Roughly speaking, movement is defined as follows: assume that at the leftmost position (spec* position) we have a $+\mathtt{x}$ and that at the rightmost (comp$^+$ position) we have a $-\mathtt{x}$: then the movement takes the whole constituent having $-\mathtt{x}$ as a head and moves it to the leftmost position (spec* position).

1.3 MP and Resource Logics

We think that Chomsky's position may be better expressed in terms of a resource consumption logic. In fact, *Suicidal Greed* may be formulated in a resource logic as: some feature *consumes* another one and after deletes.
In this paper, we try to show how the two operations which are now fundamental in Chomsky's view: *Merge* and *Move* are very conveniently recast in logical terms. This has as a consequence that these two operations no longer appear as the primitive operations: more primitive ones are arising. This is one of the advantages of the logical analysis : to make objects decomposed in more primitive parts and recomposed by very simple operations. Other researchers have presented works with a similar purpose, we may particularly notice W. Vermaat ([11]) T. Cornell ([2]), and one of the authors ([4]). They are using the framework known as Multimodal Categorial Grammar, in the spirit of [6]. In their approach, features are conceived like modalities and components are assembled by means of various composition modes. We think that the introduction of modalities and of the complex machinery associated does not pursue the goal of simplicity that we would like to reach in order to express so primitive mechanisms. It is the reason why we do not use modalities in our framework. That does not entail we should not use different composition modes provided that we introduce the fewest as possible. We shall see that in fact two composition modes seem to be necessary but one of the two remains implicit.
Vermaat's system has been shown equivalent to the Stabler's grammars. It is also the case for our system.

2 A logical analysis of Merge

The operation *Merge* is legitimated by the following observation: an object which already exists (either from the lexicon or by previous construction) has some property which can only be satisfied by another object, that will put these two objects together, and then, when the property is satisfied, we shall consider it as "inactive".
If we provisionally introduce here two connectives: $\bullet$ and $/$, with rules given like in the Lambek calculus, we can describe this fact in the following way.

[2] *Merge (2)* : $\phi/F, F \bullet \psi \vdash \phi \bullet \psi$

where $\bullet$ is *associative* and *non-commutative*. The proof is, in the sequent calculus:

$$\dfrac{\dfrac{F \vdash F \qquad \dfrac{\phi \vdash \phi \quad \psi \vdash \psi}{\phi, \psi \vdash \phi \bullet \psi}[R\bullet]}{\phi/F, F, \psi \vdash \phi \bullet \psi}[L/]}{\phi/F, F \bullet \psi \vdash \phi \bullet \psi}[L\bullet]$$

or, in the Natural Deduction style:

$$\cfrac{F \bullet \psi \qquad \cfrac{\cfrac{\phi/F \quad [F]^1}{\phi}[/E] \qquad [\psi]^1}{\phi \bullet \psi}[\bullet I]}{\phi \bullet \psi}[\bullet E]^1$$

We can provisionally conclude that / and • are primitive (more primitive than *Merge*).
We know that, for logical reasons, we also have the connective \, in such a way that:

$$A \bullet B \vdash C \Leftrightarrow A \vdash C/B$$
$$A \bullet B \vdash C \Leftrightarrow B \vdash A\backslash C$$

Of course, the \ operator is not superfluous. With the /, we were able to attach a syntactic object to the right of another. If we follow the LCA convention (Kayne 94), that corresponds to the attachment of a complement. Because it is assumed in this framework that trees are binary, if the head selects a new syntactic object, it will be attached on the left, and therefore we shall have necessarily to use this \. That simply duplicates the two rules for /, leading to analogous rules for \.

3 Phonological interpretations

According to Chomsky,

> We understand L to be a device that generates expressions EXP, EXP=<PHON, SEM>, where PHON provides the "instructions" for sensorimotor systems and SEM for systems of thought.

If the ideal picture was the case we should have only expressions consisting in <PHON, SEM> pairs, **but** uninterpretable features bring a **third** component so that we have: EXP=<PHON,UNINT,SEM>. In the following rules, we only include phonetic features: they are considered labels.

$$\frac{\Delta \vdash \beta : A \quad \Gamma, \gamma : B, \Gamma' \vdash \delta : C}{\Gamma, \alpha : B/A, \Delta, \Gamma' \vdash \delta[\alpha\beta/\gamma]C}[L/] \qquad \frac{\Delta \vdash \beta : A \quad \Gamma, \gamma : B, \Gamma' \vdash \delta : C}{\Gamma, \Delta, \alpha : A\backslash B, \Gamma' \vdash \delta[\beta\alpha/\gamma]C}[L\backslash]$$

$$\frac{\Gamma, \alpha : A, \beta : B, \Gamma' \vdash C}{\Gamma, \alpha\beta : A \bullet B, \Gamma' \vdash C}[L\bullet] \qquad \frac{\Gamma \vdash \alpha : A \quad \Delta \vdash \beta : B}{\Gamma, \Delta \vdash \alpha\beta : A \bullet B}[R\bullet]$$

$$\alpha : A \vdash \alpha : A \ [axiom] \qquad \frac{\Gamma \vdash \alpha : A \quad \Delta, x : A, \Delta' \vdash \gamma : C}{\Delta, \Gamma, \Delta' \vdash \gamma[\alpha/x] : C}[cut]$$

These rules can be restated as Natural Deduction rules:

$$\frac{\Gamma \vdash x : A/B \quad \Delta \vdash y : B}{\Gamma; \Delta \vdash xy : A}[/E] \qquad \frac{\Delta \vdash y : B \quad \Gamma \vdash x : B\backslash A}{\Delta; \Gamma \vdash yx : A}[\backslash E]$$

$$\frac{\Gamma \vdash \alpha : A \bullet B \quad \Delta; x : A; y : B; \Delta' \vdash \gamma : C}{\Delta; \Gamma; \Delta' \vdash \gamma[\alpha/xy]C}[\bullet E] \qquad \frac{\Gamma \vdash x : A \quad \Delta \vdash x : B}{\Gamma, \Delta \vdash xy : A \bullet B}[\bullet I]$$

So, let us start with the following lexicon:

$$\begin{array}{lll} \textit{reads} & ::= & \vdash \textit{reads} : ((\overline{\mathtt{k}}\backslash \mathtt{vp})/\mathtt{d}) \\ a & ::= & \vdash a : ((\mathtt{d} \bullet \overline{\mathtt{k}})/\mathtt{n}) \\ \textit{book} & ::= & \vdash \textit{book} : \mathtt{n} \end{array}$$

with $\overline{\mathtt{k}}$ a formal feature (*case*) and $\mathtt{d}$, $\mathtt{n}$, $\mathtt{vp}$ categorial features (all these features being uninterpretable).
We can have the following derivation in order to build a new syntactic object:

$$\dfrac{\dfrac{\vdash a : ((\mathtt{d} \bullet \overline{\mathtt{k}})/\mathtt{n}) \quad \vdash \textit{book} : \mathtt{n}}{\vdash a\ \textit{book} : \mathtt{d} \bullet \overline{\mathtt{k}}}[/E] \quad \dfrac{\dfrac{\vdash \textit{reads} : ((\overline{\mathtt{k}}\backslash \mathtt{vp})/\mathtt{d}) \quad x : \mathtt{d} \vdash x : \mathtt{d}}{x : \mathtt{d} \vdash \textit{reads}\ x : (\overline{\mathtt{k}}\backslash \mathtt{vp})}[/E] \quad y : \overline{\mathtt{k}} \vdash y : \overline{\mathtt{k}}}{x : \mathtt{d}, y : \overline{\mathtt{k}} \vdash \textit{reads}\ xy : (\overline{\mathtt{k}}\backslash \mathtt{vp}) \bullet \overline{\mathtt{k}}}[\bullet I]}{\vdash \textit{reads a book} : (\overline{\mathtt{k}}\backslash \mathtt{vp}) \bullet \overline{\mathtt{k}}}[\bullet E]$$

It is important to notice two things:

- in the lexicon, words are not associated with formulae (like it is the case in usual categorial grammars, and notably in standard Lambek grammars), but with sequents: they are therefore considered *extra-logical axioms*, which are labelled by the phonetic feature of the word itself,
- because $\bullet$ is non commutative, there's no kind of move here. For instance, in the rule [$\bullet$E], α substitutes for the concatenation of x and y, which label the types A and B, which are adjacent in the LHS of the sequent and which can be neither permuted nor displaced.

4 A logical analysis of Attract+Move

Still according to Chomsky: attraction (hence movement) is driven by the need to delete an uninterpretable feature F (we call it the attractor). Moreover, 'the attractor F in the label L of the target β locates the closest F' in its domain, attracting it to the multi-lexical item of F'. We can assume in our framework that attractors occur as negative polarity items ($F\backslash$), attracted features as positive ones, and that the so called Domain of F is the result of several merge steps, thus resulting in a sign like:

$$F\backslash\phi_1 \bullet \phi_2 \bullet ... \bullet F \bullet ... \bullet \phi_n$$

but dislocation needs to *relax* the order ($\bullet$) of the hypotheses. For instance,

$$\textit{reads a book} : (\overline{\mathtt{k}}\backslash \mathtt{vp}) \bullet \overline{\mathtt{k}}$$

cannot be reduced.
This enforces us to introduce a second product, $\otimes$ between types, which is commutative, the structural counterpart of which being ',' (whereas the structural counterpart of $\bullet$ is ';'). By doing so, we are working inside the calculus pCLL ('partially commutative linear logic') designed by P. de Groote ([3]). The two products are able to communicate through the entropy rule:

$$\dfrac{\Gamma[(\Delta_1; \Delta_2)] \vdash A}{\Gamma[(\Delta_1, \Delta_2)] \vdash A}$$

and we have the following $\otimes$-*elimination rule*:

$$\dfrac{\Gamma \vdash \alpha : A \otimes B \quad \Delta, x : A, y : B \vdash \gamma : C}{\Gamma, \Delta \vdash \gamma[\alpha/\{x, y\}] : C}[\otimes E]$$

where $\gamma[\alpha/\{x, y\}]$ means the *substitution of* α *to the unordered set* $\{x,\ y\}$ that is the simultaneous substitution of α for both x and y, *no matter the order between* x *and* y *is.*

In this new framework, we can assume the following lexicon:

$$\begin{array}{lcl} \textit{reads} & ::= & \vdash \textit{reads} : ((\overline{\mathtt{k}}\backslash \mathtt{vp})/\mathtt{d}) \\ \textit{a} & ::= & \vdash \textit{a} : ((\mathtt{d} \otimes \overline{\mathtt{k}})/\mathtt{n}) \\ \textit{book} & ::= & \vdash \textit{book} : \mathtt{n} \end{array}$$

and we may now build up a new syntactic object by means of the following derivation: [5]

$$\dfrac{\dfrac{\vdash a : ((\mathtt{d} \otimes \overline{\mathtt{k}})/\mathtt{n}) \quad \vdash \textit{book} : \mathtt{n}}{\vdash a\ \textit{book} : \mathtt{d} \otimes \overline{\mathtt{k}}}[/E] \quad \dfrac{\dfrac{y : \overline{\mathtt{k}} \vdash y : \overline{\mathtt{k}} \quad \dfrac{\vdash \textit{reads} : ((\overline{\mathtt{k}}\backslash \mathtt{vp})/\mathtt{d}) \quad x : \mathtt{d} \vdash x : \mathtt{d}}{x : \mathtt{d} \vdash \textit{reads}\ x : (\overline{\mathtt{k}}\backslash \mathtt{vp})}[/E]}{y : \overline{\mathtt{k}}; x : \mathtt{d} \vdash y\ \textit{reads}\ x : \mathtt{vp}}[\backslash E]}{y : \overline{\mathtt{k}}, x : \mathtt{d} \vdash y\ \textit{reads}\ x : \mathtt{vp}}[\textit{entropy}]}{\vdash a\ \textit{book reads a book} : \mathtt{vp}}[\otimes E]$$

We get an order in the label *even if we have got through entropy*. It is so because when using [/ E] and [\ E], we necessarily order the labels, and this order is then recorded inside the label and never destroyed, even when using the entropy rule: at this moment, it is only the order *on hypotheses* which is relaxed. We obtain a subsystem of pCLL by simply restricting the proof space to proofs which only contain some particular kinds of step. Let us call $\mathcal{MG}$-proofs those proofs. We have:

Definition 1 *$\mathcal{MG}$-proofs contain only three kinds of steps:*

- *implication steps (elimination rules for / and \)*
- *tensor steps (elimination rule for $\otimes$)*
- *entropy steps (entropy rule)*

We can moreover assume:

Definition 2 *A lexical entry consists in an axiom $\vdash w : \mathcal{T}$ where $\mathcal{T}$ is a type:*

$$((F_2\backslash(F_3\backslash...(F_n\backslash(G_1 \otimes G_2 \otimes ... \otimes G_m \otimes A))))/F_1)$$

where:

- *m and n can be any number greater than or equal to 0,*
- *F_1, ..., F_n are attractors,*
- *G_1, ..., G_m are features,*
- *A is the resulting category type*

But that's not all because we need to express the fact that 'the attractor F in the label L of the target β locates the *closest* F' in its domain. This simply corresponds to the following restriction.

Definition 3 (Shortest Move) *: A $\mathcal{MG}$-proof is said to respect the shortest move condition if it is such that hypotheses are discharged in a First In, First Out order.*

5 Examples

5.1 SVO languages

Let us look at a very simple example, corresponding to an elementary sentence in a SVO language:

```
every linguist speaks some language
```

Before giving the proof, we give the lexicon which is used, together with the translation from Stabler's labels into $\mathcal{MG}$-axioms.

entry	*Stabler′stype*	*label* : *type*
every	$=\mathtt{n}\ \mathtt{d}\ -\overline{\mathtt{k}}$ *every*	*every* : $((\overline{\mathtt{k}}\otimes\mathtt{d})/\mathtt{n})$
some	$=\mathtt{n}\ \mathtt{d}\ -\overline{\mathtt{k}}$ *some*	*some* : $((\overline{\mathtt{k}}\otimes\mathtt{d})/\mathtt{n})$
language	$\mathtt{n}$ *language*	*language* : $\mathtt{n}$
linguist	$\mathtt{n}$ *linguist*	*linguist* : $\mathtt{n}$
speaks	$=\mathtt{d}\ +\overline{\mathtt{k}}\ =\mathtt{d}\ \mathtt{v}$ *speaks*	*speaks* : $((\overline{\mathtt{k}}\backslash(\mathtt{d}\backslash\mathtt{v}))/\mathtt{d})$
(*tense*)	$=\mathtt{v}\ +\overline{\mathtt{k}}\ \mathtt{t}$	$((\overline{\mathtt{k}}\backslash\mathtt{t})/\mathtt{v})$
(*comp*)	$=\mathtt{t}\ \mathtt{c}$	$(\mathtt{t}\backslash\mathtt{c})$

We begin by showing the proof in ND format, and by reversing the proof, we show that we get a tree structure similar to T-markers. For reasons of size of the proof, it will be cut off into two pieces, the first piece gives a reduction of

```
speaks some language
```

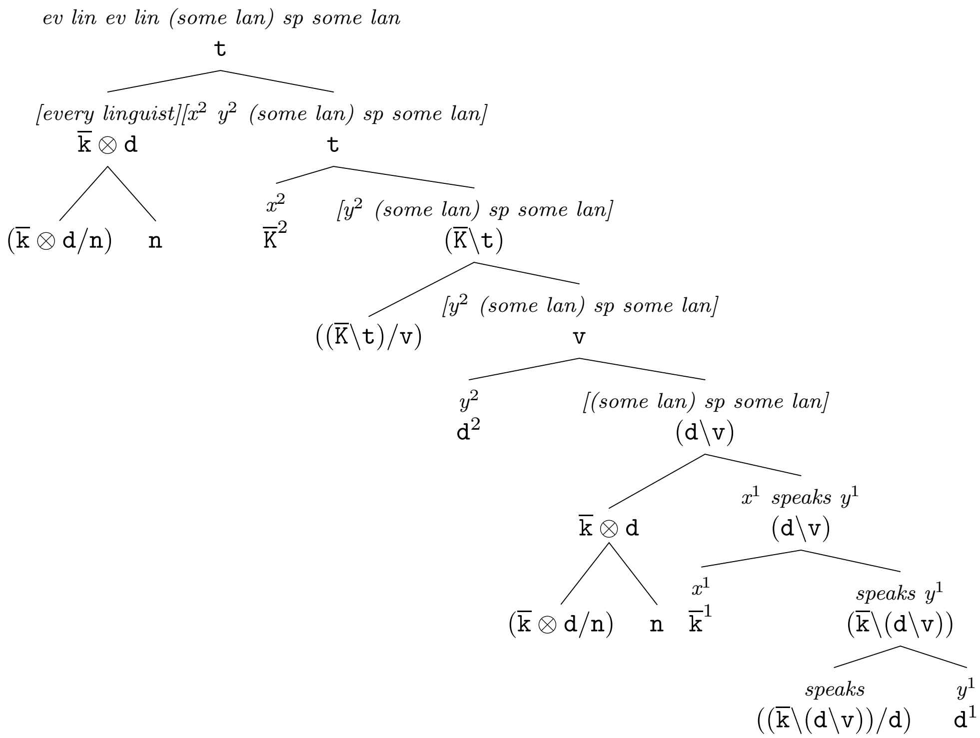
to $\mathtt{v}$, and the second piece shows the continuation of the proof, by using the conclusion thus obtained.

$$\dfrac{\begin{matrix}y^2\\ \mathtt{d}^2\end{matrix} \quad \dfrac{\dfrac{\begin{matrix}\text{some}\\ ((\overline{\mathtt{k}}\otimes\mathtt{d})/\mathtt{n})\end{matrix} \quad \begin{matrix}\text{language}\\ \mathtt{n}\end{matrix}}{\begin{matrix}\textit{some language}\\ \overline{\mathtt{k}}\otimes\mathtt{d}\end{matrix}}[/E] \quad \dfrac{\begin{matrix}x^1\\ \overline{\mathtt{k}}^1\end{matrix} \quad \dfrac{\begin{matrix}\text{speaks}\\ ((\overline{\mathtt{k}}\backslash(\mathtt{d}\backslash\mathtt{v}))/\mathtt{d})\end{matrix} \quad \begin{matrix}y^1\\ \mathtt{d}^1\end{matrix}}{\begin{matrix}\textit{speaks}\ y^1\\ (\overline{\mathtt{k}}\backslash(\mathtt{d}\backslash\mathtt{v}))\end{matrix}}[/E]}{\begin{matrix}x^1\ \textit{speaks}\ y^1\\ (\mathtt{d}\backslash\mathtt{v})\end{matrix}}}{\begin{matrix}\textit{(some language)}\\ \textit{speaks some language}\\ (\mathtt{d}\backslash\mathtt{v})\end{matrix}}[\otimes E]^1}{\begin{matrix}y^2\ \textit{(some language)}\\ \textit{speaks some language}\\ \mathtt{v}\end{matrix}}[\backslash E]$$

And the continuation of the proof is :

$$\dfrac{\dfrac{\begin{matrix}\text{every}\\ ((\overline{\mathtt{k}}\otimes\mathtt{d})/\mathtt{n})\end{matrix} \quad \begin{matrix}\text{linguist}\\ \mathtt{n}\end{matrix}}{\begin{matrix}\textit{every linguist}\\ \overline{\mathtt{k}}\otimes\mathtt{d}\end{matrix}}[/E] \quad \dfrac{\begin{matrix}x^2\\ \overline{\mathtt{K}}^2\end{matrix} \quad \dfrac{\begin{matrix}\emptyset\text{: tense}\\ ((\overline{\mathtt{K}}\backslash\mathtt{t})/\mathtt{v})\end{matrix} \quad \begin{matrix}y^2\ \textit{(some language)}\\ \textit{speaks some language}\\ \mathtt{v}\end{matrix}}{\begin{matrix}y^2\ \textit{(some language)}\\ \textit{speaks some language}\\ (\overline{\mathtt{K}}\backslash\mathtt{t})\end{matrix}}[/E]}{\begin{matrix}x^2\ y^2\ \textit{(some language) speaks some language}\\ \mathtt{t}\end{matrix}}[\backslash E]}{\begin{matrix}\textit{every linguist every linguist}\\ \textit{(some language) speaks some language}\\ \mathtt{t}\end{matrix}}[\otimes E]^2$$

Let us see the tree we obtain.



Comments: a transitive verb like *speaks* has a categorial feature looking for a $\mathtt{d}$ on the right, and a functional feature $\overline{\mathtt{k}}$ (*case*) on its left. These two demands are satisfied by two hypotheses. By elimination of / and then of \ (here analogous to *merge*), the labels of these hypotheses are incorporated into the verb. But it is only in a second step that these hypotheses are discharged by means of elimination of $\otimes$. Because objective case is weak, only the semantic part of the object is substituted to x^1, and the content of the object is substituted to y^1, thus resulting in the node (*some language*) *speaks some language* : $(\mathtt{d}\backslash\mathtt{v})$. The integration of *tense* (or *inflection*) makes $\overline{\mathtt{K}}$ to occur, and it will be cancelled (or *checked*) only by a new $\mathtt{d}$ requiring a case. But this time, because nominative case is strong, the whole content of the subject is attracted to the highest position, that means that the two variables x^2 and y^2 are substituted by the same content.
We may now call "move", the kind of line we can draw in such a tree from a variable y^i to a variable x^i which has the same index. The content that "moves" is determined by the content of the node which makes possible to substitute the two variables.

The conventions for reading the result are:

- all strings not inside parentheses or slashes must be read at the same time as phonetic and semantic features,
- any second occurrence of a string in the left-right order must be deleted (such an element is thought of having been copied)

Following these conventions, the interpretative result at the root of the previous tree, which is:

every linguist every linguist (some language) speaks /some language/

gives us two interpretative forms:

- the phonetic form:

  /every linguist//speaks//some language/

- the logical form:

  (every linguist)(some language)(speaks)

5.2 SOV languages

The SOV word order is obtained by making a verb a strong case assigner (exactly like [9]). In this case, the whole phrase some language is copied when using the product- elimination rule $[\otimes E]$, thus giving:

```
every linguist some language speaks
```

yielding:

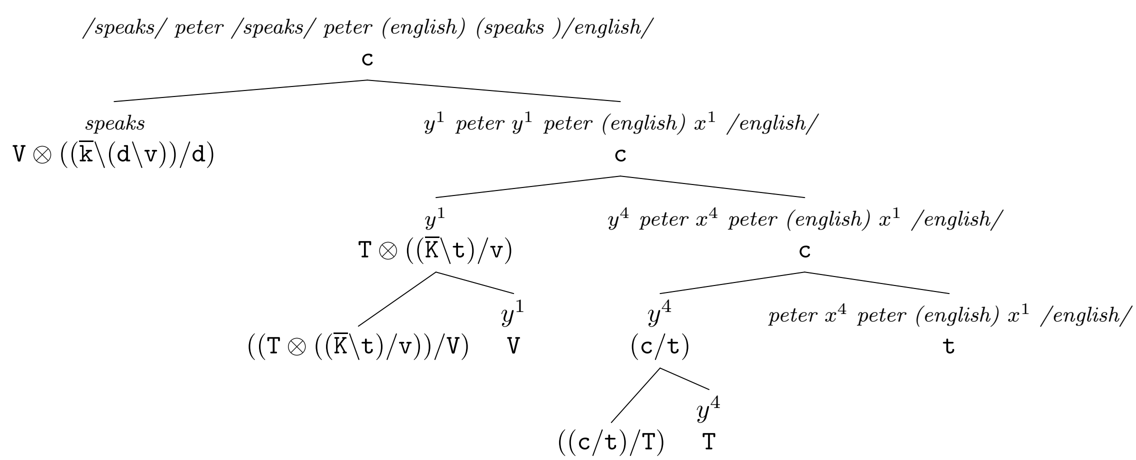
- phonetic form: */every linguist//some language//speaks/*
- logical form: *(every linguist)(some language)(speaks)*

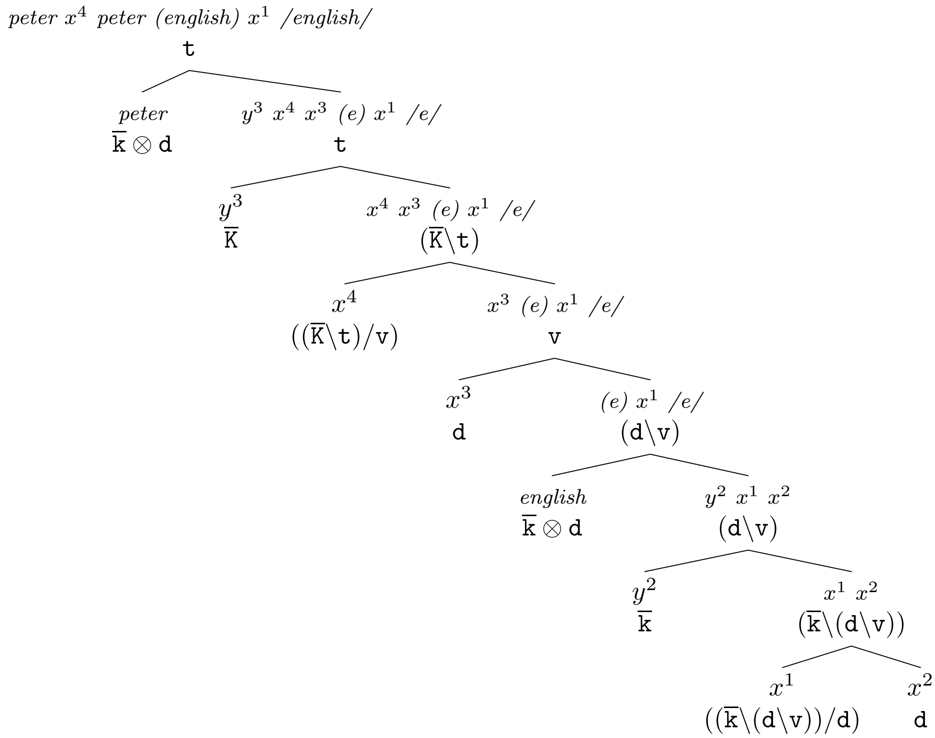
5.3 VSO languages

VSO languages are more difficult to obtain because they involve head movement. Head movement implies that categorial features too can be strong. In this case, in our setting, we duplicate the result category of a functorial type, combining the functorial type with this duplication of the result category seen as a strong feature by means of $\otimes$. The following lexicon gives an example (the resulting $\mathtt{v}$ is duplicated as $\mathtt{V}$ in the verbal entry, in order to correspond with the strong $\mathtt{V}$ demanded by the inflection category). The translation of Stabler's lexical entries is therefore the following:

entry	*Stabler'stype*	*label* : *type*
Peter	$\mathtt{d}\ -\overline{\mathtt{k}}$ *peter*	*peter* : $\overline{\mathtt{k}} \otimes \mathtt{d}$
english	$\mathtt{d}\ -\overline{\mathtt{k}}$ *english*	*english* : $\overline{\mathtt{k}} \otimes \mathtt{d}$
speaks	$=\mathtt{d}\ +\overline{\mathtt{k}}\ =\mathtt{d}\ \mathtt{v}$ *speaks*	*speaks* : $\mathtt{V} \otimes ((\overline{\mathtt{k}}\backslash(\mathtt{d}\backslash\mathtt{v}))/\mathtt{d})$
(*tense*)	$=\mathtt{V}\ +\overline{\mathtt{K}}\ \mathtt{t}$	$((\mathtt{T} \otimes ((\overline{\mathtt{K}}\backslash\mathtt{t})/\mathtt{v}))/\mathtt{V})$
(*comp*)	$=\mathtt{T}\ \mathtt{c}$	$((\mathtt{c}/\mathtt{t})/\mathtt{T})$

An example of derivation :(we represent this tree in two parts)





In the result, repetitions are omitted, thus producing:

```
/speaks//peter/(peter)(english)(speaks)/english/
```

thus providing the following PF and LF:

- */speaks peter english/*
- *(peter)(english)(speaks)*

6 Conclusion

We present here a very simple logical system which does the same job as minimalist grammars do. It is a kind of intuitionistic linear logic where two products are mixed: one is non commutative and is not directly used, it is indirectly used via its residuates / and \, the other is commutative and is directly used for discharging hypotheses corresponding to two different insertion points in a tree (T-marker) where some definite material may occur. A simple algorithm is used afterwards to clean up the result thus making phonetic forms and logical forms appear. It may be shown that such a system has the same generative power as minimalist grammars (which have been proved to be *mildly context-sensitive*[5]), and that it is polynomial [8].

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On the General Semantics of Empty Words

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I remember feeling perplexity every time when as a schoolgirl I was asked by my teacher to “analyse” a sentence “syntactically” and “morphologically”, as it was then called. I followed the given pattern of questions obediently - who, what, what is he doing, where, etc., and all was right except when I came to strange little words about which no such questions could be asked, which were vaguely and unsatisfactorily defined, and also, often neglected.

These, of course, were what we here designate as “empty” words (vs. “full” words). I am far from considering this term satisfactory - it is just one of the variety of others, such as: secondary words, auxiliaries, link words, discourse connectors, discourse items, logical signs (vs. descriptive signs - Carnap 1965), etc. I hope it will be possible to have a fuller and a better name in the future.

Empty words are the words traditionally grouped into the following parts of speech: interjections, particles, articles, prepositions, pronouns, conjunctions - not all of them are necessarily manifested in every language, though. Some adverbs also belong to these, and occasionally, in text adjectives and nouns of certain general semantics could function like “empty” words.

Our principal interest here lies in what the general functions of these “little” words are in the language. We’ll first try to show briefly the most essential differences between full vs. empty words.

Full words have the following general qualities: they are semiotic signs, i.e. they denote things, events, properties, etc., of the outer world. These signs are members of the system, which means that the meaning and value of these discrete elements are determined by the relationship to the other members within the system. Besides, full words are hierarchically built into complicated morphological structures. They belong to paradigmatic classes and change their forms accordingly. Consequently, in the process of communication full words perform two important functions - semantic and grammatical: a. full words present the semantic material for building the lexical meaning and the theme in the text; b. full words function as sentence members - they structure sentences grammatically. Every full word can be substituted by another full word of the same paradigmatic quality in the formal structure. E.g., the English word “daffodil” can have certain semantic functions in a sentence (which is the most complicated structural unit itself) and be substituted by “rose”, “chair”, “hope” etc. Owing to its denotative and connotative meaning the word “daffodil” also introduces some kind of pastoral theme into a text. (Certain expectations are aroused in a listener as to the occurrence of other lexical units in the sentence, and concerning the theme of the expected linguistic expression).

Empty words are principally uninflected forms, they are not members of the formal structure, they cannot be independent sentence members. The “substitution of the given element with another one of the same class”, as it is formulated in the case of full words, is therefore, non-relevant for empty words. Their main and essential function is of semantic character.

As we have argued elsewhere, empty words are close to indexical signs of Peirce’s terminology, i.e. they are signals of some semantic notions and relationships. While a full word names certain logical connections like, e.g., “addition”, an empty word “and” introduces it directly into the text. While a certain “John” is named in the text with a proper noun, the pronoun “he” signals its re-occurrence in the text, and so on. Thus semiotically empty words are indeces either for something in the outer world, or for something already mentioned in the text.

Comparing dictionary definitions for full words and empty words is striking, for the former presents explanations, synonyms, visual material, but the latter - only verbal context. The meaning of an empty word in a good dictionary is demonstrated with a deliberately chosen context - examples, samples of texts, which provide a reader with the knowledge of the correct usage of the given empty word. The meaning and the value of empty words are thus determined by their usage in the context.

According to Benveniste (1969), semiotic signs, i.e. full words, should be recognized, but semantic units - units of speech, should be understood. Recognizing means being conscious of the identity of the former and the latter usages, but understanding means perceiving the meaning of a new semantic unit. A lexical unit of a language system - a semiotic sign, a full word - is principally new. The justification of a system is in the possibility of transforming old meanings into new, descrete elements into continuous units, the social quality - into the individual one, the neutrality - into the subjectivity. (See also Weinrich 1976). This is the transition from recognizing to understanding.

This changing from semiotic signs and the language system to speech units (texts), according to Benveniste 1969, is complicated, and it is difficult to know what subtle processes carry out the transition. Now, to me empty words seem to be the means for the realization of these changes. In every text there is a kind of "metasemantics", says Benveniste, that is built over the lexical meaning supplied by full words. Building this "metasemantics" is, I suppose, the most general and linguistically most important function of empty words. Empty words ensure the transition from formal hierarchical structuring and from the language system with its discrete elements, to a new continuous communicative unit - a text, which is semantically structured in essence. The lexical homogeneity provided by full words is perhaps only raw material for the further semantic structuring of text. The essential features of the meaning of a communicative unit - its continuous and connected character, completeness, and coherence to its situation are ensured by the work of empty words - those "little", "secondary", "short words that help to economize", as they are often qualified (de Beaugrande and Dressler 1983).

The meanings of particular empty words in texts are being actively studied now. What we attempt here is commenting on their general semantic functions.

Three major kinds of general semantic functions are ensured by empty words in building the above-mentioned "metasemantics" in communication:

- building logical and semantic coherence,
- marking out new vs. old information,
- making the meaning cohere with the non-verbal context.

Structuring the text logically and semantically - the semantic function mentioned first - concerns the inner semantics of the content of the continuous text. Here empty words signal the semantic connections between text parts, like: causation, reason, result, inference, alternation, negative alternation, contrast, addition, generalization, condition, reformulating, negation, concession, restriction, numeration.

When we say empty words "structure" a text according to these semantic features, we mean exactly that, for the parts of such semantic constructions could be substituted without causing any destruction to the quality of the given semantic relationships. From this point of view the logical and semantic connections inside the text are of the same quality as formal linguistic structures. A simple example will suffice to illustrate the nature of semantic structuring carried out by empty words:

(1) You can't go - because - you are too young
You can't do it - because - you are not strong
I'll do it - because - I love him

Such semantic structures in texts are built by conjunctions, prepositions, particles, pronouns.

The next kind of major semantic functions of empty words is differentiating the old from new. Saying something new about something old is really basic for human communication. Not only grammatical patterning, deep structure relationships or word order, but also empty words contribute to qualifying what is old and what is new in the given context. Two kinds of old and new information could be distinguished here: one is the situationally determined new vs. old knowledge common to the speaker and the listener. New information could in this case be indicated in the text by particles universally. Particles can change the neutral old-vs-new patterning in the sentence by placing emphasis on certain parts. The meaning of definiteness expressed through definite articles, pronouns, indicate old information - something known to both participants of communication. The notion of indefiniteness underlines new information, unknown to the listener and, possibly also, to the speaker. Normally the speaker attempts to be carefully explicit about distinguishing what is situationally old and what is new in his text. If this attempt is a failure, the communication does not work.

Another kind of old vs. new information is determined by the verbal context - something has been mentioned earlier in the text, something is repeated and known. Old information is shown by co-referential devices built by pro-forms, and also by substitutional devices forming different kinds of anaphoric and cataphoric ties, relying on the sameness of reference of e.g. full nouns on the one hand and pronouns on the other; on the identity of a whole passage and a pronoun, etc. The same is to be said of the pro-verbal forms. This is in fact the most usual and very active way of reminding what is known to the interlocutors from the former samples of the text.

Another kind of co-reference - substitution - is employed when an empty words refers to different elements from the identical class of words mentioned earlier. So the sameness is only partial, the old repeated information is only partially identical. The device of substitution in English is often formed by an indefinite pronouns “one” and “else” (e.g., “another one”, “a better one”, “someone else”, etc.). Here is an example from Halliday and Hasan (1976):

(2) This is a fine hall you have here! I’ve never lectured in a finer one.

Substitutions like this are not equally usual to different languages. In Georgian, e.g., they are comparatively rare - the corresponding device used in this language is ellipsis - the zero stage of substitution.

The third major semantic quality of empty words - making the meaning coherent to the non-verbal context - can also be of two kinds. One is the subjective character of the text that rests upon speaker-oriented (also listener-oriented) modality. Attitudinal relations to the described events could express: degree of belief, evaluation, ability or necessity, permission, possibility or impossibility, hesitation, emphasis, wish, determination and the like. In Georgian, these are very actively expressed by various particles, interjections. E.g., the particle *kinagam* expresses negative modality, but *titkmis* implies positive attitude to the event (the meaning of both is “nearly, almost”).

Another situational characteristic is “spatio-temporal coordinates of the utterance”, as Lyons (1977) puts it. Fillmore 1975 talks additionally of social deixis (the choice of honorific or intimate speech levels), person deixis (the identity of interlocutors in communicative situation). Systems of expressing deictic meanings could differ strongly in languages, which is well known and wil not be discussed here. Deictic meanings are introduced into the text by personal pronouns and demonstratives, prepositions, preverbs and postpositions.

Such seem to be the three kinds of general semantic functioning of empty words in text: Logical-semantic coherence, marking out old vs. new information, building semantic ties with a non-verbal context. Some words could be added about other semantic features of empty words

Empty words can be strong semantic dominants governing the occurrence of certain lexical meanings and some grammatical features in the full words to which they are clitics. Georgian *titkmis*, *lamis*, *kinagam* are clitics to verbs and mean incompleteness or non-occurrence of the action or event. They determine the aspect - the duration or momentariness, also lexical meanings in the verbs to which they are attached; they also have positive, negative or neutral implications.

Empty words are always unambiguous in the context, expressing the one meaning that is determined by it. In fact, the correct use of empty words is one way of ruling out ambiguity from the text. Empty words never mean more than one thing at a time. Nonsense literature allows all kinds of puns, neologisms, play on words with full, but not with empty words, just because of the unambiguity of their usage.

The logical-semantic structuring ensured by empty words is what cannot be changed (or broken) within one span of the text, and the constancy of this backbone is what makes the punning acceptable and funny for the reader. English Nonsense Literature suggests plenty of examples for illustration.

Empty words are inherently polyfunctional, but in communication their functioning is quite definite and unambiguous - no wonder that their quantity in a language is both finite and small, but they work hard. There are always many empty words in any sample of text - their hard work guarantees the full use of the exceptional ability of communicating in the way only humans can. Only through empty words is the full expression of speaker's intended meaning possible, they are thus determining the quality, adequateness and appropriateness of the linguistic aspect of locution.

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GELEXI PROJECT: PROLOG IMPLEMENTATION OF A TOTALLY LEXICALIST GRAMMAR

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A new sort of generative grammar — *GASG* (Generative/Generalized Argument Structure Grammar) — is demonstrated in this paper (together with its Prolog implementation); which is more consistently and radically lexicalist than any earlier generative grammar: the lexicon plays the role of the generating component (→ *Ge(nerative) Lexi(con) Project).*[1]

The first motivation of the enterprise (Alberti 1990, 1996) was the stubborn problem of compositionality in Discourse Representation Theory (e.g. Kamp 1981). Nowadays (van Eijck – Kamp 1997; see also Karttunen 1986, Zeevat 1991a-b) some kind of unificational Categorial Grammar is held to promise the best chance for capturing the language→DRS transition in a properly compositional manner (according to the authors mentioned). The basic problem with UCG, which has amounted to the starting-point of GASG, lies in the fact that syntax, deprived of the information concerning sentence cohesion in favor of the unification mechanism and reduced to the primitive task of combining adjacent words, will produce linguistically irrelevant constituents (Karttunen 1986: 19; see his examples). Instead of the usual categorial apparatus, we argue that adjacency and order among words are to be treated by the same technique of unification as morphological cohesion (agreement, case). And the engine combining words to form sentences must be unification itself, which is capable of running Prolog programs properly.

The crucial point of our proposal is the introduction of rank parameters in the lexical formulas (substituting for the usual categorial apparatus) where adjacency requirements between words are fixed and the special simultaneously recursive definition of (the satisfaction of) these formulas.

1. Arguments for the Principle of Total Lexicalism and GASG

1.1 A Metatheoretical Argument. We are arguing (see also Alberti 1999) that studying the possibility of a totally lexicalist grammar is not only a legitimate research program but an unavoidable metatheoretical task forced upon us by the five-decade scientific tendency in the course of which the Lexicon is occupying more and more areas at the loss of syntax in *every* important branch of the family of generative theories.

We are referring to the tendency in the course of which *generative theory,* which appeared in the fifties as a radically syntax-centered linguistic theory with a very simple lexicon (Chomsky 1957), had become by the nineties — though separated into several branches (MP, CGs, LFG, HPSG, TAG, C&S)— a theory with a highly reduced syntax and a lexicon of rich content and structure. Whenever a new non-Chomskyan branch was founded, leading points of the program were almost always the extension and a more exact and thorough formalization of the area of the lexicon, and a definite ambition to store the information concerning the syntax–semantics interface in the lexicon. In the light of these facts, the radical lexicon-centrism and the non-language-specific approach to syntax characteristic of the Chomskyan Minimalist Program (Chomsky 1995) are of even greater importance: "The syntactic engine itself – the autonomous principles of composition and manipulation Chomsky now labels 'the computational system' – has begun to fade into the background. Syntax reduces to a simple description of how constituents drawn from the lexicon can be combined and how movement is possible (i.e. how something other than the simple combination of independent constituents is possible). The computational system, this simple system of composition, is constrained by a small set of economy principles which Chomsky claims enforce the general requirement, 'do the most economical things to create structures that pass the interface conditions (converge at the interfaces)" (Marantz 1995: 380, section 8 *The End of Syntax).*

Our conclusion drawn from this metatheoretical discussion is that it is a grammar dispensing with Merge and Move that is derivable from the general generative philosophy as a conceptual minimum (Chomsky 1995). GASG is nothing else than an attempt to realize this conceptual minimum: to get rid of Move, to reduce Merge to a non-PS-tree-producing unification, and to store all linguistic information in the lexicon (the minimalist feature checking is also a kind of unification, so unification seems to be the final "inevitable coceptual minimum").[2]

[1]Special thanks are due to the Hungarian National Scientific Research Fund (grant no. OTKA F026658), the Széchenyi Professorial Scholarship (G. Alberti 1999–2002), the Faculty of the Humanities of the University of Pécs and the Rector of the university for their financial support. Futher, we are grateful to Henk Zeevat and other participants of the 4th Tbilisi Symposium on Language, Logic and Computation (held in Borjomi) for their valuable comments on our talk this article is based on.

[2] It is worth quoting here Dowty's (1996: 12, 53) opinion from *Toward a minimalist theory of syntax* about the legitimacy of PS trees, which Chomsky (1995: 403) seems to consider "inescapable on the weakest interface conditions": "I suspect syntacticians today have almost come to think of the "primary empirical data" of syntactic research as phrase structure trees, so firm are our convictions as to what the right S-structure tree for most any given sentence is. But speakers of natural languages do not speak trees, nor do they write trees on paper when they communicate. The primary data for syntax are of course only *strings* of words, and everything in syntactic description beyond that is part of a theory, invented by a linguist." The author's aim is "getting linguists to question our automatic assumptions about constituents and our basis for assuming as a methodological principle that languages must always have a phenogrammatical syntactic structure describable by phrase structure trees."

1.2 A Theoretical Argument. The main theoretical argument in favor of GASG is that it promises a better answer to the stubborn problem of *compositionality* as to the morphosyntax→DRS (Discourse Representation Structure; e.g. Kamp 1981) transition than PSGs (Phrase Structure Grammars). We argue (Alberti 1999) that the failure of elaborating a properly compositional solution to this language→DRS transition arises from the fundamental incompatibility of the strictly hierarchically organized generative syntactic phrase structures with the basically unordered DRSs (or ones ordered but in an entirely different way).

Nowadays (Zeevat 1987, Karttunen 1986, van Eijck and Kamp 1997) some kind of Categorial Grammar (CG) is held to promise the best chance for capturing the language→DRS transition in a properly compositional manner: a version of (Classical) Categorial Grammar with capacity increased[3] by the technique of *unification,* applied in Prolog, for instance *(UCG)*[4]. The basic problem with UCG, which has amounted to the starting-point of GASG, lies in the fact that syntax, deprived of the information concerning sentence cohesion in favor of the unification mechanism and reduced to the primitive task of combining adjacent words, will produce linguistically irrelevant constituents. According to Karttunen's (1986: 19) remark on UCG trees: they look like PS trees but they are only "analysis trees"; and he adds "all that matters is the resulting [morphological] feature set."

This general problem with Unificational CGs requires more elaboration here; which will be based on Karttunen's (1986) paper on Finnish word order variation and long-distance dependencies.[5]

In Karttunen's grammar the linguistically relevant "checking" among words relies on a mechanism in the course of which attribute–value paths attached to categories are to be unified: "The value of *argument* is unified with the feature set of the expression the functor attempts to combine with. If this unification fails, the two expressions cannot be combined; if it succeeds, the feature set of the consequent expression is the same as the value of the functor's *result*." Categories (e.g. NPs) and the categorial rules of their combination are assumed to be practically trivial: "We define a nominative [/accusative] NP in Finnish as an expression that combines with an adjacent verb phrase to yield a verb phrase (V|V) in which it plays the role of the subject [/object]." "They [these expressions of category V|V] combine with a basic verb phrase to yield a verb phrase that is the same as the original except that the content of the functor has been merged with the argument in a manner determined by the functor. This latter aspect of the analysis of course can only be expressed in a unification–based formalism. … The parsing algorithm can be extremely simple: *Always try to combine a functor with an adjacent argument.*"

The advantage of UCG lies in the fact that "…by treating subject and object noun phrases syntactically as functors we have accounted for [some generalizations about the free Finnish word order] by using only Function Application. In a CG of a more traditional sort, in which the verb is responsible for linking up with its syntactic dependents, type raising and function composition must be introduced to achieve the same result."

There is a serious (linguistic and computational) disadvantage, however, discussed by Karttunen himself: "It is convenient to represent the analyis of a phrase as a tree that shows how the resulting feature set was derived. However, the structure of the analysis tree has no linguistic significance in our system: in this respect analysis trees are different from PS trees as they are traditionally construed in linguistics. *All that matters is the resulting feature set.* Because no functor has any priority over others with respect to order of application, the same result can often be obtained in more than one way. This is potentially troublesome from a computational point of view. … From the parser's point of view, this is a "spurious ambiguity" because the alternative analyses yield exactly the same set of features. In a more complicated sentence, spurious ambiguities multiply very quickly…"

As for the way out, the author's final conclusion / conjecture is that "…a *radical lexicalist* approach is a better alternative." GASG is to be regarded as an attempt to reach the most radically —*totally*— lexicalist solution, which can be formulated informally as follows: GASG = UCG – C where the "subtrahend" refers to the categorial way(s) of combining words (i.e. Function Application in UCG), and 'C' refers to the (constituent combining) apparatus of categorial grammars.

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[3] The problem with Classical CG is that it has only a context free generative capacity (according to the Chomsky hierarchy of grammars), which is proved (Shieber 1985) to be insufficient for the description of human languages. There seem to be two ways to increase the generative capacity of CCG: to let in, in opposition to the original goals, a few combinatorial means (though non-language-specific ones such as Function Composition ($X/Y \;\; Y/Z \Rightarrow X/Z$), Commutativity ($(X/Y)\backslash Z \Rightarrow (X\backslash Z)/Y$), Type Raising ($X \Rightarrow Y/(Y\backslash X)$)) in addition to the "classical" Function Application ($X/Y \;\; Y \Rightarrow X$), or to introduce the technique of *unification,* applied e.g. in Prolog (UCG).

[4] The anonymus reviewer of this paper mentioned Muskens' "lambda" DRT as a popular option at this point; (s)he aslo mentioned further relevant alternatives to our approach (Assumption Grammars, Link Grammar, Dependency Grammars etc.). The size of this paper does not make it possible for us to provide a comparison of our system with these (and further) alternatives – we hope, however, that we will get the chance for doing that in the future. Special thanks are due to the reviewer for these useful comments helping in deciding the direction of our future research.

[5] The relevant property of Finnish (in which respect this language is similar to Japanese or Hungarian) is its ("highly") free word order, concomitant with a rich morphology.

2. GASG

GASG can be characterized generally as a distinguished monostratal grammar where there is no need for a "separate generating engine" (linguistic rules of computation) at all. The engine must be *unification* itself, which is capable of running Prolog programs properly. The rich description of a lexical sign (say, out of a group of lexical signs selected from the Lexicon in order to combine them to form a sentence) serves a double purpose. It characterizes the *potential environment* of the given sign in possible grammatical sentences in order for the sign to find the morphologically (or in other ways) compatible elements and to avoid the incompatible ones in the course of forming a sentence. And the lexical description characterizes the sign itself in order for other words to find (or not to find) it, on the basis of similar "environmental descriptions" belonging to the lexical characterizations of these other words. And while the selected words are seeking each other on the basis of their formal features suitable for unification, their semantic features are also being unified simultaneously; so by the end of a successful building it will have been verified that a particular sequence of fully inflected words constitutes a grammatical sentence, and its semantic representation, a DRS, will also have been built at our disposal.

As the special treatment of word order and the Prolog implementation of GASG are intended to be concentrated on in this paper, there is no place here to provide a detailed general demonstration. We can only sketch such an important topic as the morphological basement of GASG. Here we offer the two papers serving as the basis of this short summary: Alberti (2000b, 2001). The starting-point is a comparative analysis of the German, Hungarian and English version of a sentence:

(1) a. Mein kluger Lehrer fand eine freundliche ungarische Studentin.
my-m-sg-NOM clever-m-sg-NOM teacher found-3sg an-f-ACC friendly-f-sg-ACC Hung.-f-sg-ACC she-student

b. Az én bölcs tanár-om talál-t egy barátságos magyar diáklány-t.
the I clever teacher-poss-1sg found-3sg a friendly Hungarian student-girl-acc

c. My clever teacher found a friendly Hungarian student.

The annotations belonging to the German and the Hungarian (but not the English) words refer to a rich system of morphological "sensitivity" among words in sentences (practically case and agreement relations; see Lehmann (1988)). The task of this system can be formulated by means of a comparison between the (simplified common) DRS of the sentences, shown in (2a) below, and the "proto-DRS" in (2b), which is the result of collecting the (discourse–) semantic contributions of the lexical items evoked by the words of the German sentence (still regarded as independent elements).

(2) a. q ∧ i ∧ r ∧ e ∧ clever(q) ∧ teacher-of(q,i) ∧ I(i) ∧ find(e,q,r) ∧ past(e) ∧ friendly(r) ∧ Hung.(r) ∧ student(r)

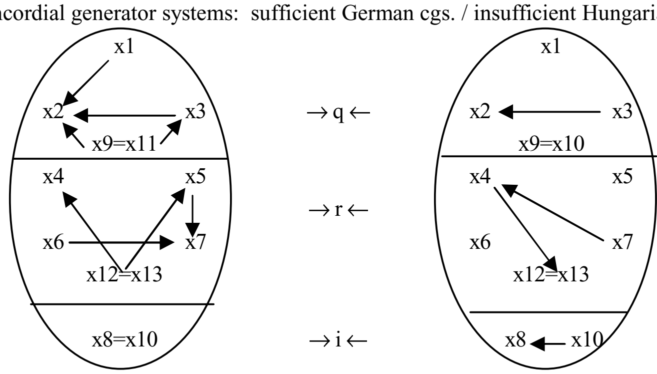
b. klug(x1), Lehrer(x2,…), fand(x3,x4), freundlich(x5), ung.(x6), Stud.(x7)
mein: x8=i, x9, X(x9,x10), x8=x10, Y(x11,…), x9=x11
eine: x12, W(x12), Z(…,x13,…), x12=x13

In a proto-DRS, thus, (proto–) referents coming from different lexical items are to be different. Now the task of the infrasentential morphological system can be formulated as follows: it should provide information enough for making the "underspecified" proto-DRS specific, by converting it into a "normal" DRS. We argue (Alberti 2000b, 2001) that this procedure is based on the following universal: if two words belonging to two different lexical items in a sentence stand in some morphological / morphosyntactic relation (agreement or case relation), then the corresponding lexical items stand in a *copredicative* relation (i.e. there are referents to be identified in the (underspecified) propositions constituting the semantic components of the given lexical items).

Arrows in the left figure in (3) below show the proto-referents in (2b) that can be identified due to some morphological sensitivity between words. The agreement between *kluger* and *Lehrer* in number, gender and case, for instance, indicates that the teacher coincides with the person claimed to be clever, which can be expressed by the identity *x1=x2.*[6] This relation on proto-referents can be regarded as a potential *generator system* of the equivalence relation represented by the final (specified) DRS — in the sense that the latter relation is the reflexive–symmetric–transitive closure (Partee *et al.* 1990) of the former one in the ideal case.

[6] Doubtful instances of copredication are discussed (interpreted appropriately) in Alberti (2000a).

(3) Concordial generator systems: sufficient German cgs. / insufficient Hungarian cgs.



Only the German sentence version exhibits this ideal case in (1), however; which can be formulated by saying that the concordial generator system that belongs to sentence (1a) is *sufficient.* The cgs. belonging to the Hungarian sentence in (1b) is *insufficient* in the same sense (because of the lack of any kind of agreement between attributive adjectives and nouns in Hungarian), and the corresponding English cgs. is simply empty. These latter two facts suggest that we should "re-discover" syntax:

A copredicative relation between two lexical items in a sentence can be expressed in human languages also by *pure syntactic relations* (e.g. adjacency and/or (immediate) precedence; see fn. 2 again) between (the) two words belonging to them. Pure syntactic relations and morphological dependence relations thus are both *formal intrasentential relations* between words suited for marking copredicative (semantic) relations between the corresponding lexical items.

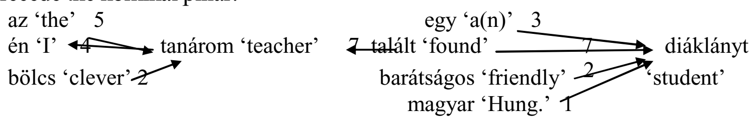
In lexical descriptions of GASG these two kinds of formal relations are represented in the same way, on the same level; there is no separation such as that in (U)CGs (and other PSGs), where unifiable attribute–value paths (or feature systems) are responsible for morphological relations and some categorial apparatus of combinatorial rules (or Merge and Move rules) are responsible for word order. The lexical item of *talált* 'found' in (1c) serves as an illustration (see Alberti 2000b, 2001):

(4) Λ5 =< {v5 = *talált},*
{fin.past(v5), verb.tr(v5), noun(α, V5.11), 3.sg(α, V5.11), arg(α, nom, V5.11, v5), ref(α, V5.12),
immprec(**α=5**, V5.12, V5.11),
immprec(**α=7,** V5.11, v5), noun(α, V5.21), ref.indef(α, V5.22), 3(α, V5.21), arg(α, acc, V5.21, v5),
immprec(**α=5**, V5.22, V5.21), immprec(**α=7,** v5, V5.21)},
{∧(α, found(X5.1, X5.2), Q5.1(X5.1)), ∧(α, found(X5.1, X5.2), Q5.2(X5.2))},
{corr(v5, found), corr(V5.11, Q5.1), corr(V5.21, Q5.2)}>

Only the most relevant details of lexical items (and this particular one) will be explained here. Lexical items consist of four components, out of which the first one shows the *own word:* a fully inflected word to look for in sentences whose parsing is supposed to require the given lexical item. The second component provides the morphosyntactic characterization of the *own word* (denoted by a lower case letter in harmony with the practice in Prolog where constants are marked in this way) and the *environmental words* — they are variables to be unified with own words of other lexical items. In the example above own word v5 is characterized as a transitive verb in the past tense and there are four environmental words because of the two arguments of 'find.' Why are there four environmental words? As categories, such as DP, are not intended to be referred to, a nominal argument should typically be referred to via its two "pillars": the nominal pillar (e.g. 'teacher': V5.11) and the "referential" pillar (the one responsible for the (usually obligatory) referentiality of the argument (Alberti 1997); e.g. 'the': V5.12). If *Peter found sy,* the two pillars of the subject coincide, i.e. both are to be unified with one and the same own word, the one belonging to the lexical item of 'Peter.' As for the two pillars of the potential object of 'found,' a crucial morphosyntactic property of the nominal pillar (V5.21) is its accusative case, and the referential pillar (V5.22) is characterized as an indefinite element (with respect to the definite conjugation of the finite verb).

The contribution of a lexical item to the determination of sentential word order basically lies in the *immprec* ('immediate precedence') requirements, whose satisfaction is defined so that the α *rank parameters* (see above in (4)) are considered too. Section 3 is devoted to this topic, so only a problem is mentioned here: if all *immprec* requirements coming from the lexical items that the parsing of the given sentence is based on are considered, the following contradictory requirements will have been collected (Alberti 2000b, 2001), where the formula *'the'---5--→'teacher,'* e.g., means that in one of the relevant lexical items there is a condition of rank 5 requiring the referential pillar of a "DP" to immediately precede the nominal pillar:

(5)



The main point in Section 3 will be the claim that requirements like the particular one above is *not violated* in sentence (1b) in spite of the word order '*the* I clever *teacher*…' but it is satisfied *indirectly* by satisfying stronger requirements (those of higher ranks denoted by smaller numbers).

To finish demonstrating the lexical item, the third component provides its discourse–semantic contribution to the DRS of the sentence parsed, and the fourth component is responsible for fixing the correspondence between elements of the morphosyntactic component (own and environmental words) and those of the discourse–semantic component (own and environmental predicates).

Computations in the course of which variables (of "word," "predicate" and other types) of lexical items belonging to particular (Hungarian, English, German and Dutch) sentences to be parsed are unified with constants (own words and own predicates) of each other (resulting in some evaluation of grammaticality and specified DRSs) are demonstrated in the following papers: Alberti (1998, 1999, 2000b, 2001).[7]

3. Rank Parameters in GASG

This short but very important section is devoted to the definition of (the satisfaction of) *immprec* relations (see (4-5) above, which the facility for dispensing with the concept of *constituents* depends on.

(6) DEF.: An *immprec*$_n$ relation (immediate precedence relation of rank n) between words w1 and w2 in a sequence of words is satisfied if

1) w1 immediately precedes w2 (indeed), or
2) there is a word w3 (between w1 and w2)[8] such that w3 is to satisfy an $immprec_k$ relation with w1 or w2 where $n \geq k$, and sequences [w1,...,w3] and [w3,...,w2] are *legitimate.*

A sequence [x1,...x2] of words is *legitimate* if

1) it consists of only two members, or
2) there is a word x3 in [x1,...x2] such that, for some number m, x3 is to satisfy an $immprec_m$ relation with x1 or x2, and sequences [x1,...,x3] and [x3,...,x2] are *legitimate.*

The informal content of the definition is that a requirement of rank n concerning the immediate precedence of word w1 relative to w2 can be satisfied either *directly* — by the fact that w1 does immediately precede w2 indeed — or indirectly — by permitting certain words to be inserted between w1 and w2, those, and only those, whose immprec requirement to w1 or w2 is stronger, or are dependents of such words, or dependents of dependents, or dependents of dependents of dependents, etc[9]. Sequences of such "dependent" words are called *legitimate*, and this relation should be checked either trivially — by recognizing that the list of words in question only consists of the two peripheral members, and hence nothing is inserted inside, or by pointing out novel and novel "dependents of dependents."

[7] Certain lexical items are supposed to have more than one own word (preverb+verb constructions (Alberti 1999), idioms (Alberti 2001)) or no own word (Hungarian focus (Alberti 1998), English interrogative operator (Alberti 2000b)).

[8] The version with parentheses is to be applied in the case of Swiss German cross-serial dependencies (see Alberti (2000b) where this theoretically very important (Shiebr 1985) construction is compared to German nested dependencies and the corresponding (regular) English nifinitival constructions).

[9] This point seems to be the best one for answering a question of the anonymus reviewer's on the treatment of long distance dependencies in GASG. The article on this topic (Alberti 2002) is unfortunately in Hungarian. The essence, however, is clearly demonstrated by (the "recursive" spirit of) the definition in (6) above. What is to be captured is the *unbounded* distance between the "real" lexical regent of an argument and its "surface" place in the neighborhood of a higher regent, which is 'higher' in the sense that the former regent is an argument of an argument of an argument… of the latter regent. One mugth think that it is hard to formalize this unbounded functional relation in a theory based on the lexical description of "local" environmental requirements of words. The recursive technique of definition, however, clearly proves the opposite: we may define the "higher regent" of an element as either its immediate (lexical) regent or the higher regent of its immediate regent. Even constrains on these chains of regents (e.g. some kind of *that* trace filter) are easy to formulate by demanding certain kinds of regent-argument relations.

The *immprec*$_5$ requirement between 'the' and 'teacher' in (5), for instance, is satisfied indirectly in sentence (1b): (possessive) 'I' is inserted between them legitimately due to its adjacency requirement of rank 4, then the ['the,' 'I'] sequence is trivially legitimate, and the ['I,' 'clever,' 'teacher'] sequence is legitimate due to the *immprec*$_2$ requirement between 'clever' and 'teacher'; and finally two trivially legitimate (two-member) sequences should be pointed out. More complicated cases will be analyzed in the course of discussing the Prolog implementation of GASG.

Examples in (7) below illustrate that in Hungarian, but not in English, certain free adverbs ('yesterday,' 'in the library') can be inserted between the finite verb and its arguments quite freely (in the case of idioms as well). This can be accounted for in GASG easily — by choosing the same rank parameter, namely 7, for both the regent–argument adjacency requirement and that between free adverbs and the finite element of sentences, at least in Hungarian. In English, however, the regent–argument adjacency requirement is to be qualified as stronger.

(7) a. Adt-am tegnap Mari-nak a könyvtár-ban egy cikk-et.
gave-1sg yesterday Mary-dat the library-ine a paper-acc
'Yesterday I gave Mary a book in the library.'
b. *I gave yesterday Mary in the library a paper.
c. *Peter kicked yesterday the bucket.
d. Péter be-adta tegnap a kulcs-ot.
Peter in-gave yesterday the key-acc
'Yesterday Peter kicked the bucket.' (lit. 'P. returned the key.')

This also shows that GASG is suitable for the treatment of "rigid" word order as well. Differences between languages can be captured by fixing different adjacency (and order) numeral rank parameters, which is the simplest and most elegant way of expressing parametric differences between human languages (as alternative possible realizations of UG).

The final comment in this section concerns the possibility for applying rank parameters in semantics as well. The ambiguity of the sentence in (8a) below, for instance, can be accounted for by fixing the same rank parameter, in semantics, for adjacency requirements between certain pieces of DRS formulas in the lexical description of *every* and *a(n);* and in this way two final DRS formulas can be computed (see (8a-b) below). Details are available in Alberti (1999).

(8) a. *Every* English boy visited *a* pretty Dutch girl.
b. ...∧**((x4 ∧ boy(x4)) ∧ english(x4)) →** [*(((x5 ∧ girl(x5)) ∧ dutch(x5)) ∧pretty(x5))* ∧ visit(x3, x4, x5)]
c. ...∧*(((x5 ∧ girl(x5)) ∧ dutch(x5)) ∧pretty(x5))* ∧ [**((x4 ∧ boy(x4)) ∧ english(x4))→** visit(x3, x4, x5)]

4. Implementation in Prolog

4.1 Purposes and parsing. As many others, we regard it as a primary purpose of an implementation work to verify the descriptive adequacy of a given theory as well as to make the theory and its practical realization to be suitable for revealing more about languages and their function.

Our program is permanently being developed, and the version that is available now can parse uncompound neutral Hungarian sentences.[10] It is practically to be regarded as a "toy grammar" which can interpret just a small — but non-trivial — fragment of the Hungarian language. In our parser we insist on the theoretically "clear" principles, but naturally we have to make some technical change because of the special features of programming in Prolog. Hence, parts of lexical items in GASG are stored in different places in the programme. The database section contains the own word and inherent properties that lexical items can reveal about themselves. Environmental conditions and properties of words that a lexical item searches are put down in `relation` predicates. This part means the "syntax" together with a checking section that contains the `immprec` relations (6). The third part of a lexical item — which is semantics — is represented in `sem` predicates, and the fourth part of "theoretical" lexical items requires no separate expression in the programme, however, because in our Prolog clauses semantic relations are already written as consequences of morphosyntactic ones.

[10] In this section the parts of the program are written by courier fonts

In our program all lexical items have two main components: the first one is the “own word” and the second one is a label with “own properties.” This label itself also consists of three components, out of which the first one is the English “translation” of the root of the verb. In the case of the second one there are various possibilities: if the given lexical item is a noun, this component contains its case, if a finite verb, the own features, and if a determiner, its definiteness feature. The third component of labels consits of the features that the word “shows” about its arguments: inflection, agreement relations etc.

The clauses from database section below are examples for lexical items:

(9) a. Adjuncts:

```
lexitem("holland",adj("Dutch")).
lexitem("barátságos",adj("friendly")).
```

b. Determiners

```
lexitem("egy",det("a","indef")).
lexitem("a",det("the","def")).
lexitem("három",det("three","indef")).
```

c. Nouns:

```
lexitem("Péter",ln("Peter","NOM",arg(f("sg","3"),s("none","0","none")))).
lexitem("Péter",ln("Peter","POSS",arg(f("sg","3"),s("none","0","none")))).
lexitem("Péternek",ln("Peter","DAT",arg(f("sg","3"),s("none","0","none")))).
lexitem("Péternek",ln("Peter","GEN",arg(f("sg","3"),s("none","0","none")))).
lexitem("lányt",ln("girl","ACC",arg(f("sg","3"),s("none","0","none")))).
lexitem("fiúra",ln("boy","OBL",arg(f("sg","3"),s("none","0","none")))).
lexitem("te",ln("you","NOM",arg(f("sg","2"),s("none","0","none")))).
lexitem("barátom",ln("friend","NOM",arg(f("sg","3"),s("sg","1","none")))).
lexitem("barátod",ln("friend","NOM",arg(f("sg","3"),s("sg","2","none")))).
lexitem("barátomat",ln("friend","ACC",arg(f("sg","3"),s("sg","1","none")))).
lexitem("barátjának",ln("friend","GEN",arg(f("sg","3"),s("sg","3","none")))).
```

d. Verbs:

```
lexitem("szeret",lf("love",feat("pres","indic"),arg(f("sg","3"),s("none","0","indef")))).
lexitem("szereti",lf("love",feat("pres","indic"),arg(f("sg","3"),s("none","0","def")))).
lexitem("szerettek",lf("love",feat("pres","indic"),arg(f("pl","2"),s("none","0","indef")))).
lexitem("szerettek",lf("love",feat("past","indic"),arg(f("pl","3"),s("none","0","indef")))).
lexitem("szeretnék",lf("love",feat("pres","cond"),arg(f("pl","3"),s("none","0","def")))).
lexitem("szeretnék",lf("love",feat("pres","cond"),arg(f("sg","1"),s("none","0","indef")))).
```

e. Participles:

```
lexitem("szerető",lif("love")).
lexitem("utáló",lif("hate")).
```

The parsing starts with the principal predicate **`gramm_semantics`**, which, after a successful morphosyntactic parsing carries out some semantic selection and gives a semantic representation formulated as a discourse-representation structure.

(10)

```
gramm_semantics(WL):-
    numberlist(1, WL, PWL), belongto(PWL, LIL2),
    finitelist(WL), norepart(WL),
    belonto2(LIL2, LIL, LIL2), satisfy(LIL, LIL2, GRALIST),
    semantics(LIL, SEMLIST, GRALIST, LIL),
    writeline(LIL), nl, writeline2(SEMLIST).
```

And there is a very similar predicate — **`gramm_morphosyntax`** — which differs only in one respect: it cannot provide semantic parsing; but it is necessary to separate semantic wellformedness from syntactic wellformedness.[11]

The input for the programme is a simple word list (variable WL), and as a first action the **`numberlist`** predicate gives serial numbers to words. This step seems to be technical but it is quite important because of unambiguous identification of the words in sentences. Then predicate **`belongto`** chooses, from the database section, the adequate lexical items for the words on the basis of *own words* which appear in the sentence (“in the surface structure”).

Predicate **`belongto2`** suggests grammatical relations on the basis of agreement and case relations observable in the sentence. This predicate refers to **`relations`** predicates, in which the environmental conditions of the given lexical items can be found.

[11] In the future it will be of great importance in translation to be able to parse sentences that are not grammatical according to semantics but grammatical according to morphosyntax (e.g. the sentence **The apple loves Mary*, where verb *love* has a semantic condition that the object must have a *+human* feature).

(11)
```
relations(N,SZERET,lf(X,F,A),[gr("subj","d",N,K),gr("subj","n",N,M),
gr("obj","d",N,L),gr("obj","n",N,J)],LIL2):-
   subjd(Q,SZERET),in(li2(K,Q,LAB),LIL2),
   prec(li2(K,Q,LAB),li2(N,SZERET,lf(X,F,A)),LIL2),
   subjn(R,SZERET,LIL2),in(li2(M,R,LAB2),LIL2),
   prec(li2(M,R,LAB2),li2(N,SZERET,lf(X,F,A)),LIL2),
   objd(S,SZERET,LIL2),in(li2(L,S,LAB3),LIL2),
   prec(li2(N,SZERET,lf(X,F,A)),li2(L,S,LAB3),LIL2),
   objn(Z,SZERET,LIL2),in(li2(J,Z,LAB4),LIL2),
   prec(li2(N,SZERET,lf(X,F,A)),li2(J,Z,LAB4),LIL2),
   prec(li2(K,Q,LAB),li2(M,R,LAB2),LIL2),
   prec(li2(L,S,LAB3),li2(J,Z,LAB4),LIL2).
```

Variable N is the serial number of the lexical item, which plays an important role later. The following variables are the own word (from the sentence) and the label, and after that the most important component of this predicate comes, which is a list demonstrating the grammatical relations that the given lexical item demands. Because of unambiguity, these relations include the serial numbers mentioned above, and their formula is an ordered quadruple, e.g. `gr("subj","d", N, K)`, where the first string shows the name of the relation, the second string is the pillar, then the first number is the own serial number and the second number is that of the other word in the given relation: its environmental word.

In the example (11) shown above there is a transitive verb that has two arguments, a subject and an object, and four pillars — both arguments has a "nominal pillar" and a "determiner pillar." Between the verb and a pillar stands a grammatical relation that has conditions which are traced back to other predicates; see (12) below as an example:

(12)
```
subjd(X, Z):-
   ref(X), verb(Z).
subjn(X, Z, LIL2):-
   in(li2(_,X,ln(_,"NOM",arg(Y,_))),LIL2),in( li2(_,Z,lf(_,_,arg(Y,_))),LIL2).
objd(X, Z, LIL2):-
   in(li2(_,X,det(_,Y)),LIL2),in( li2(_,Z,lf(_,_,arg(_,s(_,_,Y)))),LIL2).
objn(X, Z, LIL2):-
   in(li2(_, X, ln(_, "ACC", _)), LIL2), transitive(Z).
```

At this point the programme executes a "local search" — in the sense that every element is to find environmental words satisfying the appropriate grammatical relations. This is far from enough, however, because in this way a sequence of words like **Péter Mari alszik* 'Peter Mary sleeps,' for instance, would be predicted to be grammatical, because the verb can find a subject for itself and the two nouns can also find the verb for themselves. That is why some kind of mutual search is required, which means that members of a pair of words in a grammatical relation must find each other but no further words can be found for the same grammatical relation. The **`satisfy`** predicate in (10) carries out two important checking actions. First it has to consider the (quite simple) mutual search between lexical items: if there is a grammatical relation `gr("name", "pillar", N, M)`, there must be a relation with the same name and numbers `gr("name", "pillar", M, N))`. As a second operation, predicate **`immprec`** checks the right word order through the rank parameters ("weighing" the "intensity" of adjacency requirements between words in certain constructions; see (5)) in argument positions of predicate **`grimmprec.`**

(13)
```
grimmprec(7,"subjn",li2(A,_,_),li2(B,_,_),GRALIST):-
   in2(gr("subjn", A, B), GRALIST).
```

If all predicates are satisfied, the sentence is grammatical "according to" morphosyntax, and the last predicate writes the analysis. Let us demonstrate a running, where the question in the `goal` section is (14) below, and the answer is (15):

(14) `gramm_morphosyntax (["Péter","szeret","egy","barátságos","holland","lányt"]).`

(15)
```
["Péter","szeret","egy","barátságos","holland","lányt"]
li2(1,"Péter",ln("Peter","NOM",arg(f("sg","3"),s("none","0","none"))))
[gr("subj","d",1,2),gr("subj","n",1,2)]
li2(2,"szeret",lf("love",feat("pres","indic"),arg(f("sg","3"),s("none","0","indef"))))
[gr("subj","d",2,1),gr("subj","n",2,1),gr("obj","d",2,3),gr("obj","n",2,6)]
li2(3,"egy",det("a","indef"))
[gr("det","n",3,6),gr("obj","d",3,2)]
li2(4,"barátságos",adj("friendly"))
[gr("adj","none",4,6)]
li2(5,"holland",adj("Dutch"))
[gr("adj","none",5,6)]
li2(6,"lányt",ln("girl","ACC",arg(f("sg","3"),s("none","0","none"))))
[gr("det","n",6,3),gr("obj","n",6,2)]
```

4.2 Semantic representation. If a sentence has a right morphosyntactic output, predicate `semantics` carries out semantic selection, and if it also successful, it can provide the semantic representation: a DRS.

According to DRT, determiners (and proper names) provide referents, common nouns predicate something of them, and finite verbs provide a situation referent besides predicating something (of other referents). The following sentence in (16) below, for instance, is assigned the Prolog representation shown in (17) — collected in SEMLIST (10) —, which is practically the same as "box-representation" in DRT:

(16) Péter szeret egy barátságos holland lány-t.
Peter love-SG3 a friendly Dutch girl-ACC
'Peter loves a friendly Dutch girl.'

(17)
```
[provideref("the",ref(1)), pred("Peter",[ref(1)])]
[provideref("state",ref(5)),
    pred("love",[ref(5),ref(1),ref(6)]),pred("past",[ref(5)])]
[provideref("a",ref(6))]
[pred("friendly",[ref(6)])]
[pred("Dutch",[ref(6)])]
[pred("girl",[ref(6)])]
```

r1, r2, r3
Peter(r1) friendly(r2) dutch(r2) girl(r2) love(r3, r1, r2)

In the semantic representation, determiners provide a referent in the following way:

(18) `sem(N,_,det(A,_),_,[`**`provideref(A,ref(N))`**`],_,_).`

During the creation of DRSs, the checked morphosyntactic output is used as an input. As it can be seen above in (18) the identifier of the provided referent can be the serial number that we used during the morphosyntactic parsing. The provided referent gets a parameter A which can take determiners 'the' or 'a' as its value (among others) to be given from the (input) label. As for nouns, sentence (16) above demonstrates the ground case, when they are DPs whose determiner provides a referent. In this case we use the grammatical relations shown below in (19) in order for the common noun to get its referent from its determiner ('K' below):

(19)
```
sem(N, _, ln(A, _, arg(f(_, "3"), s("none", "0", "none"))),
 [gr("det", "n", N, K), _], [pred(A, [ref(K)])], _, _).
```

4.3 Possessive relations in Hungarian. In this subsection some practical parsing problems are demonstrated. Possessive relations in the Hungarian language raise a lot of parsing questions concerning morphosyntax and semantics, from agreement relations between possessions and possessors, through the "short" and "long" possessor, as far as multiple possessive relations.

The most interesting problems concern possessive relations containing determiners which do not appear in the (surface) sentence, as we insist on the basic principle of the GASG theory that in the morphosyntax only what we can "hear" can and should be described; i.e., no empty or deleted elements are used in the course of parsing.

It will be demonstrated what kind of tools can help us to solve these problems; for example, how our system can capture that the "short" possessor looks like an NP (with no determiner) in morphosyntax but its semantic representation is undoubtedly to include a DP. After all the interesting question for a linguist concerns the source of referents in DRSs when no suitable determiner can be found "in the surface."

During the morphosyntactic and semantic description the great variety of Hungarian possessive relations (see (20-23) below) requires a principled separation of different cases. Both in morphosyntax and in semantics the "short", "long" and "possessed" possessors are to be handled separately.

(20) *"long" possessors*
a. a lány-nak a barát-ja
the girl-DAT the friend-POSS.3sg
'the friend of the girl'

b. Péter-nek a barát-ja
Peter-DAT the friend-POSS.3sg
'the friend of Peter'

(21) *"short" possessors*
a. a lány barát-ja
the girl friend-POSS.3sg
'the girl's friend'

b. Péter barát-ja
Peter friend-POSS.3sg
'Peter's friend'

(22) *the possessor is a missing personal pronoun*
a barát-om
the friend-POSS.1sg
'my friend' (or 'the friend of mine'?)

(23) *multiple possessive relations*
a. a lány barát-já-nak a kutyá-ja
the girl friend-POSS.3sg-DAT the dog-POSS.3sg
'the dog of the girl's friend'

b. Péter barát-já-nak a kutyá-ja
Peter friend-POSS.3sg-DAT the dog-POSS.3sg
'the dog of Peter's friend'

c. a lány-nak a barát-já-nak a kutyá-ja
the girl-DAT the friend-POSS.3sg-DAT the dog-POSS.3sg
'the dog of the friend of the girl'

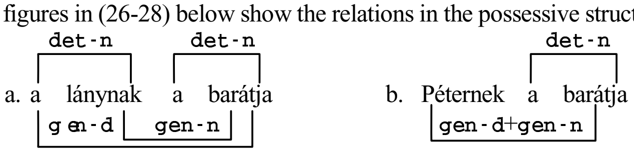
d. Péter-nek a barát-já-nak a kutyá-ja
Peter-DAT the friend-POSS.3sg-DAT the dog-POSS.3sg
'the dog of the friend of Peter'

By default, the possession searches the possessor as its argument, and it also searches the determiner pillar of this argument. However, this possessor can have a "long form" with a dative suffix *(-nAk)* or a "short form" (in the nominative case?) with no suffix. We would like to exhibit this difference by the names of the relations, too. Therefore the relation with a "short" possessor is denoted by `poss-n / poss-d`*,* and with a "long" possessor `gen-n / gen-d`. Two different lexical items may belong to the same own word in such cases, which can be distinguished in this way. For example, the own word *Péter* can be the subject or a "short" possessor in the sentence to be parsed. The database is to contain two lexical items with the same own word but different cases: *NOM(inative)* when the given word is analyzed as a subject and *POSS(essive)* in the latter case; see (24) below. What (25) demonstrates is that "long" possessors show a similar ambiguity between *DAT(ive)* and *GEN(itive):* the latter case can occur in possessive constructions.

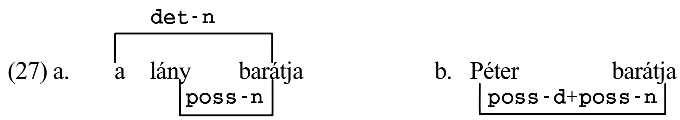
(24)
```
lexitem("Péter",ln("Peter","NOM",arg(f("sg","3"),s("none","0","none"))))
lexitem("Péter",ln("Peter","POSS",arg(f("sg","3"),s("none","0","none"))))
```

(25)
```
lexitem("Péternek",ln("Peter","DAT",arg(f("sg","3"),s("none","0","none"))))
lexitem("Péternek",ln("Peter","GEN",arg(f("sg","3"),s("none","0","none"))))
```

The figures in (26-28) below show the relations in the possessive structures in (21)-(24).

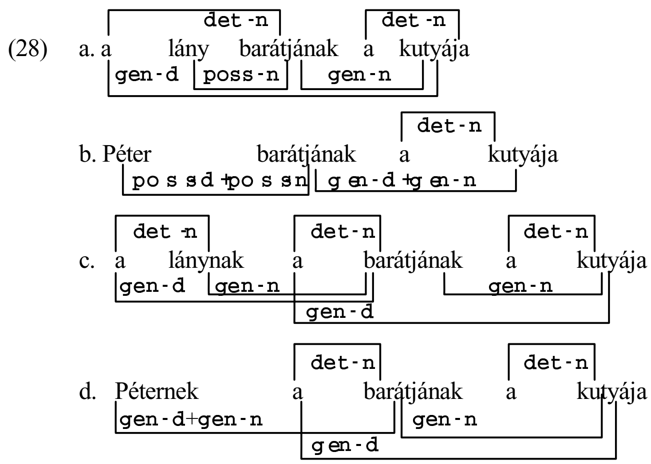
(26) a.


Grammatical relations of the possession in (26a) are a `det-n` relation with its own determiner, a `gen-d` relation with the deteminer pillar of its ("long") possessor and a `gen-n` relation with the noun pillar of the possessor. When the possessor is a proper name (26b), the possession can find both pillars (`gen-d+gen-n`) in it (i.e. this proper name).



The case of "short" possessors (see (27a-b) above) is even more complicated, because in the surface structure short possessors appear with no determiner, and — according to the basic philosophy of GASG, mentioned above, that in the morphosyntax only what we can "hear" is to be described (independently of semantic circumstances) —, in these situations the possession cannot search a separate determiner pillar, it is to be regarded as lacking a `poss-d` relation, so it has only a `poss-n` relation with the noun pillar of the possessor, which means that the short possessor is practically an NP according to morphosyntax (27a); except for the case of proper names (27b), which can serve as the noun pillar and the determiner pillar at the same time.

(28) below demonstrates the intricate morphosyntactic relations in multiple possessive constructions:



Let us turn to the question of semantic representation. The source of referents are determiners as a default. The finite verb also provides a (situation) referent and predicates something of other referents which are provided by determiner pillars — just as common nouns, which get their referent through its own determiner, and adjectives, which get their referent indirectly from the determiner pillar of the noun that they belong to.

In the case of possessive structures the starting-point is also that arguments (i.e. possessors) are DPs, so it has two pillars, when providing referents causes no special problem because determiners are available. Examples (26a,b), (27b), (28b-d) are representatives of this basic case. As for the type shown in (27a) and (28a), our proposal is based on "false NPs," which are arguments analyzed as NPs in the course of morphosyntactic parsing but regarded as DPs in semantics. How is it possible, i.e. what is the source of the determiner of the possessor when this argument is regarded as consisting of only the word *lány* 'girl'? The determiner pillar appears in the semantic characterization in the lexicon of the regent, that is, the possession *(barátja),* and its type (its definiteness: `"an"` or `"the"`) is to be fixed as follows: it has to be the same as the type of the referent of the possession word. GASG supports this solution due to the fact that connections between morphosyntactic and semantic features are precisely fixed in the lexicon but the two levels of description need not be true reflections of each other at all.[12]

[12] There are also "true NPs" in Hungarian, where the referential type of a nominal expression is to be chracterized as "unspec" rather than "a" or "the": e.g. *vendég érkezett* 'guest arrived' in the sense that 'one or more guests have arrived.' The source of the determiner of the argument ('guest') in cases like this is also the lexical item of the regent ('arrived'). Predicative NPs (e.g. *Péter most* ***vendég*** 'Peter now guest' ('Now Peter is a guest.') mean another type of "true NPs"; where the common noun ('guest') needs a special lexical item in whose semantic component a referent is introduced.

5. Summary

A new sort of generative grammar, GASG, which is more consistently and radically lexicalist than any earlier generative grammar due to the fact that the lexicon plays the role of the generating component, has been demostrated in this paper.

Section 1 contains arguments in favor of the basic principle behind this enterprise, which can be called the Principle of Total Lexicalism. It also provides comments on the embedding of our grammar in the family of (non-transformational) generative grammars: GASG is to be regarded as a "radically lexicalist alternative" to Unificational Categorial Grammars, whose representative is Karttunen's CUG (1986), among others.

Section 2 provides a sketchy demonstration of GASG, especially the structure of (monostratal) lexical items, which play a central role in the theory in accordance with its lexical nature. Afterthat a separate section is devoted to the demonstration of rank parameters and the special way of satisfying lexical requirements on the basis of these parameters, because we regard this element of our grammar as promising a new chance to cope with stubborn problems of (free) word order.

Section 4 is about our first steps towards the Prolog implementation of GASG. In addition to grammaticality evaluation of sequences of words, our program can assign DRSs to grammatical sentences in a very quick and efficient, immediate, way (where 'immediate' means: not through sentence-level morphosyntactic representations). Practical parsing problems are discussed, too, in the intricate area of Hungarian possessive constructions (4.4), where "disappearing" articles should be accounted for by means of the totally lexicalist toolbox. The solution, here and elsewhere, is based on the characteristic property of GASG that connections between morphosyntactic and semantic features are precisely fixed in the lexicon but the two levels of description need not be true reflections of each other at all — that is why we can "afford" to insist on another basic principle of total lexicalism that in the morphosyntax only what we can "hear" can and should be described.

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Topical Domain Restrictions

Maria Aloni and Robert van Rooy

Abstract

We extend Gawron's (1996) dynamic analysis of domain restriction with an analysis of questions. The obtained semantics allows us to account for the impact of questions on subsequent domains of quantification – notably the domain of focusing operators like *only* – improving on existing analyses of interrogative sentences.

1 Introduction

In Rooth's (1985, 1992) semantic theory of focus, sentences in which focused constituents occur are always interpreted relative to a set of alternatives. The set of alternatives for a sentence like (1) can be derived by substituting the meaning of the focused constituent by the meaning of an alternative of the same category.

(1) John introduced $[\text{Bill}]_{\text{F}}$ to Mary.

For our sentence, the alternatives to Bill are the other elements of D and thus the set of alternatives is $\{\{w: \text{ John introduced } d \text{ to Mary in } w\} : d \in D\}$. For bare focus constituents, this alternative set is used to determine the *appropriateness* conditions of the sentence with the focal constituent, while for cases where the sentence contains a focus sensitive operator like *only*, this alternative set can be used to determine the *truth conditions* of the sentence. Sentence (2a) is correctly predicted to entail that John introduced only one person to Mary, whereas sentence (2b) lacks such entailment.

(2) a. John only introduced $[\text{Bill}]_{\text{F}}$ to Mary.

b. John only introduced Bill to $[\text{Mary}]_{\text{F}}$.

But as Von Stechow (1990) has shown, Rooth's account is not flexible enough to determine the right truth conditions of sentence (3b) in the context of question (3a).

(3) a. Which gentlemen did John already introduce to which of their diner partners?

b. John introduced only $[\text{Bill}]_{\text{F}}$ to $[\text{Mary}]_{\text{F}}$.

Rooth (1985) wrongly predicts that (3b) is false if John already introduced the gentlemen of the diner to each other, but has only started introducing the gentlemen to their lady diner partners. The reason for the false prediction is that in Rooth (1985) the quantification involved by *only* is only sensitive to the alternatives determined by the *focus value* of the sentence, and this focus value is determined in a *context-independent* way with the result that the 'diner-partner' relation is ignored. Rooth (1992) responded by saying that focus values can only put constraints on domains of quantification of *only*, but need not determine it. How this domain of quantification is determined is left to pragmatics.

Large parts of pragmatics have become semantics by looking at things from a *dynamic* point of view. Groenendijk (1999) has recently implemented Groenendijk and Stokhof's (1984) partition theory of questions into dynamic semantics. According to their partition theory, the meaning of a question is the set of meanings of its *complete* (possible) answers. Thus, the meaning of a question like (4) is identified with the set of propositions $\{\{v \in W : \{d| \ d \text{ smokes in } v\} = \{d| \ d \text{ smokes in } w\} \ \& \ w \in W\}$.

(4) Who smokes?

In Groenendijk (1999), questions partition the context, represented as a subset of the set of relevant alternatives, while answers eliminate cells of this partitioned context. Though appealing, this analysis is not quite satisfactory. First, it fails to account for the intuition that questions influence the domain of quantificational expressions used later on (e.g. *only* in example (3)). Second, if more questions are asked but not yet resolved in the discourse, the partitioned context cannot determine what the influence is of the individual questions, although this is needed to determine whether the answers are appropriate or not. Furthermore, partitions are too *coarse grained* to account for focused and constituent answers. For example, Groenendijk's theory cannot account for the fact that answer (5c) would express a different content as a reply to (5a) than to (5b), for both induce the same partition.

(5) a. Who smokes?

b. Who does not smoke?

c. John

Krifka (2001) has shown that related problems arise also for theories in the Hamblin/Karttunen/Rooth tradition. The standard treatment of alternatives as sets of (propositional) answers is not fined-grained enough and to properly account for questions (and focus) we need something like the *abstracts* that underlie questions: structured propositions. Utilizing the close correspondence between information states in dynamic semantics – sets of world-assignment pairs – and structured propositions, we will show how all the above problems can be solved by introducing questions into Gawron's (1996) dynamic analysis of domain restriction: abstracts correspond to sets of world-assignment pairs associated with variables represented in a Gawronian environment. The obtained analysis of interrogative sentences will combine the empirical strength of the structured meaning approach with the logical appeal of Groenendijk's dynamic theory.

2 Formal Framework

In Gawron's (1996) formalism, the introduction of a quantificational operator is separated by the introduction of the quantificational domain. The latter is allowed to be fixed non locally. The intuition is that domains of quantification are constructed by combining constraints that arise from different sources. These constraints are encoded in so called *environments* which map variables to sets of possible assignments encoding information about which values are possible for them. We propose to interpret the semantic contribution of *questions* in term of extensions of these Gawronian environments. In our formalism an environment will map variables x to sets $e(x)$ of world-assignment pairs. We will take these sets to represent the *topics* under discussion in the current context. Interrogative sentences will be analyzed as setting up new topics, or expanding on previously introduced ones.

On the one hand, these topics correspond to abstracts, therefore our analysis of questions will be fine-grained enough to properly account for phenomena like constituent answers. On the other hand, since from a topic $e(x)$ we can recover the partition it would induce on the information state, we will be able to define all of the logical notions which are relevant for a theory of questions and answers. Finally, since, as in Gawron's original formalism, topics encode domain restrictions we will be able to account for the impact of questions on the domain of subsequent quantificational expressions (like *only* in von Stechow's dining example discussed above).

2.1 The Dynamics of Domain Restrictions

The proposed semantics is an extension of Aloni, Beaver, Clark (1999) (built on Gawron 1996) with an explicit analysis of questions.[1]

The *syntax* of our language is the same as that of standard first-order predicate logic with identity, but with the addition of a question operator '?'. We say that if $x_1, ..., x_n$ is a sequence of variables and ϕ a formula of first order predicate logic, then $?x_1, ..., x_n\phi$ is a formula in our language (see Groenendijk (1998)).

As for the *semantics*, formulae are associated with context change potentials. A context is a pair consisting of an environment e and an information state s. An information state consists of a set of world-assignment pairs, while environments are modeled as partial functions from variables to information states in which these variables are defined. States encode what is known and what antecedents are available for future anaphora; environments encode information about what is merely under discussion. If $c = s_e$ is a context, we will write $S(c)$ for s and $E(c)$ for e.

Elements of a state are called *possibilities*, given a possibility $i = \langle w, g \rangle$, we will write $i(\alpha)$ to refer to the denotation of α with respect to g_i and w_i. As in Dekker (1993), possibilities are ordered by an extension relation $\prec$: j *extends* i, $i \prec j$ iff $w_i = w_j$ & $g_i \subseteq g_j$. This extension relation carries over to an ordering relation between information states: s is a *substate* of t, $s \prec t$ iff $\forall i \in s: \ i \prec t$, where $i \prec t$ iff $\exists j \in t : i \prec j$.

Now we can give a recursive definition of the context-change potential of the formulae of the language. The basic formulae are defined as expected: they can only influence the state parameter s and eliminate possibilities in s in which the formulae are false:

1. $s_e[Pt_1, \ldots, t_n] = s'_{e'}$, if $\forall i \in s$ and $\forall k: \ 1 \leq k \leq n \Rightarrow \ i(t_k)$ defined

   (a) $e' = e$

   (b) $s' = \{i \in s \mid < i(t_1), \ldots, i(t_n) > \in i(P)\}$

   undefined otherwise.

2. $s_e[t_1 = t_2] = s'_{e'}$, if $\forall i \in s : i(t_1)$ and $i(t_2)$ are defined

   (a) $e' = e$

   (b) $s' = \{i \in s \mid i(t_1) = i(t_2)\}$

   undefined otherwise.

In the interpretation rule of *negation*, we make crucial use of the relation $\prec$. Just like atomic formulae, also negation influences only the state:

3. $s_e[\neg\phi] = s'_{e'}$.

   (a) $e' = e$

[1] Zeevat (1994) and van Rooy (1997) defend similar ideas in somehow different formalisms.

(b) $s' = \{i \in s \mid i \not\prec S(s_e[\phi])\}$

Conjunction is defined as standard in dynamic semantics as sequential update:

4. $s_e[\phi \wedge \psi] = s_e[\phi]\ [\psi]$

Until now the environments played virtually no role. They are crucial, however, for the semantic analysis of *quantified* sentences. If x is already defined in environment e, the update of context with $\exists x\phi$ is defined in terms the *merge* of an information state and the state resulting from applying e to x. The *merging* of information state s with information state s', $s \wedge s'$, is defined as the 'least upper bound' of s and s' (see again Dekker (1993)):

$$s \wedge s' = \{i \mid \exists j \in s:\ \exists j' \in s':\ dom(i) = dom(j) \cup dom(j')\ \&\ j \prec i\ \&\ j' \prec i\}$$

If we define random assignment, $s[x]$, as $\{\langle w, g[x/d]\rangle :\ \langle w, g\rangle \in s\ \&\ d \in D\}$, we can define the update of s_e with $\exists x\phi$ in terms of this merge-operator as follows:

5. $s_e[\exists x\phi] = s'_{e'}$, if x is not defined[2] in s.

   (a) $e' = e$

   (b) $s' = S((s[x])_e[\phi])\ \wedge e(x)$ if $e(x)$ is defined;
   $s' = S((s[x])_e[\phi])$ otherwise.

   undefined otherwise.

Quantificational sentences make use of the environment, but have no influence on these environments themselves. Only *questions* have. The effect of updating context s_e with question $?x\phi$ is that the output environment assigns to variable x a set of possibilities that verify ϕ.[3] Let $e[x_1,...,x_n]e'$ hold iff $dom(e') = dom(e) \cup \{x_1,...,x_n\}$ and $\forall x \in dom(e) : x \notin \{x_1,...,x_n\}\ \Rightarrow\ e'(x) = e(x)$.

6. $s_e[?x_1,...,x_n\phi] = s'_{e'}$

   (a) $e[x_1,...,x_n]e'$ and $e'(x_i) = S(s_e[\exists x_1,...,x_n\phi])$ for all $i \in n$.

   (b) $s' = s$

Finally, as in Gawron, we define an operation which copies topics under different labels.

7. $s_e[y \leftarrow x] = s'_{e'}$, if $y \notin dom(e)$

   (a) $e[y]e'$ and $e'(y) = \{i[y/g_j(x)] \mid \exists j \in e(x)\ \&\ i = \langle w_j, g_j \setminus \{\langle x, g_j(x)\rangle\}\rangle\}$

   (b) $s' = s$

Disjunction and implication are defined as standard in terms of conjunction and negation, '$(\phi\vee\psi)$' and '$(\phi \rightarrow \psi)$' stand for '$\neg(\neg\phi\wedge\neg\psi)$' and '$\neg(\phi\wedge\neg\psi)$', respectively. The universal quantifier is defined in terms of the existential quantifier and negation: $\forall x\phi = \neg\exists x\neg\phi$.

[2]As in Heim 1982, variables cannot be reset. So, in addition to formulas containing free variables, quantified sentences are partial updates as well. Since this issue is not directly relevant to the topic of this article, we will pass over it in what follows.

[3]Clause (6) only applies to constituent questions, but our analysis can also be extended to polar questions as shown in Aloni and van Rooy (2002).

Topics and quantification The crucial clauses in the previous definition are the ones of the question operator ? and the existential quantifier $\exists$.

The introduction of a question in a context only modifies the environment parameter. $s_e[?x\phi]$ yields a context $s_{e'}$ where $e'(x)$ is a state which merges the information contained in s and in ϕ with the information possibly contained in $e(x)$. This will be a state in which x is restricted to individuals who are ϕ.

The update of a context with a quantified sentence modifies only the state parameter, but it crucially depends on the value assigned by the environment to the quantified variable. $s_e[\exists x\psi]$ yields a context s'_e where s' depends on $e(x)$ which encodes all of the restrictions previously placed on x.

Topics restrict quantification. In context s_e, question $?xP(x)$ yields a context $s_{e'}$ such that $\langle w, g\rangle \in e'(x)$ only if $g(x) \in w(P)$. After question $?xP(x)$, the update with $\exists xQ(x)$ yields an information state containing only pairs $\langle w, g\rangle$ such that $g(x) \in w(P)$ and $g(x) \in w(Q)$.

2.2 The Logic of Interrogation

Groenendijk (1998, 1999) presents an update semantics for questions. The information change potential of an interrogative is viewed as a way of structuring information (see also Hulstijn (1997)). Questions add structure to an information state by raising an issue. An issue is modeled as an equivalence relation on the possibilities in a state and raising an issue amounts to disconnecting possibilities. Ordinary information states (sets of possibilities) are turned into *structured* states which are equivalence relations on ordinary states. So structured states σ are sets of pairs of possibilities expressing a partition of an ordinary state. If two possibilities i and j are connected in σ this means that the difference between i and j is irrelevant in σ. In this section we will see that topics structure information states. The topics under discussion in a context s_e uniquely determine a partition of the information state s. This fact will enable us to incorporate Groenendijk's logic of interrogation in our framework.

Topics, abstracts and partitions From topics $e(x)$ represented as discourse referents in an environment we can recover both (i) *abstracts*, i.e. functions from possibilities (possible worlds or in our case world-assignment pairs) to sets of (n-tuples of) individuals; and (ii) *partitions*, i.e. equivalence relation over the set of relevant possibilities.

Topics define *abstracts*. Abstracts are functions from states to sets of individuals. The abstract $A_x^{s_e}$ determined by topic $e(x)$ on state s is defined as follows:[4]

Definition 1 [Abstract] Let s_e be a context and x be a variable defined in e. $A_x^{s_e}$ is a function from s into D such that for $i \in s$:

$$A_x^{s_e}(i) = \{g_j(x) \mid i \prec j \;\&\; j \in (s \wedge e(x))\}$$

Intuitively abstract $A_x^{s_e}$ is a function mapping each possibility i in s to the set of individuals which satisfy in i the information encoded by $e(x)$.

From a topic $e(x)$ we can recover the partition $P_x^{s_e}$ it would induce on the current information state s.

Definition 2 [Partition] Let s_e be a context and x be a variable defined in e.

$$P_x^{s_e} = \{\langle i, j\rangle \mid i, j \in s \;\&\; A_x^{s_e}(i) = A_x^{s_e}(j)\}$$

[4]This definition can be easily generalized to the case with n-tuples of variables.

As an illustration consider the topic $e(x) = \{i \mid g_i(x) \in w_i(\text{SPY})\}$ set up by question *Who is a spy?* ($?x$ SPY(x)). The abstract determined by such a topic in context s_e will be the function $A_x^{s_e}$ assigning to each possibility i in s the set of spies in w_i: $A_x^{s_e}(i) = w_i(\text{SPY})$ for all $i \in s$. The partition $P_x^{s_e}$ of s induced by such a topic will contain those possibilities i and j in s such that for all individuals d: d is a spy in w_i iff d is a spy in w_j: $P_x^{s_e} = \{\langle i, j\rangle \mid w_i(\text{SPY}) = w_j(\text{SPY})\}$.

Entailment Building on Groenendijk (1998, 1999), we define entailment in term of subsistence between structured states. By $P(s_e)$ we will denote the partition induced on s by all the topics in e.

Definition 3 $P(s_e) = \bigcap_{x \in dom(e)}(P_x^{s_e})$

Partitions $P(s_e)$ assigned to contexts s_e are equivalent to the structured states σ defined in Groenendijk (1998). Structured states are sets of pairs $\iota = \langle i, j\rangle$ of world-assignment pairs. Groenendijk defines subsistence between structured states in terms of the notion of $\prec$ between world-assignment pairs defined above. A pair $\langle i, j\rangle$ subsists in $\langle i', j'\rangle$, $\langle i, j\rangle \prec \langle i', j'\rangle$ iff $i \prec i'$ & $j \prec j'$. This relation between pairs of possibilities carries over to a relation between structured states: $\sigma \prec \tau$ iff $\forall \iota \in \sigma : \iota \prec \tau$, where $\iota \prec \tau$ iff $\exists \iota' \in \tau : \iota \prec \iota'$.

We can now turn to entailment. We denote by $\min_\phi$ the context of *minimal* information in which an update with ϕ is defined.[5] Following Dekker (1993) we say that a state t is an *update* of s, $s \leq t$ iff $\forall i \in t : \exists j \in s : j \prec i$. This update relation can be carried over to environments and contexts in an obvious way: (i) $e \leq e'$ iff $dom(e) \subseteq dom(e')$ and $\forall x \in dom(e) : e(x) \leq e'(x)$; (ii) $s_e \leq s'_{e'}$ iff $s \leq s'$ and $e \leq e'$. We define $\min_{\phi_1,\ldots,\phi_n} = s_e$ iff $s_e[\phi_1]\ldots[\phi_n]$ exists and $\forall s'_{e'}$: if $s'_{e'}[\phi_1]\ldots[\phi_n]$ exists, then $s_e \leq s'_{e'}$. Entailment between sentences is defined in terms of these minimal contexts.

Definition 4 [Entailment]

(i) $s_e \models \phi$ iff $P(s_e) \prec P(s_e[\phi])$

(ii) $\phi_1, \ldots \phi_n \models \psi$ iff $\min_{\phi_1,\ldots,\phi_n,\psi}[\phi] \models \psi$

A sentence ϕ is entailed in a context s_e iff the structured state induced by s_e survives in the structured state induced by $s_e[\phi]$. While checking on entailment between sentences we only look at the minimal context in which the sentences are defined.

The defined notion of entailment applies uniformly to both interrogative and indicative sentences exactly as in Groenendijk (1998-99). In this section we will use $\phi?$ and $\phi!$ to denote interrogative and indicative sentences respectively.

An indicative is entailed in a context, $s_e \models \phi!$, if it does not bring about new information. An interrogative is entailed in a context, $s_e \models \phi?$, if the issue it raises is already present in the context, although possibly under a different label.

An entailment $\phi \models \psi$ means that ψ is superfluous after ϕ. When applied to indicative sentences, $\models$ corresponds to entailment in D(ynamic) P(redicate) L(ogic) (see Groenendijk and Stokhof (1991)). $\phi! \models \psi!$ means that ψ after ϕ does not bring about new information. The following is a classical example of a valid entailment in DPL: $\exists x P(x) \models P(x)$. [*A man*]$_i$ *smokes* entails *He*$_i$ *smokes* since the anaphoric pronoun in the second sentence is co-indexed with the indefinite in the previous sentence.

When applied to interrogative sentences, $\models$ corresponds to the classical Groenendijk and Stokhof's (1984) notion of entailment between questions. $\phi? \models \psi?$

[5] Note that there is always a unique minimal context with this property.

means that whenever the issue raised by $\phi?$ is resolved, the issue raised by $\psi?$ must also be resolved, so answering $\phi?$ yields a complete answer to $\psi?$. An example of a valid entailment between questions is $?xyR(x,y) \models ?yR(y,a)$. If we give a complete answer to question *Who invited whom?* we cannot fail to give a complete answer to the entailed question *Who invited Adam?*. Note further that we have that $?xP(x) \models ?yP(y)$, so the label under which an issue is encoded is irrelevant in this case. It is not irrelevant in general, as it is evident from the following facts which illustrates one of the crucial features of this formal system which will be exploited later on: a quantified sentence is automatically restricted by a preceding question if the two are co-indexed.

(i) $?x\phi \wedge \exists x\psi \;\models\; \exists y(\phi[x/y] \wedge \psi[x/y])$

(ii) $?x\phi_1 \wedge \ldots \wedge ?x\phi_n \wedge \exists x\psi \;\models\; \exists y((\phi_1[x/y] \wedge \ldots \wedge \phi_n[x/y]) \wedge \psi[x/y])$

(iii) $?x\phi \wedge \forall x\psi \;\models\; \forall y(\phi[x/y] \rightarrow \psi[x/y])$

(iv) $?x\phi_1 \wedge \ldots \wedge ?x\phi_n \wedge \forall x\psi \;\models\; \forall y((\phi_1[x/y] \wedge \ldots \wedge \phi_n[x/y]) \rightarrow \psi[x/y])$

Note that $\models$ is not monotonic.

(v) $\forall xPx \models \forall yPy$, but

(vi) $?xPx \wedge \forall xPx \not\models \forall yPy$

Finally, $\phi! \models \psi?$ can be read as $\phi!$ gives a *complete answer* to $\psi?$. For instance $\forall x(S(x) \leftrightarrow x = a) \models ?yS(y)$ is a valid entailment. Sentence *Only Adam smokes* completely answers question *Who smokes?*.

Following Groenendijk (1999) we can also define other logical notions, notably the notion of licensing which applies non-trivially only to indicatives in his system.[6]

Definition 5 [Licensing]

(i) $s_e \propto \phi$ iff $\forall \langle i,j\rangle \in P(s_e) : \langle i,i\rangle \notin P(s_e[\phi]) \Rightarrow \langle j,j\rangle \notin P(s_e[\phi])$

(ii) $\phi \propto \psi$ iff $\min_{\phi\wedge\psi}[\phi] \propto \psi$

An indicative is licensed after a question iff it exclusively addresses the issue raised by the question. Groenendijk uses licensing to characterize Grice's maxim of relation. See van Rooy (1999) and ten Cate (2002) for more sophisticated characterizations of the notion of relevance, which apply to both interrogatives and indicatives and which also could be implemented in the present semantics.

To conclude, since from topics we can recover partitions, we can define, as in Groenendijk (1999), the logical notions which are relevant for a theory of questions and answers. But, since topics correspond to abstracts, we can improve on Groenendijk's theory with respect to a number of phenomena, for instance focused and constituent answers, as we will see in the following section.

3 Applications

In this section, we will show how the formalism presented in the previous section allows us to solve the problems discussed in the introductory part of the present article. The first application concerns constituent answers.

[6]See Aloni & van Rooy (2002) for an extension of this notion which also applies to questions.

3.1 Constituent answers

As already mentioned in the introduction, a partition theory of questions cannot account for the fact that one and the same constituent answer, say *John*, can express different contents if used as reply to different questions like:

(6) Who smokes?

(7) Who does not smoke?

This is because questions (6) and (7) induce exactly the same partition, as it is easily checked. G&S (1984) solved the problem by associating questions with both a partition and an abstract. The formalism introduced in the previous section avoids the problem with no need of this double coding. Let us have a closer look.

We propose to represent a constituent answer as an existential sentence the domain of which is crucially restricted by the preceding question. Constituent answers like (8a) and (9a) are analyzed as in (8b) and (9b):

(8) a. John.

b. $\exists y_x(j = y)$

(9) a. John and Mary.

b. $\exists y_x(j = y) \wedge \exists z_x(m = z)$

where $\exists y_x\phi$ is short for $y \leftarrow x \wedge \exists y\phi$.

It is easy to see that, given this representation, we can account for the fact that constituent answers express different contents if used as reply to questions like (6) and (7) above. This is because although the two questions induce the same partition on the current state, they crucially set up different topics in the environment.

Fact 1

(i) $?xS(x) \wedge \exists y_x(j = y) \wedge \exists z_x(m = z) \models S(j) \wedge S(m)$

(ii) $?x\neg S(x) \wedge \exists y_x(j = y) \wedge \exists z_x(m = z) \models \neg S(j) \wedge \neg S(m)$

In order to account for quantified answers like *all men* or *most men*, which lack a proper first order representation we need a higher order theory exactly as in the structured meaning account of these phenomena.

3.2 Focus

In the present analysis focus expresses conditions on the kind of context in which the associated utterance can occur. Focus is presuppositional (see for instance Roberts (1996)). Focal sentences presuppose questions.

We use Beaver's partial operator ∂ to denote *presupposition*. We extend the language in such a way that only questions can occur in the scope of ∂. As is standard in dynamic semantics (see Stalnaker, Heim and Beaver) presuppositions are analyzed as conditions on the input context which has to be satisfied before the sentence can be interpreted with respect to that context. A presupposed question expresses a condition on the environment part of the context. $\partial[?x\phi]$ expresses the presupposition that the ϕs are among the topics under discussion in the current environment under label x.

Definition 6 [Presupposed topics]

$$s_e[\partial?x_1,...,x_n\phi] = s_e \quad \text{iff} \quad (s \wedge e(x_i))_e \models \phi \text{ for all } i: 1 \leq i \leq n$$

$\partial[?x\phi]$ is defined in a context s_e only if the state obtained by merging s with the topic assigned by e to x verifies ϕ. Provided this condition is met, the update of a context with $\partial[?x\phi]$ yields the input context as result.

Accommodation is built in the notion of entailment (see Beaver (1995)). According to the definition given in the previous section, a sentence ϕ entails another sentence ψ iff $\min_{\phi,\psi}$, i.e., the minimal context in which updates with the sentences are defined, updated with ϕ is a context which supports ψ. So while checking on entailment we restrict ourselves to contexts which minimally satisfy the presupposition of the sentences involved. The following are typical examples of valid entailments:

(i) $\partial[?x\phi] \wedge \exists x\psi \models \exists y(\phi[x/y] \wedge \psi[x/y])$

(ii) $\partial[?x\phi] \wedge \forall x\psi \models \forall y(\phi[x/y] \rightarrow \psi[x/y])$

We propose to represent focus in terms of presupposition of topics under discussion which can be accommodated. Focus triggers the presupposition that the so-called 'background' is among the topics under discussion at the moment of the utterance.

A sentence like (10a) with the item 'Mary' in focus is represented as in (10b).

(10) a. John loves $[\text{Mary}]_{\text{F}}$.

b. $\partial[?xL(j,x)] \wedge \exists x(x = m)$

(10b) presupposes the previous introduction, as topic, of a set of individuals (those loved by John) and, since topics restrict quantification, it asserts that the item in focus (Mary) belongs to that set. A typical example of a sentence introducing such a topic would be the explicit question

(11) Who does John love?

Note that from (10b) we can recover the ordinary meaning of the sentence:

Fact 2 $\partial[?xL(j,x)] \wedge \exists x(x = m) \models L(j,m)$

(10b) entails the ordinary meaning of (10a), but it also represents the information structure of the sentence.

3.3 Association with focus

As in Aloni et al (1999) we assume that focus has two roles: the first role is that of introducing a certain presupposition; the second role is to identify the focused variable to focusing operators such as *only*. We treat *only* as an indexed sentential operator only_x, where x is the associated focused variable. The interpretation of only_x involves a universal quantification automatically restricted by the presupposition expressed by the associated focus.

Definition 7 $s_e[\text{only}_x(\phi)] = s'_{e'}$

1. $e = e'$
2. $s' = \{j \in s \mid \{i \mid j \prec i \;\&\; i \in (s \wedge e(x))\} \subseteq \{i \mid i \prec S(s_e[\phi])\}\}$

As an illustration consider the following example. The sentence in (12a) receives the representation in (12b).

(12) a. John only loves $[\text{Mary}]_F$.

b. $\text{only}_x(\partial[?xL(j,x)] \wedge \exists x(x = m))$

Note that although an existential binds the focused variable x, the operator *only* has the effect of changing the quantificational force, in much the same way as in the standard dynamic analysis of (un)selective binding. (12b) is interpreted as asserting that each x is equal to Mary. Since the sentence is defined only in contexts in which the topic $?xL(j,x)$ has already been introduced, the domain of quantification will contain only individuals who John loves. In a neutral context, (12a) means 'John loves nobody but Mary', as in Rooth (1985).

Fact 3 $\text{only}_x(\partial[?xL(j,x)] \wedge \exists x(x = m)) \models \forall x(L(j,x) \rightarrow x = m)$

3.4 Context sensitivity

Among the advantages of a treatment which represents focus by presupposing the background as a domain restriction is that we have a straightforward account of the fact that the quantificational domain of *only* can be further restricted by the context, in particular by a preceding question. Consider the following dialogues:

(13) a. Who is wise?

b. $?xW(x)$

c. Only $[\text{Socrates}]_F$ is wise.

d. $\text{only}_x(\partial[?xW(x)] \wedge \exists x(s = x))$

(14) a. Which Athenians are wise?[7]

b. $?x(A(x) \wedge W(x))$

c. Only $[\text{Socrates}]_F$ is wise.

d. $\text{only}_x(\partial[?xW(x)] \wedge \exists x(s = x))$

The relevant observation due to Gerhard Jäger in (1996) is that the answers in (13) and (14) although identical at the surface have different meanings. (13c) means 'nobody but Socrates is wise'; (14c) means 'there are no wise Athenian but Socrates'. We have a straightforward account of this contrast, as illustrated by the valid entailments in the following fact.

Fact 4

(i) (13d) $\models \forall y(W(y) \rightarrow y = s)$

(ii) (14d) $\models \forall y((W(y) \wedge A(y)) \rightarrow y = s)$

In the first dialogue, the quantification is automatically restricted to wise men, in the second to wise Athenians. Our system matches Jäger's (1996) predictions on these cases. Example (14) shows that domain restrictions can be constructed by combining constraints that arise from different sources. The restriction on x triggered by the focus is combined with the restriction placed by the which-clause in the preceding question. This same context-dependence of interpretation also allows us to account for Rooth's problem, as noted by von Stechow, discussed in the introduction.

[7] The simple analysis of which-interrogatives assumed in (14) is not satisfactory, but it suffices to make the point here. See the next application for a better analysis.

3.5 Which-questions

In this last section we will discuss a further application of the present semantics which concerns which-questions. According to Groenendijk & Stokhof's (1984) question semantics, a question like (15a) is represented as (15b).

(15) a. Which men are bald?

b. $?x(M(x) \wedge D(x))$

This gives rise to the prediction that (15a) is equivalent to (16):

(16) Which bold persons are men?

However, this seems wrong for *indirect questions*, i.e. questions embedded under verbs like *know*, as observed by Bäuerle & Zimmermann (1991), but also for *direct* questions as illustrated by the following pair due to Stanley Peters (see also Higginbotham (1996)):

(17) Which men are bachelors?

(18) Which bachelors are men?

Intuitively, these two questions are different. While the first question is answered by a characterization of the bachelors among the men, the second is a trivial question. As Groenendijk & Stokhof (1997) note, the 'search routines' for (17) and (18) seem different: whereas (17) suggests to search among the men to find the bachelors, (18) suggests to search among the bachelors to find the men and therefore it is a trivial question. This difference in search procedure is closely related with the syntactic notion of *D-linking* (Comorovski (1985) and Pesetsky (1987)). A Discourse-linked (D-linked) *wh*-phrase is a phrase whose denotation ranges over a contextually given set. It is commonly assumed that *which*-phrases are always D-linked. To account for the difference between (15a) and (15b), we will assume that a D-linked *wh*-phrase – so a *which*-phrases in particular – gives rise to the presupposition that the set over which it ranges is already given as a *topic*. To account for this we will represent *which*-questions as follows:

(19) a. Which men are bald?

b. $?y_x(\partial[?xM(x)] \wedge D(y))$

Where $?y_x\phi$ is short for $y \leftarrow x \wedge ?y\phi$. We can now distinguish between (17) and (18) which are represented as follows:

(20) a. Which men are bachelors?

b. $?y_x(\partial[?xM(x)] \wedge B(y))$

(21) a. Which bachelors are men?

b. $?y_x(\partial[?xB(x)] \wedge M(y))$

Although the two questions determine the same partition, under the assumption that in all worlds all bachelors are men, (21) is vacuous whenever defined. In distinction with (20) which is not a trivial question.

Fact 5

(i) $\not\models ?y_x(\partial[?xM(x)] \wedge B(y))$

(ii) $\models ?y_x(\partial[?xB(x)] \wedge M(y))$

4 Conclusion

In this article we have analyzed within dynamic semantics how questions can restrict the domain of quantificational sentences used later in a discourse. We have done this by extending Gawron's (1996) dynamic model of domain restriction with an explicit analysis of questions. The obtained semantics incorporates Groenendijk's (1999) Logic of Interrogation, but improves on it by introducing (basically) the abstracts underlying the questions to the discourse. In this way we are able to account for the context-dependent meaning of constituent-answers, free focus, association with focus and how the domain of focus sensitive operators like *only* can be determined by a previous question.

Traditionally, linguists have thought in two different ways of a topic: either as the question under discussion, or as the individual talked about. By analyzing questions as discourse referents in an environment, our dynamic analysis suggests that both ways of thinking are two sides of the same coin.

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Conceptual Structure of Reflexive and Middle (On the Georgian Data)

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Abstract

Formal unification of semantically different verb forms – passive, potential, reciprocal, reflexive, subjective version (resp. middle) – is characteristic for the Georgian language. It is also well known for: Indo-Germanic, Indo-Iranian, Ancient Greek and others. Thus, the question naturally arises what is the cognitive background for such cross-linguistically isomorphous interjections of categories and does there exist any conceptual proximity between them? This paper examines the morphosyntactic characteristics and semantic properties of these categories on Georgian data and argues for a conceptual structure which clarifies these issues. The categories are described as the whole continuum of adjacent conceptual structures which are variously divided by the marked-unmarked binary formal oppositions and are represented differently in the linguistic structures of natural languages. The main semantic features in the suggested conceptual structures are 'introversion' and 'extraversion'. According to these features, the conceptual proximity of the above mentioned categories is considered to be the result of either conceptual extension via alienability of the object or conceptual contraction via inherence of the object. This is directly connected with the process of extension or contraction of space relations in the process of linguistic cognition of the world. Different stages of conceptual extension (or contraction) are grammaticalized variously in the natural languages and the process creates the basis for the different marked formal models.

Keywords: conceptual structure, deponenses, extraversion, introversion, linguistic cognition, middle, passive, potential, reciprocal, reflexive, version

I. Introduction

Georgian is one of the Caucasian languages with the oldest literary traditions. The Georgian script was devised around 400 AD in order to facilitate the dissemination of Christian literature. It should therefore offer us the unique opportunity to study a history of 1.500 years of polysynthesis. The `Georgian` population from the last Soviet census of 1989 is 3.787.393. Georgian and its related languages, Laz, Megrelian and Svan are known as the South Caucasian or Kartvelian languages. Georgian (as far as Laz, Megrelian or Svan) personal verb-marking mirrors the system of the Kartvelian protolanguages and mostly has been stable over this time.

II. The structure of the Georgian verb

Georgian has very complex and complicated verb categories. Structurally a Georgian verb may incorporate the following elements:

(1) Preverb (s) - (2) S/O agreement prefix - (3) Version vowel - (4) Root - (5) Passive formant (-d-) or Causative suffix (in) - (6) Thematic suffix - (7) Past marker - (8) Tense/mood vowel - (9) S agreement suffix - (10) Plural suffix.

E.g. da - g - a - c'er - in - eb - d - e - s
prev - OII- vers - write - cause-them- past- mood- SIII
'It would be great if s/he makes me write it'

da - g - a - c'er - in - eb - d - a - t
prev- OII - vers - write - cause - them - past - SIII - pl(O)
'S/he would make you (pl) write it'

III. Version forms

The specific category of Version distinguishes the orientation of subject action:

a. If the subject is acting for somebody: a result of his action belongs to or is intended for the indirect object , the verb has prefixes u- (in case indirect object is III person) or i- (in case indirect object is either I or II person):

m - i - xat' - av - s "(s)he draws it for me"
indO-version-draw-thematic suf.-SIII
g - i - xat' - av - s "(s)he draws it for you"
indO-version-draw-thematic suf.-SIII
u - xat' - av - s "(s)he draws it for him/her"
version-draw -thematic suf.-SIII

This type of version is called the Objective Version.

b. If subject is acting for itself: a result of his action belongs to or is intended for the subject itself, the verb has prefix i- :

v - i - xat' - av " I draw it for myself"
SI - version - draw -them.suf.
i - xat' - av "You draw it for yourself"
version-draw-them. suf..
i - xat' - av - s "(s)he draw it for him(her)self"

This type of version is called the Subjective Version.

c. If there is not such orientation of the subject action, the verb does not have prefixes or sometimes vowel a- appears:

xat' - av. - s "(s)he draws it"
draw -them. suf. -SIII
a - shen - eb - s "(s)he builds it"
version-draw-them.suf.-SIII

Such forms are neutral according to "possessive-intended" relations and, consequently, are called the Neutral Version.

IV. Reflexive forms

The definition of subjective version shows that this category is compatible with the Reflexive, which usually is represented by the reflexive pronoun tav- (grammaticalized lexical entity for body-part "head") preceded by the appropriate possessive pronoun:

me v - xat' - av chem - s tav - s " I draw myself"
I(nom) SI - draw-them. my-dat head-dat
shen xat' - av shen - s tav - s "You draw yourself"
You(nom) draw-them. your-dat head-dat
is xat' - av - s tav-is tav - s "(S)he draw him(her)self"
(s)he(nom) draw-them-SIII head's head-dat

1. When S=indO, the subjective version is obligatory, while the reflexive phrase does not appear:

me v - u - xat' - av chem-s tav - s surat - s ----- me v - i - xat' - av surats
I(nom) SI-vers.- draw- them my-dat self-dat picture-dat ---- I(nom) SI-vers.-draw-th.
"I draw a picture for myself"

2. When S=indO= dirO, the subjective version appears together with the reflexive pronoun:

me v - i - xat' - av (chem-s) tav - s "I draw myself (for myself)"
I(nom) SI-vers.- draw- them. (my-dat) self-dat

me v - i - k - eb (chem-s) tav-s "I appreciate myself (for myself)"
I SI - vers- appr. - them (my-dat) self- dat

3. When S=dirO, the reflexive phrase is necessary:

me v - xat' - av chem-s tav-s "I draw myself"
I(nom) SI-draw-them my-dat self-dat

If dirO is part of the subjects body or belongs to the subject, the subjective version is obligatory and there is no reflexive phrase:

me v - i - ch'ri (xels) " I cut (my hand)"
shen i - p'ars - av (c' vers) " You shave (your beard)"
is i - ban - s (p'irs) "S/he washes (the face)", (but: is i - ban - s "S/he bathes")

The subjective version is typologically compatible with Indo-Germanic Middle: It has similar functions and is used when a part of subject's body or his clothes, shoes, jewels and so on -- are objects affected by the verbal action. But there are important differences. These differences could be the result of the monopersonality of the Indo-European verb which does not allow the formation of a category similar to the Georgian Version. In Georgian on the basis of polypersonal verb forms the reflexive which is widened to the subjective version gives the opposition with objective and neutral versions and, thus, has the particular features we noted.

V. Functions of i - prefix

Generally speaking, verbal prefix i- is polyfunctional. It shows different grammatical categories:

1. Monopersonal passives

bavshvi i - zrd-eb- a "A child grows up"
q'vavili i - shl - eb - a "A flower blooms"
bich'i i - mal - eb - a "A boy conceals himself"

2. Potential

i - ch'm - eb - a "It is eatable"
i - sm - eb - a "It is drinkable"

In the constructions of monopersonal passives and potentialis the conceptual Agent disappears.

E.g.: mshoblebi zrdian bavshvs -- bavshvi i-zrdeba mshoblebis mier -- bavshvi i-zrdeba (' Parents bring up a child' --' A child is brought up by the parents' -- ' A child grows up');

bich'i c'ers c'erils -- c'erili i-c'ereba bich'is mier -- c'erili i-c'ereba(' A boy writes a letter' -- 'A letter is written by the boy' -- ' A letter is written');

stumari svams ghvinos -- ghvino i-smeba stumris mier -- ghvino ismeba (' A guest drinks the wine'-- 'The wine is drunk by the guests' -- 'The wine is drinkable';)

gogo ch'ams sach'mels -- sach'meli i-ch'meba (goos mier) -- sach'meli i-ch'meba ('A girl eats some food' -'Some food is eaten by the girl' -- 'The food is eatable').

The process of Agent demotion (and sometimes its disappearance) is the main general feature of passivization. So, the i-prefix here could be considered as the marker of Agent disappearance.

3. Deponenses

i - gin - eb - a "S/he uses bad language"
i - q'ep - eb - a "A dog (s/he) barks"
i - kbin-eb - a "A dog (s/he) bites"

In these constructions verbs have the forms which are characteristic for the monopersonal passives in present: i-prefix - root - thematic suffix (-eb). The verb is active in meaning, but passive in form (so called, deponenses). Structurally these constructions are far from prototypical passives -- there are no respective active constructions from which these forms could be derived by Agent demotion and Patient promotion. The only feature uniting them with passives is the 'disappearance of the main semantic role': in case of passives the Agens disappears and in case of deponenses the Addressee disappears.

E.g. k'aci k'acs aginebs – k'aci i-gineba (The man abuses the man – A man uses bad language)

dzaghli k'bens bavshvs – dzaghli i-k'bineba (The dog bites the child – A dog bites)

Thus, i-prefix here could be regarded as a functional marker, which formally represents the disappearance of the main semantic role and by this functional approach deponenses are compatible with passives.

4. Reflexives

i -varcxn - i -s "S/he does her(his) hair"
i - ban - s " S/he washes"

In the reflexive constructions Patient, which is the strongest role from the view point of disappearing, is functionally distinguished, as far as it is always represented by the reflexive phrase (one's self), which triggers only III person agreement on the verb form. This means, that the Patient in reflexive constructions is highly restricted and, therefore, actually has functionally disappeared.

5. Subjective version

i - zrd - i - s "S/he brings up (the child) for her(him)self"

These constructions are formed as the result of Addressee's disappearance:
E.g. ashenebs k'aci saxls xalxistvis (neutral version) // ushenebs k'aci saxls xalxs (objective version) (' A man builds a house for people')

When S=indO (that is, Agens and Addressee have the same referent), Addressee (= indO) usually disappears:
ashenebs k'aci saxls tavistvis (neutral version) // ushenebs k'aci saxls tavis tavs (objective version) // i-shenebs k'aci saxls (subjective version) ('A man builds a house for himself')
In the Georgian language, constructions with the subjective version are used with preference. In these constructions the Addressee has fully disappeared.

6. Objective version:

m - i - zrd - i - s "S/he brings up (the child) for me"

Concluding from the semantic analysis of above mentioned morphosyntactic processes and on the basis of the functional approach such kind of generalization could be suggested:

'i-prefix is a marker of functional disappearance of one of the main semantic roles (Ag, P or Ad).'

VI. Conceptual proximity of passives, reflexives and middle

Formal unification of semantically different categories -- passive, potentialis, reflexive, subjective version (resp. middle) -- is not specific to the Georgian language. It is well-known in the Indo-Germanic and Indo-Iranian languages, in Ancient Greek and other languages. Thus, the question naturally arises what is the cognitive background for such cross-linguistically isomorphous interjections for these categories and does there exist any conceptual proximity between them?

The passive construction reflects the situation where the pragmatic subject is not the semantic Agent: from the communicational point of view Agens is demoted and Patient is promoted (that is, it is given the function of subject). Promoted subjects are not prototypical subjects -- they are not active and their 'action' is not directed or oriented to objects. They are introversive: the subject acts within its own space.

Introversion is the main feature for reflexive and middle subjects, but their introversion is realized through objects: the subject crosses its space, affects an object and this object belongs to it (in case of subjective version) or is the subject itself (in case of reflexive).

Such conceptual proximity of these categories according to the feature 'introversion' could be considered to be the result of either conceptual extension via the alienability of the object or conceptual contraction via inherence of object. This is directly connected with the process of extension or contraction of space relations in the process of linguistic cognition of the world and could be represented by the following scheme:

passives reflexives reciprocal subjective objective version

I II III IV V VI VII

I/II III

Introversion

Extension of cognition -- crossing subject's space through inherent objects to external objects

Extraversion

Contraction of cognition -- narrowing of space through external objects to inherent objects (resp. subjects)

The process of extension goes step by step from passives ('Subject acts within its own space') (I stage) through I-reflexives ('Subject acts on his own body part') (II stage) and II-reflexives ('Subject affects an object which denotes subject's own things: clothes, shoes, jewelry and so on) (III stage) to middle ('Subject affects an object which is not in its space (that is, subject crosses its space) , but it tries to put the object in its own space') (V stage).

The IVth stage reflects the intermediate step from reflexive to middle -- Reciprocals ('Subject affects an object and the object similarly affects the subject'. E. g. 'I love you' and 'You love me' gives 'We love each other')

This process of extension continues and as a result the forms of objective version arise (‘Subject affects an object which is not in its space (subject crosses its space). Affected object belongs to or is appointed to somebody (or something).) Introversion turns to extraversion.

The process of extension could be considered as the opposite process of contraction from VII stage to I stage. It depends on the language type.

These stages could be regarded as the different stages of diachronic language change, but this is not obligatory: the scheme merely represents the dynamic process of development of linguistic cognition.

VII. Georgian data

Every language chooses its own way of coding these conceptual structures.

In Georgian the I-II-III-V-VI stages are united into one conceptual class and make an opposition to VII stage: passives, I-II reflexives, (no reciprocal), subjective version and objective version (when O is either I or II person) have i-prefix in the verb form, while other form of objective version (when O is III person) has u-prefix.

It seems interesting, that the objective version shows the person dichotomy I/II. III. VI stage in conceptual representation might reflect the fact that introversion is defined within this dichotomy and opposition S:O is not relevant for I or II persons: If action is oriented to I/II person (disregarding their roles -- Agens, Patient or Addressee; or functions -- S, dirO or indO), it is regarded as introversive and the verb, consequently, has i-prefix:

m-i-xatav-s ‘S/he draws it for me’, g-i-xat’v-s ‘S/he draws it for you(sing)’

According to these structures Georgian linguistic cognition defines actions as extraversive iff the subject affects a IIIrd person Patient.

Such peculiarity proceeds from the typological characterization of the Georgian language:

Georgian data clearly show that I/II - III person dichotomy is dominant in the system of grammatical categories. This dichotomy governs case shift and verb concord, also arguments competitive situation constraints. It defines polypersonality of the verb form, the category of ‘direction’ and so on. The Georgian language strives for formal markedness of I/II - III person dichotomy and within such tendencies the VI stage of our conceptual relations seems to be necessarily distinguished from VII stage.

In Georgian IV stage (= reciprocal) is not grammaticalized in the verb form; instead, the special constructions with the pronoun ‘ertmaneti’ (each other) are used. Reciprocal pronoun ‘ertmaneti’ is derived from the numeral ‘erti’:

ert-man-ert-i (one-erg-one-nom ‘each other’)

E.g. bavshv-eb-s uqvar-t ertmanet-i ‘Children love each other’
child-pl-dat love-SIIIpl each +other-nom

megobr-eb-i   icn-ob-en ertmanets ‘Friends know each other’
friend-pl-nom know-them.-SIIIpl   each+other-dat

st’udent’-eb-i she-xvd-nen ertmanet-s ‘Students met each other’
student-pl-nom pev-meet-SIIIpl   each+other-dat

The main feature which makes distinction between reciprocals and all other constructions is the interruption of introversion (note IV stage in the scheme). Thus, i-prefix is characteristic for the verb forms which denote uninterrupted introversive actions.

The continuum of I--VII stages is divided as well into I: II-III-V-VI-VII opposition by S-III person suffixes in present: passive are marked by the a-suffix, while the s-suffix is characteristic for the active verb forms.

Thus, the Georgian language distinguishes the marginal stages of a conceptual continuum: I stage by suffixes and VII stage by prefixes.

VIII. Generalization

On the base of above functional and conceptual approaches one common generalization could be formulated:

‘the i-prefix is present in the verb form when it denotes continuous introversive situations reflected by the linguistic structures where one of the main semantic roles (Agent, Patient or Addressee) has disappeared.’

IX. Different possibilities of coding

There are other possibilities of marking of these conceptual relations.

In Russian I-II-III-IV stages of continuum are distinguished from other stages by the ending   -sya:

I stage: pishetsya ‘ It is written’
II stage: moetsya ‘S/he washes her/himself’
III stage: odevaetsya ‘S/he puts smth. on’
IV stage: vlyublayutsya ‘They fall in love’

In English only I stage is distinguished by the passive constructions: It is written, It is built, It is known, etc.

In Greek I-II-III-IV-V stages are united in one conceptual class and marked by the suffix -sthai:

ap-hik-e-stai ‘arrive’
thanei-sthai ‘die’
phy-e-sthai ‘grow’
koima-sthai ‘be sleep’
tatte-e-sthai ‘arrange onself’
parakeva-sa-sthai ‘prepare (for) oneself’
komi-sa-sthai ‘bring/carry for oneself’

According to linguistic cognition different stages of object alienability (or inherence) are distinguished from the continuum of conceptual relations and, consequently, different models of formal markedness have been observed.

X. Typological prospects of the conceptual representations

We suppose that our conceptual explanation clarifies the cognitive background of cross-linguistically isomorphous processes of interjection of the different categories such as passive, potentialis, reflexive, reciprocal, middle (=Georgian subjective version).

Methodologically it seems more effective to describe grammatical categories as the whole continuum of adjacent conceptual structures which is divided in different ways by marked - unmarked binary formal oppositions and is represented differently in the linguistic structures of various languages.

Explanatory theoretical approaches could be the basis for further typological studies.

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Logical Construction Games

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Abstract

We define logic games for model construction tasks, inspired directly by Beth's semantic tableaus, and discuss their general properties and potential impact for logic.

1 Logic games

Many logical tasks can be cast in the form of games between two players. Well-known examples are so-called evaluation games between Verifier and Falsifier, model comparison games between Duplicator and Spoiler, of proof games between Proponent and Opponent. Logic games are discussed extensively in van Benthem 2001, whose Chapter 3 is the main source of this paper. Important logical notions then emerge as *winning strategies* of player in such games. Examples are a *proof* for Proponent in an argumentation, a *distinguishing formula* for Spoiler in a comparison between two models, or a *way of refuting* an assertion under evaluation in some model for Falsifier. The idea in designing such games is to take some logical task, such as evaluating a formula in a model, and pull it apart into two opposing roles that interact. Say, someone is trying to show that the formula is true, and someone else tries to show that it is false.

In the case of argumentation and proof, such attempts have met with some obstacles, though. Paul Lorenzen's pioneering work on logical dialogue dates back to the mid 1950s – but his system is somewhat messy, and in need of ad-hoc procedural conventions. Now Lorenzen corresponded with Evert Beth around 1955, and noted the similarity between what he was trying to achieve, and Beth's *semantic tableaus* (cf. van Ulsen 2001). In this note, we will look at semantic tableaus as an instance of yet one more logic game, whose structure is more elegant than that of pure proof games. The reason is this. In Lorenzen games, as in many people's thinking about argumentation, there is just one brave guy, viz. Proponent sticking his neck out and making a claim – whereas the other player is dragging his heels, or even engaging in sabotage to prevent Proponent from achieving his lofty purpose. But this asymmetry is misleading. In practice, two equally 'positive' roles are involved in argumentation, viz. *inference* and *consistency management*.

Consider a case in court. All the evidence is in, and the prosecutor must now show that the accused has committed the crime. That is his *burden of proof*. Inside that same game, however, it is the lawyer's task to come up with a *scenario* in which all the evidence is compatible with her client's not having committed the crime. The latter can be a positive activity as well. Indeed, even in Lorenzen games, it is much easier to understand what Opponent is doing by thinking of some intuitionistic model which she has in mind, where her moves in the game correspond to stepping form some world in that model to another, when shifting the focus of the discussion. Precisely this duality lies at the heart of tableaus – so much so, that we will even give the scenario player the priority in our description. After all, in real life, we are more often a person telling stories to an audience that have to stay consistent than a prover deriving consequences from given discourse. Moreover, model construction seems closer in spirit to the task of making sense of discourse than proof search – although it must be admitted that the latter paradigm also works, e.g., in the parsing-as-deduction, or the interpretation-as-abduction communities.

2 Tableau construction games

Semantic tableaus are a wide-spread method testing for logical validity of an inference by means of a systematic search for potential counter-examples making the premises true and the conclusion false. We assume that the reader is familiar with some version adequate for first-order logic (cf. Hodges 1977). We will cast tableaus as a game between two players, to be dubbed *Builder* and *Critic*. Both of them will have important roles to play.

> At each stage of the game, there are two finite boxes of formulas
> representing Builder's current tasks. One named YES contains
> the formulas to be made true, and NO those to be made false.

Think of Builder as creating *Paradise*, a situation with existential formulas in YES listing forthcoming attractions like apples and trees, and those in NO the snakes to be avoided. The moves of the game decompose complex formulas, or tasks. Play proceeds in *rounds*, each of which starts as follows:

> Critic schedules some formula for current treatment.

After that, Builder must respond in some manner to be defined. There are several options for gamifying tableaus, and we choose just one of these here. But first, some moves may be considered *automatic*, without any player involved at all:

if $\neg A$ sits in one box,	it changes to A in the other box
if $A\&B$ sits in YES,	it is replaced by both A, B separately
if $A \vee B$ sits in NO,	it is replaced by both A, B separately

if $\exists x\phi$ sits in YES,	it is replaced by $\phi(d)$ for some new object d not yet used in any formula in YES or NO
if $\forall x\phi$ sits in NO,	it is replaced by $\phi(d)$ for some new object d not yet used in any formula in YES or NO

In another version, we could give Builder more control over the latter two moves, allowing her to also choose from already available objects. This makes sense, e.g., if we are interested in finding smallest models, and make that part of the evaluation of the game. (As an incentive, we might give Builder a higher pay-off for smaller models.) Next, we list the rules that require deliberate actions by players:

(1) Disjunctions in YES and conjunctions in NO are a *choice* for Builder.

(2) For existential formulas $\exists x\phi$ in NO , Critic mentions some object in the domain under construction so far, and adds $\phi(d)$ to the NO box.
For universal formulas $\forall x\phi$ in YES , Critic mentions some object in the domain under construction so far, and adds $\phi(d)$ to the YES box.

The latter rule is not precise yet. In standard tableaus, Critic's moves do not *replace* the formulas $\exists x\phi$, $\forall x\phi$: these remain available for further calls later in the game. In that case, there is no special advantage to carefully choosing 'the right challenge', and case (2) may then also be considered 'automatic'.

Next, here is the *winning convention* of this game,

> A stage is a *loss* for Builder if some formula occurs *in both boxes,*
> Builder *wins* a complete run of the game if no such loss occurs at any stage.

The reason for this stipulation is clear. No model construction can make a formula both true and false: these are conflicting tasks. Finally, one might stipulate some *procedural conventions* – e.g., restricting Critic's scheduling of the game. Here is one that we will adopt, in order to avoid everybody's waste of time:

> Critic *may not repeat schedulings* that he has already made.

Every semantic tableau for a propositional or predicate-logical satisfiability problem may be viewed as a record of a game like this – but see below for some divergences. Finally, Builder and Critic are not necessarily antagonistic roles (strife may not always be helpful as a way of understanding a logic game). One can also think of Critic as a *manager* making sure that Builder forgets no tasks, and eventually creates a building of the highest quality.

3 Examples: tableaus and game trees

Here are two logical construction problems demonstrating these ideas.

Example 1 Propositional logic

Consider the following initial boxes for Builder:

YES $\neg A \vee B$, $\neg B \vee C$ NO $B \vee \neg(A \vee C)$

First, there is an automatic simplification in the NO box to the disjunction, and then automatic box transfer for one negation, resulting in:

YES $\neg A \vee B$, $\neg B \vee C$, $A \vee C$ NO B

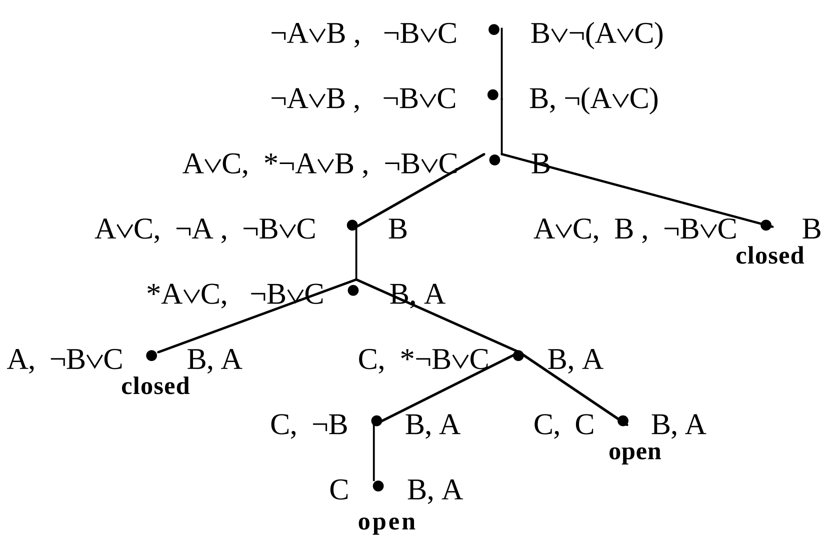
Now, Critic has to schedule assertions in the YES box, but it does not matter how he does this. (We analyze this below.) Say, he selects the first (with the *):

YES $* \neg A \vee B$, $\neg B \vee C$, $A \vee C$ NO B

Builder must choose A or B , giving a branching in the game tree. The second choice is an immediate loss, however, because of the conflict with the NO box. Therefore, the more interesting choice is $\neg A$, which leads automatically to

YES $\neg B \vee C$, $A \vee C$ NO B, A

Suppose Critic now schedules the second disjunct. Builder can choose $\neg B$ which puts B in the NO box: no new task; so perhaps the best move. Alternatively, Builder can put C in the YES box. After that, the last choice is scheduled, which puts either A in the YES box, which is a loss for Builder – or C in the YES box, which ends the game in a win for Builder. Which player (if any) has a *winning strategy* here? We display the complete semantic tableau, which is virtually the same as a game tree:



This tableau remains open, witness the open branches. These yield models solving the original task, making all formulas in YES true, and those in NO false. These models are at the same time Builder's winning strategies in the game tree corresponding to this tableau. Next comes a familiar predicate-logical example.

<u>Example 2</u> Shifting quantifiers

The YES box has the formula $\exists x\ \forall y\ Rxy$ and the NO box has $\forall y\ \exists x\ Rxy$. This should be a hopeless task for Builder, as the inference from $\exists x\ \forall y\ Rxy$ to $\forall y\ \exists x\ Rxy$ is valid. The semantic tableau might unfold as follows, starting with two automatic moves, and then two moves that can be seen as instigated by Critic:

$$\exists x\ \forall y\ Rxy \bullet \forall y\ \exists x\ Rxy$$
$$\forall y\ Rd_1 y \bullet \forall y\ \exists x\ Rxy$$
$$\forall y\ Rd_1 y \bullet \exists x\ Rxd_2$$
$$\forall y\ Rd_1 y,\ Rd_1 d_2 \bullet \exists x\ Rxd_2$$
$$\forall y\ Rd_1 y,\ Rd_1 d_2 \bullet \exists x\ Rxd_2,\ Rd_1 d_2$$

closed

In this case, Critic obviously has the winning strategy.

Tableaus are not yet complete games. In particular, complete game trees for the above examples would have to list *other* admissible *scheduling orders* for Critic. So, let us consider scheduling. First, players must keep moving when they still can. In particular, this forces Critic to examine every task in any finite game, as happens indeed in the propositional case. This feature explains why his scheduling order is not crucial to the outcome of the game. Eventually, every relevant task will be performed. But predicate-logical tableaus can be *infinite*, as some formulas have only infinite models, inducing infinite runs of the game. Tableaus normally impose some task scheduling making sure that every formula is eventually considered along such a branch of the tableau. Otherwise, the branch might remain open even though there is a hidden inconsistency. Nevertheless, we can also leave this open. We are mostly interested in players' winning strategies. The fact that Critic can be sloppy, so to speak, does not affect his winning strategies, which will obviously be more harsh qua scheduling.

This freewheeling attitude toward scheduling might suggest that Critic's precise moves are largely irrelevant. This would make construction games ceremonial disputes with one player doing all the work. There is more to these games, however – and we will return to this later.

4 Theory of tableaus and game theory

The usual adequacy result for semantic tableaus states that the method correctly tests for consistency and validity:

Theorem The following two assertions are equivalent in first-order logic:

(a) The set of formulas $\{A_1, .., A_k, \neg B_1, \ldots, \neg B_m\}$ is satisfiable

(b) There is an *open* tableau with top node $A_1, .., A_k \bullet B_1, \ldots, B_m$

Alternatively, one can also state this equivalence in terms of valid consequence. The following two assertions are also equivalent in first-order logic:

(a) $\&\{A_1, .., A_k\}$ logically implies $\vee\{B_1, \ldots, B_m\}$

(b) There is a *closed* tableau with top node $A_1, .., A_k \bullet B_1, \ldots, B_m$

Proofs are in the standard literature. In particular, they involve the observation that closed tableaus for first-order logic are always *finite*: a fact whose game-theoretic import is stated below. As we said, however, open tableaus may have irreducibly infinite branches. This feature reflects the complexity of the satisfiability or consequence problem for first-order logic, which is *undecidable*. Stated purely in terms of construction games, the above results express an equivalence between the following assertions:

(a) The set of formulas $\{A_1, .., A_k, \neg B_1, \ldots, \neg B_m\}$ is satisfiable

(b) Builder has a winning strategy in the above construction game starting with the A's put in the YES box and the B's in NO

Now let us look at the general situation from a game-theoretic point of view. The Adequacy Theorem implies an important property. As the tableau is either closed or open, either Critic of Builder has a winning strategy:

Proposition Model construction games are *determined*.

This can also be seen on general grounds. Even though tableaus can be infinite, the winning condition for Critic (i.e., finite failure by Builder) defines an *open* set of runs in the sense of the *Gale-Stewart Theorem* – which therefore predicts the determinacy. The core observation follows reflects a general game-theoretic

Fact Either Critic has a winning strategy, or Builder has a strategy making sure that only branches result at no stage of which Critic gets a winning strategy from there on.

The latter strategy is bound to produce a set of branches that remain open at every stage (since no win for Critic occurs): i.e., it is winning for Builder.

What players' winning strategies encode by way of independent logical objects becomes clear from a more concrete analysis of tableaus. Builder's winning strategies are related to open branches, which themselves generate *models* satisfying all initial requirements. By contrast, Critic's winning strategies allow him to end every branch of the game tree at some finite stage by a failure for Builder. Thus, the subtree of the complete game tree played according to that strategy *has no infinite branches*. One can think of this as a 'barrier' through the game tree. But then, Critic must have a *finite procedure* for achieving this – by contraposition of the following well-known mathematical result:

König's Lemma   Each finitely branching infinite tree has an infinite branch.

This explains why winning strategies for Critic are associated with finite objects, namely 'closed tableaus'. And we know what the latter correspond to. They are proofs of the positive sequent corresponding to the initial formula

$$A_1 \,\&\, \ldots \,\&\, A_k \;\Rightarrow\; B_1 \vee \ldots \vee B_m$$

Summing up, we have found the following connection:

Theorem   Construction games have an explicit correspondence between

(a) winning strategies for Builder,

(b) *models* for the given formulas

Likewise, construction games have an effective correspondence between

(a') winning strategies for Critic,

(b') *proofs* for the initial sequent.

Thus, the single notion of strategy for two players in one single game 'unifies' the notions of *model* and *proof*. These are normally taken to represent vastly different aspects of logic – while here, they meet directly. This double aspect was also present in Beth's original description of tableaus, where he related the two directions to 'analysis' and 'synthesis' as general practices in mathematics, emphasizing the complementary methods involved.

Our own view is that tableaus are eventually a kind of *task decomposition games*. Their general game-theoretic properties presuppose very little about the details of these tasks – but we shall now discuss some specifics.

5 Making Critic more essential: fragments and variations

In the propositional case, tableau games seem rather lifeless - as Critic is not really doing much. Nothing depends on his precise choice of scheduling. This is borne out by the *complexity* of the propositional satisfiability task, which is in **NP**. This means that, in order to win, Builder has to choose 'something good', which is of course, an open branch, or a lucky valuation satisfying all initial YES and NO demands. In that sense, we might just as well see the problem as a one-person game, where a winning strategy is one branch with this **NP** character. But things are different with full predicate logic, as Critic has to schedule in the right way in order to make every branch end at some finite stage (if possible at all).

But more interesting are intermediate cases. Consider *modal logic*, viewed as a bounded-quantifier fragment of the full first-order language (cf. Andréka, van Benthem & Németi 1998.). We know that the complexity of satisfiability is **PSPACE** for the basic modal logic, which is also the complexity of many genuine two-player games. The reason is as follows. In tableaus for modal logic, quantifiers are treated in a special fashion. Stated in terms of the universal modality only – for convenience – the instruction is as follows:

> Analyze the tableau at any world stage w as far as possible through propositional rules. If no closure occurs, then look at the remaining set of box formulas $[]\phi$ in YES, and the box formulas $[]\alpha$ in NO:
>
> For each of the latter $[]\alpha$, pick a new R–successor t of w, starting there with α in NO, and all ϕ of the first kind in YES

This fan-out of successors is a universal requirement for Builder, and the game gives Critic the choice which new world to open. This time, being able to cope with any challenge by Critic is essential – as is the fact that each challenge be handled separately, without interference from other parts of the tableau.

But also in full first-order tableaus, it is quite possible to let Critic do real work in a finite setting. Consider the following natural variation on the game. All rules become *one-shot*. In particular,

> For existential quantifiers in NO or universal ones in YES, Critic gets to issue one challenge: an object to be plugged in at some stage. The original formula is then discarded.

In purely propositional tableaus, this makes no difference, and neither does it for our modal tableaus – where the above rule had this one-shot character already.

But in full predicate logic, we see great differences now. Note that the second example of Section 3 now depends crucially on Critic's choices. Critic would lose in the following play:

A

$$
\begin{array}{rcl}
\exists x\, \forall y\, Rxy & \bullet & \forall y\, \exists x\;\, Rxy \\
\forall y\, Rd_1\, y & \bullet & \forall y\, \exists x\;\, Rxy \\
\forall y\, Rd_1 y & \bullet & \exists x\;\, Rxd_2 \\
Rd_1 d_2 & \bullet & \exists x\;\, Rxd_2 \\
Rd_1 d_2 & \bullet & Rd_2 d_2
\end{array}
$$

open

Here is are two very simple illustrations in predicate logic that used to be valid, but are no longer guaranteed wins for Critic in the one-shot regime:

B

$$
\begin{array}{rcl}
 & \bullet & \exists x\, (\, \exists y\, Ay \rightarrow Ax) \\
 & \bullet & \exists y\, Ay \rightarrow A\, d_1 \\
\exists y\, Ay & \bullet & Ad_1 \\
Ad_2 & \bullet & Ad_1
\end{array}
$$

open

In standard tableaus, one would *repeat* the instruction for $\exists x\, (\, \exists y\, Ay \rightarrow Ax)$, plugging in the new object d_2 to close the tableau. Here is a similar illustration:

C

$$
\begin{array}{rcl}
\exists x\, \exists y\, \exists z\, (\, Ax\ \&\ Rxy\ \&\ Ryz\ \&\ \neg Az) & \bullet & \exists u\, \exists v\, (\, Au\ \&\ Ruv\ \&\ \neg Av) \\
Ad_1\ \&\ R\, d_1 d_2\ \&\ R\, d_2 d_3\ \&\ \neg Ad_3 & \bullet & \exists u\, \exists v\, (\, Au\ \&\ Ruv\ \&\ \neg Av):
\end{array}
$$

Whatever Critic chooses for u and v here, Builder can always maintain a consistent claim. To demonstrate the classical validity, one needs to make a case distinction for the middle object , and depending on that, make a choice for u and v . In a standard tableau, this intuitive point is washed away by instantiating u and v a number of times, so that Builder will drown in the total requirements.

One-shot games represent a finer level of predicate-logical validity. In particular, it is easy to see that tableaus of this sort remain *finite*. This demonstrates a

<u>Theorem</u> The validities of one-shot predicate logic are *decidable*.

These one-shot validities can be axiomatized as follows. Proof-theoretically, the repetitive character of the quantifier rules has to do with the structural rule of *Contraction*, allowing us to treat more occurrences of the same formula as a single one. This rule is crucial here. E.g., consider an earlier example, but now duplicated. The following tableau does have a winning strategy for Critic!

$$
\begin{array}{rcl}
 & \bullet & \exists x\, (\exists y\, Ay \to Ax), \exists x\, (\exists y\, Ay \to Ax) \\
 & \bullet & \exists y\, Ay \to A\, d_1\, , \exists x\, (\exists y\, Ay \to Ax) \\
\exists y\, Ay & \bullet & Ad_1\, , \exists x\, (\exists y\, Ay \to Ax) \\
Ad_2 & \bullet & Ad_1\, , \exists x\, (\exists y\, Ay \to Ax) \\
Ad_2 & \bullet & Ad_1\, , \exists y\, Ay \to A\, d_2 \\
\exists y\, Ay, Ad_2 & \bullet & Ad_1\, , A\, d_2 \\
 & \textbf{closed} &
\end{array}
$$

Thus, the valid sequents can be axiomatized with a Gentzen calculus using all the usual 'logical rules' of predicate logic, plus all the standard 'structural rules' minus Contraction Tableau games stay finite, decidable, and axiomatizable with a fixed finite bound on repetitions of the same call in a game (Prijatelj 1995).

One-shot validity has a number of interesting features. First, there are quite a few cases where it does not diverge from classical validity. One example are inferences between existential formulas with only conjunctions, as in 'conceptual graphs' (Kerdiles 2001). In that case, a conclusion of the form

'$\exists x_1 \ldots \exists x_k$: conjunction of atoms'

follows from a set of premises of the same shape iff it holds in the minimal graph model of the premises – which takes just a one-shot choice of witnesses. But even atomic negations already spoil the picture: see the above example C. Another interesting case where one-shot calls or bounded finite numbers of calls are sufficient is *modal logic*. This is in fact clear from the above tableau instruction for modal operators (cf. Andréka, van Benthem & Németi 1998).

Now, one objection to all this might be that one-shot validity has *no semantics*. But this criticism is question-begging. It presupposes that standard semantics is the ultimate test for a game-theoretic account of validity – even though that order of priorities is precisely what is at issue in game-theoretic analysis of basic logical notions. In any case, there is an independent general concept behind one-shot predicate logic, viz. the view of formulas as *resources* that can be consumed only once – in this case, game tasks. This interpretation is close to *linear logic* (cf. Abramsky & Jagadeesan 1994), and indeed, the parallels are striking.

As in linear logic, our new look at the classical instruction for treating quantifiers in tableaus suggests *redesign of the first-order language*. If a task can be rescheduled indefinitely often, this should be marked explicitly by a

iteration operator $\mathbf{!}\phi$

creating special versions of ϕ that can be scheduled an arbitrary finite number of times, depending on Critic's decision. This iteration operator makes a decidable system like one-shot predicate logic undecidable – as it leads to the resources for faithfully embedding full predicate logic. The new syntax is richer in effect than the standard first-order language, as we can 'iterate' arbitrary formulas – not just quantified ones, and that in both boxes. But this does not seem to increase expressive power. Thus, games suggest new logical operators.

On the other hand, the operator **!** *need* not produce undecidability. Added to the modal fragment, it yields a version where modalities with universal force keep recurring through successor worlds, and so on. This leads to a still decidable bimodal system with one minimal modality, and one K4-modality accessing the transitive closure of the alternative relation R. Similar roles for iteration operators arise in *intuitionistic* propositional logic (again of **PSPACE-**complexity with a genuine role for Critic), regulating the action of implications and negations as one proceeds through a Kripke model.

6 Conclusion and further directions

The game view of tableaus provides a nice unified framework for discussing two logical tasks that are intuitively intertwined: proof, and model construction. It is also less ad-hoc than proof games of earlier varieties. Moreover, the framework suggests interesting variations that make sense from a game perspective, such as one-shot predicate logic. This paper has only scratched the surface though. E.g., a closer comparison with the procedural details of Lorenzen games seems of interest – treating Proponent in a gentler guise as a scheduler for Opponent, who is following hunches as to models of the situation. Next, at several points, a richer *pay-off structure* would make sense. For instance, if termination is our main concern, we could reward players for reaching outcomes along shortest paths. By making players prefer wins or losses that are reached quickly to those which take longer, one gets more interesting *strategic equilibria* for logic games. If you have a winning strategy, but I can choose where my defeat occurs, I will go for the quickest one, exercising a real power which I have in the game. On the other hand, infinite branches represent uninterrupted communicative interaction between two players, which would represent a good by another pay-off measure. (Cf. the linear logic games of Abramsky & Jagadeesan 1994.) Finally, infinite runs suggest more complicated pay-offs, as in repeated games with discounts of outcomes (Osborne & Rubinstein 1994). It would be of interest whether this makes sense for tableau games, or other logic games allowing infinite runs.

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Relevance semantics: an approach to intensionality *

Howard Gregory

Abstract

This paper introduces a semantic system based on relevance logic (hence "relevance semantics"). It is motivated by the inability of a Montague-style semantics based on the modal notion of strict implication to distinguish between mutually entailing expressions, which becomes a problem in "hyperintensional" contexts. Relevance logic isolates implication (rather than the necessity operator) as the locus of the problem. By dropping the structural rule of Weakening, it provides a consequence relation which is not truth-functional, thus avoiding the "paradoxes of strict implication". In the semantics, the absence of Weakening gives rise to partial and inconsistent points of evaluation ("worlds"). Hence, logical truths do not hold at all indices and mutually entailing expressions do not have the same extensions. The paper includes an application of this semantics to attitude reports, which is intended to preserve the insights of existing approaches, while avoiding the problem of hyperintensionality.

1 Introduction

1.1 The background

It is well known that the possible worlds semantics of Montague (1974) fails to distinguish expressions which have the same extension at all indices. This may arise because the expressions are logically equivalent or because they are made equivalent by constraints (meaning postulates).

This problem of "hyperintensionality" has been approached in a variety of ways. Following Lappin and Pollard (2000), I assume that when sufficiently formalized, the only promising approaches boil down to two. One is

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to reverse the Montagovian perspective by taking propositions as primitive and making indices and other extensional objects derivative. Algebraically, this involves declining to equate logically equivalent sentences under the equivalence relation of mutual entailment, thus working with a pre-algebra instead of the familiar Lindenbaum algebra. This approach is developed in Fox, Lappin and Pollard (2001). The other approach is to refine the notion of possible worlds so that logically equivalent propositions no longer have the same denotations. Thus situation semantics (Barwise and Perry (1983)) introduced partial worlds (situations) to model incomplete information. Muskens (1995) extends the idea from partial to impossible worlds (formalized using multiple truth values). Orthogonally to this distinction, situation semantics is based on an independent theory of information termed "ecological realism" rather than being the interpretation of a logical language, while Muskens reverts to a logical perspective closer to Montague's.

The present proposal is located in the second of these two traditions. In particular it is closely related to the programme of Muskens, in that it is both logic-based and invokes a notion of impossible worlds. However, both the logic and its semantics are set up differently. The logic adopted is **relevance logic** (Anderson and Belnap (1975), Dunn (1986)). This provides a non-truth-functional notion of implication, avoiding the paradoxes of both material and strict implication. It is interpreted here using a frame semantics (the Routley-Meyer semantics[1]). This is related to the frame semantics used for modal or intuitionistic logic, but contrasts with them in that logical truths do not hold at all indices and mutually entailing expressions do not have the same extensions. As will be described, this arises in a principled way from the logic. The semantics is more convenient than that of Muskens in a number of respects: first, because of its relation to frames for other logics, which facilitates comparison; and also because of an important connection with channel theory, a development from situation semantics (Barwise (1993), Barwise and Seligman (1997)). This connection has been noted in the literature, and is explored especially in work by Restall (e.g. 1994) and Mares (1997). These writers, however, advocate the use of weaker substructural logics, while relevance logic differs from full intuitionistic logic only in rejecting the single structural rule of Weakening. The perspective of this paper is that this is the heart of the intensionality issue.[2]

[1]Due to Routley and Meyer (1973).

[2]For Contraction, see section 4 below.

1.2 The problem of intensionality

Natural language contains certain intensional or "opaque" contexts, where although A may imply B, it is not the case that an $\Phi(A)$ implies $\Phi(B)$ (where $\Phi(B)$ is identical to $\Phi(A)$ but with B substituted for at least one occurrence of A). Typically, the embedding context Φ represents some predicate of propositional attitude.

Using material implication (1), if A is true then it is implied by any B (**irrelevance**), while if A is false it implies any B (**explosion**). However an agent accepting or rejecting A should not be committed to any such implications, and any report that embeds such an implication inside a propositional attitude will give the wrong truth conditions.

(1) Material implication: $A \supset B = \neg A \vee B$

(2) Strict implication: $A \prec B = [](A \supset B)$

Montague's approach to this problem is to adopt an (S5) modal logic, in which propositions are modelled as sets of possible worlds, and a notion of strict implication (entailment) is obtained by prefixing a necessity operator to a material implication (2). The problem is that the same problems of irrelevance and explosion reappear, whenever (respectively) the consequent is a *necessary* truth or the antecedent a *necessary* falsehood. Traditionally Montague's semantics is called "intensional", and these problems which it fails to solve are described as "hyperintensional".

(3) If John believes [colourless green ideas sleep furiously] then John believes [Mary is pregnant or Mary is not pregnant].

(4) If John believes [colourless green ideas sleep furiously] then John believes [if Mary is pregnant then Mary is pregnant].

(5) If John believes that [Mary is pregnant and Mary is not pregnant] then John believes [the earth is flat].

In (3), the embedded proposition in the consequent is a tautology (the Law of the Excluded Middle, **LEM**, is a tautology in normal modal logic), and is entailed by anything. A partial answer to the problem may lie in taking the content of John's belief to be a situation - a point of evaluation in a partial logic, where not all issues are decided at all points. This would be the approach of situation semantics, or of intuitionistic logic. However, a modified form of the same problem would be (4). Each situation still supports logical truths - it just happens that in situation semantics (or intuitionistic logic) LEM is not among them. Furthermore (5) is a problem for all these approaches. No world or situation can support contradictory information, hence contradictions denote the empty set of situations and anything follows (ex contradictione quodlibet, or **ECQ**).

Thus, neither partial nor modal systems as such seem to be able to fix the problem. I assume that the notion of necessity, as formalized in modal logic, is a useful one in natural language semantics (especially epistemic and doxastic applications). The problem seems to lie not with the notion of necessity, but with the notion of implication. The modal operator merely quantifies over worlds in which formulae with material implication and Boolean or intuitionsistic negation are evaluated, and hence there is no way out of inferences such as (4, 5).

What is needed is a logic in which logical truths do not hold in all situations, and which furthermore allows impossible situations. This requires, first, a notion of implication which is not truth-functional, so that $A \rightarrow A$ does not follow from arbitrary antecedents, and second, a treatment of negation such that contradictions do not entail arbitrary consequents. For the latter we have to "weaken the connection between truth and falsity" (Muskens (1995)) to give a paraconsistent negation. The result in the semantics will be to admit both non-normal worlds where logical truths fail and inconsistent worlds where contradictions can hold. These "impossible worlds" will not (perhaps) be part of the real world, but may correspond to confused epistemic states[3]. In such a model each agent is allowed to be confused in their own way, without all such confused states being identified[4].

Thus we need both an intensional notion of implication and a paraconsistent notion of negation. The following sections will comprise an outline, first, of the way these notions are formalized in relevance logic, and then of the frame semantics for that logic. They are based on Dunn (1986), Gabbay and Olivetti (2000) and Restall (2000). Section 4 makes some suggestions as to how this semantics can be applied to propositional attitudes, using the idea of channels as connections between agents and the situations in which propositions are evaluated.

Note that the implication arrow $\rightarrow$ will henceforth be reserved for relevant implication, and $\sim$ for the corresponding negation (De Morgan nega-

[3]This association of impossible worlds with irrationality may be understating the case for them. Barwise (1997) and Restall (2000) both have interesting discussion of the role of impossibilities in rational investigation.

[4]cf. Landman (1986), who attempts to effect this by different means, and Lappin and Pollard (2000) for criticism.

tion), while the "standard" connectives will be notated $\supset$ and $\neg$.

2 Relevance logic

2.1 Relevant implication

The central concept of relevance logic is a notion of implication which is not reducible to a truth function. It encodes the principle that the premises in a deduction must combine in a non-trivial way to produce the conclusion. The natural deduction rules for relevant implication are given in (6). In $\rightarrow$ elimination (modus ponendo ponens), the conclusion "follows from" precisely the combination X,Y of the premise contexts X and Y from which the major and minor premises follow. Conversely, $\rightarrow$ introduction involves deducting the premise context consisting of the single formula A (equivalent to *non-trivially* discharging an assumption).

(6)
1. ($\rightarrow$ **Elim**): If $X \vdash (A \rightarrow B)$ and $Y \vdash A$, then $X,Y \vdash B$
2. ($\rightarrow$ **Intro**): If $X,A \vdash B$, then $X \vdash (A \rightarrow B)$

This non-triviality condition prevents implication from collapsing into material implication. In terms of structural rules, it amounts to ruling out Weakening. Extra premises which have no role in the deduction cannot be added at will, as in classical or intuitionistic logic where it leads to "irrelevant" deductions such as $A \supset (B \supset A)$, or $\neg A \supset (A \supset B)$.[5]

The implicational fragment of relevance logic rejects Weakening, but retains the other structural rules of implicational intuitionistic logic.[6] Other rules have to be added for extensional connectives, for negation and for modal operators, which will be discussed shortly.

The resulting notion of implication[7] is not truth-functional - it provably depends only on content sharing between antecedent and consequent, not on their truth or falsity - and does not validate the "paradoxes" of material or of strict implication. It is intensional also in that it involves considering more than one premise context; this will be directly reflected in the semantics by evaluation over several worlds using an accessibility relation.

[5] In terms of the correspondence between proof types and lambda terms, a relevant proof must correspond to a lambda term where the variable bound by each lambda actually occurs free in its scope. Weakening corresponds to vacuous abstraction.

[6] Associativity, Permutation and Contraction.

[7] That it is a genuine notion of implication is shown by the existence of a deduction theorem (see Dunn (1986)). The deduction equivalence holds for relevant implication as long as the combination of premises (notated by the comma) is understood in the sense described above, not as conjunction.

2.2 Other connectives

For reasons of space, the logic will be summarized here by a list of axioms[8]. The implicational axioms (7) correspond to structural rules. Connectives other than implication (8-10) are discussed in the following paragraphs. The modal axioms in (11) are simply the familiar axioms for S4.

(7)
1. **Identity**: $A \to A$
2. **Prefixing**: $(A \to B) \to ((C \to A) \to (C \to B))$
3. **Permutation**: $(A \to (B \to C)) \to (B \to (A \to C))$
4. **Contraction**: $(A \to (A \to B)) \to (A \to B)$

(8)
1. **Conjunction elim.**: $A \wedge B \to A$, $A \wedge B \to B$
2. **Conjunction intro.**: $((A \to B) \wedge (A \to C)) \to (A \to B \wedge C)$ (*not* $A \to (B \to A \wedge B)$), which leads to Weakening)
3. **Disjunction intro.**: $A \to A \vee B$, $B \to A \vee B$
4. **Disjunction elim.**: $((A \to C) \wedge (B \to C)) \to (A \vee B \to C)$
5. **Distribution**: $A \wedge (B \vee C) \to (A \wedge B) \vee C$

(9)
1. **Logical truth**: $\mathbf{t} \to (A \to A)$
2. **Trivial truth**: $A \to \top$

(10)
1. **Reductio**: $(A \to \sim A) \to \sim A$
2. **Contraposition**: $(A \to \sim B) \to (B \to \sim A)$
3. **Double negation**: $\sim\sim A \to A$

(11)
1. **$\wedge$-Distribution**: $\Box A \wedge \Box B \to \Box(A \wedge B)$
2. **$\to$-Distribution**: $\Box(A \to B) \to (\Box A \to \Box B)$
3. **Reflexivity**: $\Box A \to A$
4. **Transitivity**: $\Box A \to \Box\Box A$

The connectives $\wedge$ and $\vee$ are "extensional" in the sense that they presuppose a single premise context for both juncts (corresponding to being evaluated in a single world).

This distinction between intensional and extensional extends to nullary connectives (constants). In relevance logic there is an important distinction between the logical truth constant $\mathbf{t}$ and the trivial truth constant $\top$ (top).

[8] With the rules of modus ponens, adjunction and necessitation.

The first represents the conjunction of theorems, and corresponds to the empty premise structure (henceforth 0). $\top$, by contrast, is the trivial truth constant which holds everywhere (follows from all premises)[9]. In standard logic these coincide, but in the absence of Weakening they are crucially distinct[10]. Their negations are written **f** and $\bot$ respectively.

Negation is strict De Morgan negation, characterized by double negation and contraposition, but without supporting either of the complementation laws (Law of the Excluded Middle, LEM, and the Law of Contradiction, which corresponds to ECQ). That is, the logic as set up does not support either of the laws in (12), only the corresponding weaker statements in (13). Thus, LEM holds only at 0; it is a logical truth, but $A \vee {\sim}A$ is not decided by all situations. (This is as in situation semantics.) Moreover, contradictions do not entail any arbitrary proposition but only **f**; the only thing that follows from them is that somebody is being "illogical".

(12)
1. $\top \vdash A \vee \neg A$ - (Strong) Law of the Excluded Middle, LEM
2. $A \wedge \neg A \vdash \bot$ - Ex Contradictione Quodlibet, ECQ

(13)
1. $\mathbf{t} \vdash A \vee \neg A$
2. $A \wedge \neg A \vdash \mathbf{f}$

3 Frame semantics

The standard semantics for relevance logic is a frame semantics similar to the Kripke frames for modal and intuitionistic logic (of which however the present frames are a proper generalization).

A relevance frame comprises a point set $\langle P, \sqsubseteq \rangle$ ($\sqsubseteq$ being a partial order), together with a *ternary* accessibility relation R on P, and a distinguished point $0 \in P$ (which corresponds to the "empty premise structure" 0, and supports logical truths).

A relevance model for a language $\mathcal{L}$ is a relevance frame equipped with a supports relation $\models$ on $P \times Var(\mathcal{L})$, the variables of the propositional language $\mathcal{L}$. Validity in a model is not computed as truth in all worlds, but as truth at 0.

The point set is understood as the set of *non-trivial prime theories* of $\mathcal{L}$, ordered by inclusion. Because of the way the logic has been set up, such

[9] The disjunction of all propositions, hence the top element of the propositional lattice.

[10] Weakening is used in the proof that they are equivalent: $0 \vdash t$ (by definition of t); but then $0, \top \vdash t$ (by Weakening); therefore $\top \vdash t$. In the other direction, $t \vdash \top$ immediately. (Restall (2000))

theories do not have to be complete or consistent. In logics with Weakening and ECQ, they collapse into consistent theories, and with (strong) LEM into complete ones - the situation in intuitionistic frames and normal modal frames respectively[11]. In relevance logic, which rejects Weakening and ECQ, this is not the case. (Although points may be incomplete and inconsistent, they will often be referred to informally as "worlds".)

Apart from the "extensional" connectives $\wedge$ and $\vee$ (and $\top$ and $\perp$), the evaluation clauses for all connectives involve more than one world, and hence invoke some accessibility relation. There are actually several of these (though they are all expressible in terms of the ternary relation R, the key to the whole system).

The ternary relation Rxyz represents combination of worlds (corresponding to the combination of premise structures in the rules for intensional connectives). If the first argument x supports the implication $A \rightarrow B$, and y supports A, then z supports B.

How does this help? In frames for normal modal logic, an implicational formula is evaluated truth-functionally in a single world. Because of this the paradoxes of material implication surface in a modified form, leading to the problem of hyperintensionality. Logical truths hold everywhere, logical falsities nowhere, and logically equivalent expressions at the same indices. Although the evaluation clause for intuitionistic implication is different, the antecedent and consequent are still evaluated relative to the same worlds. In relevance frames, by contrast, implication is evaluated using separate worlds for the antecedent and consequent. Thus $A \rightarrow A$ is not guaranteed to hold in all worlds. Moreover the bi-implication $A \leftrightarrow B$ does not involve A and B picking out the same set of worlds. In general, there is no entailment from a contradiction to an arbitrary formula, nor from an arbitrary formula to a logical truth. Thus the paradoxes of strict implication and the identification of mutually entailing formulae are avoided.

Rxyz can be written $y \sqsubseteq_x z$, emphasizing the idea of x as a connection between y and z. In the case where $x = 0$, the subscript is omitted and we have $y \sqsubseteq z$, the relation of informational extension.

R is constrained to interact correctly with $\sqsubseteq$, by (14). Note that x and y are downward closed, while z is upward closed. The evaluation condition for implication, using R, is (15).

[11] The non-triviality condition means that every theory contains $\top$ and no theory contains $\perp$. With Weakening, $\mathbf{t} = \top$ and therefore every theory contains $\mathbf{t}$. With ECQ, any inconsistent theory supports $\perp$, and so cannot be non-trivial. Similarly, with strong LEM, any theory containing $\top$ (i.e. every non-trivial theory) must contain either A or $\sim$A.

(14) If Rxyz then for every $x' \sqsubseteq x$, $y' \sqsubseteq y$, $z' \sqsupseteq z$, $Rx'y'z'$

(15) $x \models A \to B$ iff, if Rxyz and $y \models A$ then $z \models B$

Other constraints on R correspond to structural rules. Those corresponding to the structural rules required for relevance logic are given in the appendix, as they are not transparent at first sight. The rule corresponding to Weakening is simply: R00x (or $0 \sqsubseteq x$) for all x. This makes the point set into a rooted graph (with 0 as root), which corresponds to the situation in intuitionistic logic. 0 supports only logic, while points above 0 add information monotonically.

The other important relations between worlds are the modal accessibility relation used for necessity, and the compatibility relation used for negation. These are both binary. As already mentioned, they can be defined in terms of R, though it will be convenient first to discuss them independently. Their interaction with $\sqsubseteq$ is constrained by (16) and (17). In the first case, x is downward closed and z is upward closed, while in the second case both arguments are downward closed. The evaluation clauses for the modalities which use S and C are given in (18) and (19).

(16) If Sxz then for every $x' \sqsubseteq x$, $z' \sqsupseteq z$, $Sx'z'$.

(17) If Cxy then for every $x' \sqsubseteq x$, $y' \sqsubseteq y$, $Cx'y'$.

(18) $x \models \Box A$ iff for every z such that Sxz, $z \models A$

(19) $x \models {\sim}A$ iff for every y such that Cxy, $y \not\models A$

The accessibility relation S superimposes a system of modal necessity on relevance logic. S can be constrained further in the ways familiar from normal modal systems. The modal system usually added to relevance logic is S4, where S is reflexive and transitive. Modality will not be discussed in this paper, though there is more detail in Gregory (2001).

The compatibility relation governs negation. Besides the basic constraint in (17), other constraints are added to obtain particular kinds of negation. Those required for De Morgan negation are again given in the appendix[12]. It is assumed that C is symmetric, and also directed (each world has some compatible world). The constraint corresponding to ECQ is that it is also

[12]They amount to the requirement that for each world there is a maximal consistent world (with respect to $\sqsubseteq$), though the formulation is again not immediately transparent. (This corresponds to the "Routley star" operation (Routley and Routley (1972)).) The approach adopted here is due to Dunn (1993).

reflexive. For the De Morgan negation desired here, this condition is of course rejected. Every world must be compatible with some world, but it is not required to be compatible with itself.

4 Channels and attitudes

4.1 Channel theory

The extension of situation semantics to channel theory (Barwise (1993), Israel and Perry (1990)) was intended to give a more satisfactory model for implicational constraints on situations, the basis of the situation-theoretic theory of meaning. A channel bears the same relation to a constraint as a situation does to an infon, namely the $\models$ relation. The relation between the two is given by (20). Thus a relation between situations is set up which mediates a relation between infons (Barwise (1993)).

(20) $\mathrm{c} \models \sigma \Rightarrow \tau$ iff, if $\mathrm{s} \models \sigma$ then $\mathrm{t} \models \tau$

Most constraints are not logical entailments, but hold only given certain background conditions. This leads to the suggestion that they should themselves be situated, perhaps in some kind of ambient situation. However, while infons are generally understood as being upward persistent (with respect to situation inclusion), constraints behave differently; they are *downward* persistent both with respect to the "ambient situation" and to the situation supporting the antecedent (s in the example), though not with respect to t. This behaviour is characteristic of accessibility relations, particularly the ternary relation R in relevance frames. Thus it seems possible to redefine the "channel" c in terms of an accessibility relation. A connection between situations is set up which mediates the relation between infons in a constraint. However, this connection is itself a "part of the world", and may be reasonably be considered a situation in its own right.

As has been observed[13], frames for substructural logics provide a logical interpretation of this conception of a channel. If intuitionistic logic were used, the framework would have essentially the same properties as situation semantics, including its inability to deal with hyperintensionality. The rejection of Weakening leads immediately to the recognition of impossible situations. Dropping other structural rules would have further consequences[14].

[13] cf. Mares (1997) and references therein.

[14] Work in this tradition (Restall (1996), Mares (1997)) has generally advocated very weak substructural logics; in particular, Restall advocates the rejection of Contraction,

4.2 Attitude problems

Channels provide a convenient representation of connections between the situations supporting an embedded proposition and the belief state of an agent. I take the verbs "believe" and "doubt" as examples of positive and negative attitudes, whose interrelated behaviour it is desired to capture. It is well known that positive attitudes show the basic entailment patterns characteristic of necessity operators, while negative attitudes (unsurprisingly) have the basic properties of negation operators. These basic patterns are exemplified below (23ff). At the same time, the further conditions on their behaviour as necessity or negation operators varies between different classes of attitide predicate. The following paragraphs are intended to indicate how the basic entailment patterns can be related to channels, while the "tuning" of frames for different attitudes is deferred to further work.

I make two basic assumptions. The first is that for each agent x there is a world $x \in P$ which supports the set of propositions believed by x. Recall that worlds are theories. They are not in general complete and consistent, but they are required to be closed under modus ponens[15], so that if an agent believes A and $A \rightarrow B$ (but not otherwise), he may be reported as believing B. The second is that "believe" and "doubt" are interpreted as operators (written B_x, D_x), parametrized according to agent, on an embedded proposition. The whole attitude report is evaluated using a parametrized accessibility relation defined in (21), using the semantic clauses in (22).

(21)
1. $S_x yz$ iff $Ryxz$ [cf. Syz iff $Ry0z$]
2. $C_x yz$ iff $Ryzx$ [cf. Cyz iff $Ryz0$]

(22)
1. $y \models B_x A$ iff for all z such that $S_x yz$, $z \models A$
2. $y \models D_x A$ iff for all z such that $C_x yz$, $z \not\models A$

The intuition which this notion is trying to capture is that modalities such as belief and doubt are generalizations of necessity and negation, into which they collapse when $x = 0$ (as indicated in the square brackets in (21)). The relationship between their respective accessibility relations and the ternary relation R, and hence with implication, corresponds to the idea that believed statements are those implied by a given belief state, while

as pointed out by a reviewer. In the present application, the presence of Contraction is important to ensure that belief states are closed under modus ponens. Other consequences of this choice, both desirable and undesirable, are discussed in Gregory (2001).

[15] As noted above, this follows from Contraction.

doubted statements are those which imply something incompatible with the particular belief state. The evaluation of an attitude statement involves a channel between the belief state of the agent and the worlds where the embedded proposition is evaluated.

Thus $B_x A$ corresponds to an implication $X \rightarrow A$, where the antecedent $X = \bigwedge\{P\colon x \models P\}$ - the conjunction of all propositions supported by a given world x associated with the agent x. $B_x A$ is true at y if A is true at every z such that Ryxz. Conversely $D_x A$ corresponds to an implication $A \rightarrow \sim X$, where X this time is in the consequent. $D_x A$ is true at y if A is not true at any z such that Ryzx (otherwise A would be true at x and would be believed not doubted by x). Essentially the meaning postulate linking "believe" and "doubt" (and other such pairs) should consist in fixing x for any given agent.

Much of the behaviour expected from the two verbs then falls out from the proposed semantics. Some of the more important semantic patterns are set out in (23ff). "Positive" attitudes such as belief share important properties with necessity operators, such as conjunction distribution (24)[16]. These properties are not shared by "negative" attitudes such as "doubt", which by contrast have properties characteristic of negation operators: contraposition (28) and the de-Morgan laws (27)[17]. However, these operators do not satisfy ECQ (29) or irrelevance (30).

(23) Mary believes A and Mary believes B $\vdash$ Mary believes A and B
cf. $[]A \wedge []B \vdash [](A \wedge B)$

(24) Mary believes A and B $\vdash$ Mary believes A
cf. $[](A \wedge B) \vdash []A$

(25) Mary doubts A and B $\nvdash$ Mary doubts A
cf. $\sim(A \wedge B) \nvdash \sim A$

(26) Mary believes that if A then B $\vdash$ If Mary believes A then Mary believes B
cf. $[](A \rightarrow B) \vdash []A \rightarrow []B$

(27) Mary doubts A and B $\vdash$ Mary doubts A or Mary doubts B
cf. $\sim(A \wedge B) \vdash \sim A \vee \sim B$

[16] $[]A \wedge []B \vdash [](A \wedge B)$ and distribution over implication (26), which is a property of necessity operators in normal modal systems. The other basic property of such operators, $\top \vdash []\top$, simply means here that every agent must believe some proposition - it seems reasonable to require this of any agent.

[17] This is the most "difficult" of the De Morgan laws (the one that is not immediately true of virtually all forms of negation).

(28) Mary doubts B, Mary believes A implies B $\vdash$ Mary doubts A
cf. $\sim$B, A $\rightarrow$ B $\vdash$ $\sim$A

(29) Mary believes A and not A $\nvdash$ Mary believes B
cf. $[](A \wedge \sim A) \nvdash []B$

(30) Mary believes B $\nvdash$ Mary believes if A then B
cf. $[]B \nvdash [](A \rightarrow B)$

These basic patterns of behaviour rely only on properties of implication where either the antecedent or the consequent (for necessity and negation operators respectively) is kept constant (Restall (2000)).

What does all this mean intuitively? Attitude reports are modelled as constraints on connections between the belief states of cognitive agents and situations where the embedded propositions are evaluated. These constraints are formulated as (relevant) implications. Relevant implication is an intensional operator requiring an accessibility relation for its evaluation, and this is used to give a treatment capturing important semantic affinities between attitude predicates and necessity and negation operators. The correspondence between positive attitude predicates and necessity operators ties in with standard doxastic modal treatments, while at the same time avoiding the paradoxes of strict implication inherent in standard modal systems. The interpretation of negative attitude predicates by means of a compatibility relation is strong enough to capture the similarity of such predicates with negation, while at the same time avoiding the problem of explosion (ECQ).

5 Conclusion and further work

This research attempts to provide a semantics which avoids the problem of hyperintensionality, which has proved intractable on standard approaches. It draws on two traditions of work whose convergence has been the subject of some interest in the literature, though not in connection with the same problem. It adopts the notions of intensional implication and paraconsistent negation offered by relevance logic, and seeks to interpret these in terms of channels. In work in progress, this is being worked out in terms of the more elaborate channel theory of Barwise and Seligman (1997), based on mappings between infon algebras. Other priorities for further work include the extension of the logic to higher order systems, and the treatment of identity of objects in intensional contexts.

6 Appendix

The following extra conditions on accessibility relations are required for relevance logic (as opposed to the "basic" substructural logic discussed in Restall (1995) and Mares (1997)). Only a few of them are mentioned in the text. For ease of comparison, I append the rejected conditions which, if added, would give standard logics.[18]

(31) Conditions on R (for implication)

1. Notational convention: definition of R^2
   (a) $R^2(ab)cd := \exists e(Rabe \wedge Recd)$
   (b) $R^2a(bc)d := \exists e(Raed \wedge Rbce)$
2. (a) $R^2(ab)cd \rightarrow R^2a(bc)d$ [$a\circ(b\circ c) \sqsubseteq (a\circ b)\circ c$] (Associativity)
   (b) $Rabc \rightarrow R^2(ab)bc$ [$(a\circ b)\circ a \sqsubseteq a\circ b$] (Contraction)
   (c) for **R**: $Rabc \rightarrow Rbac$ [$a\circ b \sqsubseteq b\circ a$] (Permutation)
3. For intuitionistic logic: $R00a$ [$0 \sqsubseteq a$] (Weakening)

(32) Conditions on S (for necessity)

1. Notational conventions:
   (a) $R(xS)yz := \exists w(Sxw \wedge Rwyz)$
   (b) $Rx(yS)z := \exists w(Syw \wedge Rxwz)$
   (c) $Rxy(Sz) := \exists w(Swz \wedge Rxyw)$
2. Conditions for S4 necessity
   (a) Saa (modal axiom **T**)
   (b) $Sab \wedge Sbc \rightarrow Sac$ (axiom **4**)
   (c) $Rcd(Sa) \rightarrow R(cS)(dS)b$ (axiom **K**)
   (d) $S0a \rightarrow a = 0$ (Necessitation)
3. For S5 modal system: $Sab \rightarrow Sba$ (axiom **B**)

(33) Conditions on C (for negation)

1. Extra conditions for De Morgan negation

[18] The conditions on R (31) are more transparent if they are thought of in terms of a fusion operation $\circ$ on worlds, which is like R except of course that R is not a function. (Dunn (1986), Gabbay and Olivetti (2000)). The modal component (32) is based on Read (1988) and Restall (1994), while negation (33) is based on Restall (1995, 2000).

   (a) $\forall a \exists b(Cab \wedge \forall c(Cbc \rightarrow c \sqsubseteq b))$ (Double negation)
   (b) $\exists a(Rbcd \wedge Cad) \leftrightarrow \exists e(Rbde \wedge Cce)$ (Contraposition)

2. For ECQ: Caa
3. For LEM: $Cab \rightarrow b \sqsubseteq a$

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On the Morpho-Syntax and Semantics of the Georgian Passive Constructions

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The category of voice has been widely discussed in the linguistic literature (Haspelmath M.1990; Langacker R., Munro P. 1975; Perlmutter D.M.1978; Perlmutter D.M., Postal P.M.1977; Shibatani M. 1985; Shibatani M.(ed.) 1988, and others). It is one of the important verbal categories and at the same time it is not a simple one because studying a voice category in any language involves the data from the syntactic, morphological and semantic levels. Nowadays the voice category, like some other grammatical categories, is considered as being a continuous one (Shibatani M. 1985), in distinction from the traditional view, where it was considered as being discrete. But we do not discuss the theoretical problems connected with the voice category here, our main goal is to present the analysis of certain Georgian data, and we hope that that will prove helpful in solving some theoretical problems.

Three voice forms are traditionally differentiated in Georgian: active, passive, and medium. Each of them has its own model of conjugation, and only passive has its own morphological markers: the suffix –d, the prefixes i-/e-, and there are passive voice forms without any marker as well (Shanidze A.1953; Jorbenadze B.1975).

Verb forms, according to their syntactic structure, are divided into three series. Opposition between the voice forms is well expressed in the forms of the first series, as its syntactic structure is nominative (i.e. the subject is expressed by the nominative case, and the object by the dative case). Identification of voice of a given verb form depends on how the semantic roles connected to the verb are encoded on the syntactic or morphological level.
Sharing the same root (resp. the same general semantics) the morphological forms of voice construct:
Triplets, e.g.:

(1) a-gor-eb-s (act.) (sb) rolls (sth)[1]
CAUS-to roll-TH.SUFF-S3

gor-av-s (med.) (sth/sb) rolls
to roll-TH.SUFF-S3

gor-d-eb-a (pass.) (sth/sb) begins to roll / (sth/sb) is being rolled
to roll-PASS-TH.SUFF-S3

Pairs, e.g.:

(2) a-R-eb-s (act) (sb) opens (sth)
CAUS-to open-TH.SUFF-S3

i-R-eb-a (pass.) (sth) is being open/ is being opened

(3) dg-a-s (med.) (sb/sth) stands
to stand- ST.SUFF-S3

dg-eb-a (pass.) (sb/sth) is getting up

(We do not consider directed forms, e.g. like version forms or others.)
And there are some single forms that are thought to be the voice forms only due to their morphological structure, e.g.:

(4) kvd-eb-a (pass.)
to die-TH.SUFF-S3[2]

Our research aims to study the triplets, pairs, and singles of voice forms and to establish semantic, syntactic and morpho-syntactic relations between them. In the first stage of our research, we have studied only the triplets of voice forms and their analysis is presented in the paper.

The voice form triplets are divided into three main groups, and as passive constructions are the main subject of our interest, these groups are defined according to their passive forms:

Triplets where passive is expressed by the suffix (-d);
Triplets where passive is expressed by prefixes (i-/e-);
Triplets where passive has no marker.

Each group is further divided into subgroups.

In the first group there are the subgroups defined by the morphological model of active and medium forms.
The first subgroup has the following morphological structure:

(5) a-R-eb-s (act.) ~ R-eb/ob-s (med.) ~ (a)-R-d-eb-a (pass.)
CAUS-root-TH. SUFF-S3 ~ root-TH. SUFF-S3 ~ (PrV-)root-PASS-TH.SUFF-S3

(The triplets are given in the third person form of the present tense.)
This structure is characteristic for :
The verbs with doubled onomatopoetic roots, e.g.

(6) a-qaTqaT-eb-s (act.)
CAUS-very, very white-TH.SUFF-S3

qaTqaT-eb-s (med.) - (sth) is in the state of being very, very white
very, very white- TH.SUFF-S3

qaTqaT-d-eb-a (pass.)
very, very white-PASS-TH.SUFF-S3[3]

a-barbac-eb-s (act.)
CAUS-to stagger-TH.SUFF-S3

barbac-eb-s (med.) – (sb) staggers
to stagger-TH. SUFF-S3

barbac-d-eb-a (pass.)
to stagger-PASS-TH.SUFF-S3

Onomatopoetic roots with –ial, -al, -um, -an, -in, ur-/-ul endings, e.g.

(7) a-Srial-eb-s (act.)
CAUS-to rustle-TH.SUFF-S3

Srial-eb-s (med.) – (sth) rustles
to rustle-TH.SUFF-S3

a-Srial-d-eb-a (pass.)
CAUS-to rustle-PASS-TH.SUFF-S3

Verb forms derived from either noun or adjective roots, e.g.

(8) a-Ror-eb-s (act.)
CAUS-pig-TH.SUFF-S3

Ror-ob-s (med) – (sb) acts like a pig
pig-TH. SUFF-S3

Ror-d-eb-a (pass.)
pig-PASS-TH.SUFF-S3

a-zarmac-eb-s (act.)
CAUS-lazy- TH.SUFF-S3

zarmac-ob-s (med.) – (sb) acts like a lazy one
big-TH.SUFF-S3

zarmac-d-eb-a (pass.)
lazy-PASS-TH.SUFF-S3

It is easy to see that medium forms are morphologically the simplest ones, and therefore the medium form is considered as being correspondent to the semantic frame of the verb. So the active and passive forms are considered as derived from the medium ones. What kind of semantic relations are there between the opposite forms? At first we try to give general characteristics of medium form semantics within this subgroup: the medium forms mainly express some kind of prominence, generally it is an action which is not limited in time (there is neither the beginning nor the ending reflected in the semantics of the verb forms), and therefore they could be thought as a state that has the certain degree of activity (this degree differs through the verbs). So the medium form is generally used to express the state characteristic for a certain thing/person, e.g.:

(11) mTvrali kaci barbacebs - A drunk (man) staggers.
qva goravs - A stone rolls.

The medium forms are syntactically mono-personal in this subgroup, while the active forms are bi-personal. The last ones are syntactically very close to the causatives (they are even called primary causatives) as they express external initiation or causation of the state expressed by the medium forms, and accordingly the initiator/causer of the state is added in the semantics of the active form is the subject.

(12) qari foTlebs aSrialebs
wind (NOM) leaves (DAT) rustles (ACT)
The wind rustles the leaves = The wind causes that the leaves rustle.

So the initiator/causer of the active voice form is encoded as a subject on the syntactic level while the causee (the state of which is expressed by the medium form) is encoded as a direct object. It seems that a- prefix in the active voice forms is connected with the semantics of initiation/causation.

Now it is time to define the relation between the medium forms and the passive ones. While the medium form expresses some kind of a state, in passive the bounds of the state is set, generally the changing point is expressed. So the passive voice form contains the semantics of changing, that results in the state expressed by the medium form.

The second subgroup has the following morphological structure:

(13) a-R-eb-s ~ R-av/i/Ø-s ~ (a-)R-d-eb-a
CAUS-root-TH.SUFF-S3 ~ root-TH.SUFF-S3 ~ (PrV-)root-TH.SUFF-S3, e.g.

a-gor-eb-s (act.)
CAUS-to roll-TH.SUFF-S3

gor-av-s (med.) – (sb/sth) rolls
to roll-TH.SUFF-S3

gor-d-eb-a (pass.)
to roll-TH.SUFF-S3

Syntactic and semantic relations between those forms are the same as mentioned above, but we would like to mention that some active and passive forms, corresponding to the same medium ones with i- thematic suffix, have not got simple stems (the root is complicated with the participle ending), e.g.:

(14) a-knav-l-eb-s (act)

CAUS-to meow-PART-TH.SUFF-S3

knav-i-s (med.) – (sb) meows
to meow-TH.SUFF-S3

a-knav-l-d-eb-a (pass.)
PrV-to meow-PART-TH.SUFF-S3

There are a few triplets here with medium forms being bi-personal (accordingly active forms are tri-personal and passive ones are bi-personal), but the semantic relations are nearly the same, e.g.:

(15) a-Zul-eb-s (act.)
CAUS-to hate-TH.SUFF-S3

s-Zul-s (med.) – (sb) hates (sb/sth)
S3-to hate-O3

s-Zul-d-eb-a (pass.)
S3-to hate-PASS-TH.SUFF-O3

As for the semantics of the medium form here, it denotes the state, i.e. one has the feeling of hatred, but this feeling, for its part, has an object that is syntactically the object of the medium form, and it is morphologically expressed by the nominative case. The person, bearing the feeling, is encoded as the subject on the syntactic level, and morphologically it is expressed by the dative case.

The second group contains triplets with passive marked by prefixes (i-/e-). Diversity of the relations between the voice forms is a characteristic feature for this group. Generally, the passive with e- prefix is syntactically bi-personal, i.e. the verb form has a subject as well as an indirect object on the syntactic level, and the action, expressed by the passive voice form, is in some way directed to the indirect object, e.g.:

(16) i-wer-eb-a - (sth) is being written
PASS-to write-TH.SUFF-S3

e-wer-eb-a - (sth) is being written to/for (sb/sth)
PASS-to write-TH.SUFF-S3

But within the triplets of this group, one can find mono-personal e-prefix passive.
The first subgroup here has the following morphological structure:

(17) a-R-eb-s ~ R-eb/ob-s ~ e/i-R-eb-a
CAUS-root-TH.SUFF-S3 ~ root-TH.SUFF-S3 ~ PASS-root-TH.SUFF

Some triplets have e-prefix passive forms in this subgroup, and this is the only form for passive here, unless it is impersonal, e.g.:

(18) a-Cxub-eb-s (act.)
CAUS-to argue-TH.SUFF-S3

Cxub-ob-s (med.) - (sb) argues
to argue-TH.SUFF-S3

e-Cxub-eb-a (pass.)
PASS-to argue-TH.SUFF-S3

Medium voice forms are morphologically the simplest here, and they denote the state with comparably high degree of activity, which is of the kind, that it needs a partner to be performed. This partner is included in the semantics of passive forms. The semantics of direction is connected with e- prefix here. So the passive forms are semantically close to the reciprocals. The semantic relations between the active and medium forms are the same as mentioned above.

As for the syntactic relations, the medium forms are mono-personal, and accordingly the active forms are mainly bi-personal (the initiator/causer is added, which syntactically is encoded as the subject) and the passive forms are bi-personal as well (the partner of the action is added, which syntactically is encoded as the indirect object).
Some triplets in this subgroup have e- prefix passive as well, but the medium form is syntactically bi-personal and intransitive, e.g.:

(19) a-mter-eb-s (act.)
CAUS-enemy-TH.SUFF-S3

mtr-ob-s (med.) – (sb) is in the state of being an enemy of (sb)
enemy-TH. SUFF-S3

e-mter-eb-a (pass.)
PASS-enemy-TH.SUFF-S3

Some triplets in this subgroup have i- prefix passive, which is impersonal, generally having the meaning of potential, e.g.:

(20) a-cxovr-eb-s (act.)
CAUS-to live-TH. SUFF-S3

cxovr-ob-s (med.) – (sb) lives
to live-TH. SUFF-S3

i-cxovr-eb-a (pass.) - It is possible to live
PASS-to live-TH. SUFF-S3

The e- prefix forms could be derived from the i- prefix ones, and the former carries semantics of giving direction to the action expressed by the i- prefix forms, e.g.:

(21) e-cxovr-eb-a (pass.) – It is possible for sb to live = Possibility to live is directed to sb.
PASS-to live-TH. SUFF-S3

A few triplets in this group have e- prefix passive, which is mono-personal, e.g.:

(22) a-mgzavr-eb-s (act.) – (one) sets (sb) off
CAUS-to travel-TH. SUFF-S3

mgzavr-ob-s (med.) – (sb) travels
to travel-TH. SUFF-S3

mi-e-mgzavr-eb-a (pass.) – (sb) is departing

The passive voice form here clearly contains the semantics of direction, but the action is not directed to a person, but to a place, and accordingly the passive voice form remains mono-personal.
There are few triplets in this subgroup where the active voice forms are real (morphological) causatives, so-called secondary causatives with special morphological marker, e.g.:

(23) a-varaud-ebin-eb-s (act.)
CAUS-to suppose-SEC.CAUS-TH.SUFF-S3

varaud-ob-s (med.) – (sb) supposes (sth).
to suppose-TH.SUFF-S3

i-varaud-eb-a (pass.)
PASS-to suppose-TH.SUFF-S3

In these triplets medium forms have the direct object on the syntactic level, and so syntactic structure of them is the same as characteristic for bi-personal active forms, but the conjugation is that of medium voice forms. The active forms, as they are derived from the transitive medium, have the forms of morphological causatives (secondary causatives), and they are tri-personal. As for the passive forms, since the medium is syntactically transitive, i- prefix passive is derived from the medium by the conversion.

2. The second subgroup has the following morphological structure:

R-av/i-s ~ R-(PART)-ob/Ø-s ~ i/e-R-eb-a
Root-TH.SUFF-S3 ~ root-(PART)-TH.SUFF-S3 ~ PASS-root-TH.SUFF-S3

It is clear that these are the triplets where the active forms are the simplest, the passive forms are derived from them (by the conversion), and the passive ones prove to be deponents as well. It is interesting that medium forms do not have a simple root (it has the ending of the participle), and their semantics differs from that of the corresponding active/passive forms. The different semantics for the active/passive forms on the one hand, and of the medium forms on the other hand, derives from the roots, which themselves carry those different meanings, e.g.:

(24) loc-av-s (act.) – (one) blesses
to pray/to bless-TH.SUFF-S3

loc-ul-ob-s (med.) – (sb) prays
to pray/to bless-PART-TH.SUFF-S3

i-loc-eb-a (act.) – (sb) is praying (for himself/herself)/(sb) is being blessed
PASS-to pray/to bless-TH.SUFF-S3

The third group consists of triplets with passive forms that have not special morphological marker. This group is not numerous, and one of the examples of it is the following:

(25) a-krT-ob-s (act.)
CAUS-to flinch-TH.SUFF-S3

krT-i-s (med.) – (sb) flinches
to flinch-TH.SUFF-S3

krT-eb-a (pass.)
to flinch-TH.SUFF-S3

As in the most part of the voice form triplets, the medium form is the simplest here and it denotes the state, which the action expressed by the active/passive form results in. Semantics of the active voice form includes the external causer (syntactically the subject, morphologically expressed by the nominative case), while in the semantics of passive voice form this causer is backgrounded or even deleted.
So the analysis of all the groups of voice form triplets in Georgian is presented in this paper, and when the pairs of voice forms are studied, general conclusions on the voice category in the Georgian language will be admittedly maid.

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Grammatical abbreviations:

ACT – active voice
CAUS – causation
DAT – dative case
NOM – nominative case
O3 – the third person object
PART – participle ending
PASS – passive voice
PrV – pre-verb
R – root
S3 – the third person subject
SEC.CAUS – secondary (morphological) causatives
TH.SUFF – thematic suffix

Footnotes

1. In Georgian verb forms containing personal (subject/object) markers can be used without any pronouns that correspond either the subject or the object. That is why the pronouns are given in brackets in the translation.

2 . It becomes clear from the examples above (1,2,3) that passive voice has -eb-a ending in the present tense forms.

3. Here and further translations are given only of the medium forms, semantics of active and passive forms are derived from it (see below).

On Relational Interpretation of Multimodal Categorial Logics

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Abstract

Several recent results show that the Lambek Calculus **L** and its close relative **L1** are sound and complete under (possibly relativized) relational interpretation. This paper transfers these results to **L**$\diamond$, the multimodal extension of the Lambek Calculus that was proposed in Moortgat (1996). Two simple relational interpretations of **L**$\diamond$ are proposed and shown to be sound and complete. The completeness proofs make heavy use of the method of relational labeling from Kurtonina (1995). Finally, it is demonstrated that relational interpretation provides a semantic justification for the translation from **L**$\diamond$ to **L** from Versmissen (1996).

1 Introduction

In recent years several results have been obtained that show that the associative Lambek calculus **L** (Lambek 1958) and its close relative **L1** (i.e. **L** without the ban on empty premises) can be given a fairly natural relational semantics. Kurtonina (1995) showed that **L1** is sound and complete if every formula is interpreted as a binary relation over some set of states, the product operator is interpreted as relational composition, and the two directed implications $\backslash$ and $/$ as left and right residuation respectively, i.e. $\|A \backslash B\| = \overline{\|A\|^{\cup} \circ \overline{\|B\|}}$, and analogously for $\|A/B\|$. Pankrat'ev (1994) and Andréka and Mikulás (1994) prove that the same semantics is sound and complete for **L** if the interpretation of formulas is relativized to a certain transitive relation $<$ on the set of states.

Recent linguistic applications of categorial logics make heavy usage of multimodal extensions of **L** (cf. Moortgat 1997 for an overview), and it is thus an interesting question whether the mentioned results for **L** carry over to multimodal logics. The present paper contains two results pertaining to this issue. We provide two relational semantics for **L**$\diamond$—the extension of **L** with pairs of unary residuation modalities (cf. Moortgat 1996). We establish soundness and completeness of these semantics. Finally, we point out some proof-theoretic repercussions of these model-theoretic results.

2 Relational semantics for the Lambek Calculus

Formulas of the Lambek Calculus **L** are defined by the closure of a set of primitive types under the three binary connectives $\bullet$, $\backslash$, and $/$. Derivability is given by the following sequent rules, where A, B etc. range over formulas and X, Y etc. over finite sequences of formulas. As an additional constraint, premises of sequents must not be empty.

Definition 1 (Sequent Calculus):

$$\frac{}{A \Rightarrow A}[id] \qquad \frac{X \Rightarrow A \quad Y, A, Z \Rightarrow B}{Y, X, Z \Rightarrow B}[Cut]$$

$$\frac{X \Rightarrow A \quad Y, B, Z \Rightarrow C}{Y, X, A \setminus B, Z \Rightarrow C}[\setminus L] \qquad \frac{A, X \Rightarrow B}{X \Rightarrow A \setminus B}[\setminus R]$$

$$\frac{X \Rightarrow A \quad Y, B, Z \Rightarrow C}{Y, B/A, X, Z \Rightarrow C}[/L] \qquad \frac{X, A \Rightarrow B}{X \Rightarrow B/A}[/R]$$

$$\frac{X, A, B, Y \Rightarrow C}{X, A \bullet B, Y \Rightarrow C}[\bullet L] \qquad \frac{X \Rightarrow A \quad Y \Rightarrow B}{X, Y \Rightarrow A \bullet B}[\bullet R]$$

In Pankrat'ev (1994) and Andréka and Mikulás (1994) it is shown that **L** is sound and complete with respect to the following semantics. Let a model consist of a set of possible worlds W, a transitive relation $<$ on W, and a valuation function V that maps atomic formulas to sub-relations of $<$. The semantics of complex formulas is given by the following clauses:

Definition 2 (Relational semantics):

$$\begin{array}{rcl}
\langle a, b\rangle \models p & \textit{iff} & \langle a, b\rangle \in V(p) \\
\langle a, b\rangle \models A \bullet B & \textit{iff} & a < b \wedge \exists c(\langle a, c\rangle \models A \wedge \langle c, b\rangle \models B) \\
\langle a, b\rangle \models A \setminus B & \textit{iff} & a < b \wedge \forall c(\langle c, a\rangle \models A \Rightarrow \langle c, b\rangle \models B) \\
\langle a, b\rangle \models B/A & \textit{iff} & a < b \wedge \forall c(\langle b, c\rangle \models A \Rightarrow \langle a, c\rangle \models B) \\
\langle a, b\rangle \models A, X & \textit{iff} & a < b \wedge \exists c(\langle a, c\rangle \models A \wedge \langle c, b\rangle \models X)
\end{array}$$

A sequent $A_1, \dots, A_n \Rightarrow B$ is valid iff for all models M and possible worlds a, b, if $\langle a, b\rangle \models A_1, \dots, A_n$, then $\langle a, b\rangle \models B$. If we identify the relation $<$ with $W \times W$, we arrive at a notion of validity that corresponds to derivability in **L1** (which is **L** without the restriction to non-empty premises), as shown in Andréka and Mikulás (1994) and in Kurtonina (1995)—this correspondence between frame conditions and proof theoretic characterizations of the corresponding logic is akin to analogous results in the real of modal logic.

3 Multimodal extension

L can be extended to its multimodal version **L**$\Diamond$ by adding a finite family of pairs of unary connectives $\Diamond_i$ and $\Box_i^{\downarrow}$, and by extending the sequent calculus with the following rules (taken form Moortgat (1996), who proves Cut Elimination and Decidability):[1]

Definition 3 (Sequent Calculus for **L**$\Diamond$):

$$\frac{X, ({}_i A)_i, Y \Rightarrow B}{X, \Diamond_i A, Y \Rightarrow B}[\Diamond_i L] \qquad \frac{X \Rightarrow A}{({}_i X)_i \Rightarrow \Diamond_i A}[\Diamond_i R]$$

$$\frac{X, A, Y \Rightarrow B}{X, ({}_i \Box_i^{\downarrow} A)_i, Y \Rightarrow B}[\Box_i^{\downarrow} L] \qquad \frac{({}_i X)_i \Rightarrow A}{X \Rightarrow \Box^{\downarrow} A}[\Box_i^{\downarrow} R]$$

The premise of a sequent is now a bracketed sequence of formulas, i.e. a finite labeled tree. The subscript i will be dropped in the remainder of the paper if no confusion arises.

[1] A note on terminology: The term "mode" is used as referring to a family of residuated operators here, not to the indices in the syntactic representation. So **L**$\Diamond$, which just comprises one family of binary and one family of unary connectives, would also qualify as "multimodal".

There are two ways how the relational semantics given above can be extended to the multimodal calculi. The first option is inspired by the way modal formulas are interpreted in Kripke semantics. If we use a procedural metaphor, to verify a formula $\Diamond A$ in a world a, we (i) make a transition from a to some other world b that is related to a via the accessibility relation R, (ii) we verify A in b, and (iii) we make a transition in the reverse direction back to a. The main novelty in a genuinely dynamic interpretation is the fact that verifying A may lead us to a world c that is distinct from b, and accordingly, making a R^{-1}-transition from c may lead us to a world d that is distinct from a. The static and the dynamic picture is given schematically in figure 1.

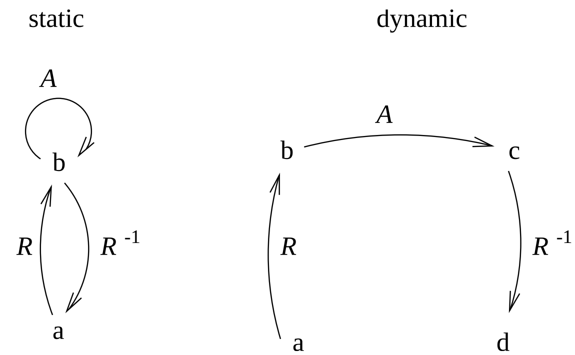

Fig. 1: Static and vertical dynamic interpretation of $\Diamond A$

Note that the input-output pairs $\langle a, d\rangle$ and $\langle b, c\rangle$ have to be related by the ordering relation $<$, while there is no such restriction for the R-relation. Inspired by the picture we might say that formulas relate points horizontally, while the accessibility relation R is vertical. Following this suggestion, we call this semantics vertical relational semantics.

Formally, a vertical relational model for $\mathbf{L}\Diamond$ is a model for $\mathbf{L}$ enriched with a family of binary relations R_i on W. The recursive truth definition is given below.

Definition 4 (Vertical relational Semantics for $\mathbf{L}\Diamond$):

$$
\begin{array}{rcl}
\langle a, b\rangle \models_v p & \textit{iff} & \langle a, b\rangle \in V(p)\\
\langle a, b\rangle \models_v A \bullet B & \textit{iff} & a < b \wedge \exists c(\langle a, c\rangle \models_v A \wedge \langle c, b\rangle \models_v B)\\
\langle a, b\rangle \models_v A \setminus B & \textit{iff} & a < b \wedge \forall c(\langle c, a\rangle \models_v A \Rightarrow \langle c, b\rangle \models_v B)\\
\langle a, b\rangle \models_v B/A & \textit{iff} & a < b \wedge \forall c(\langle b, c\rangle \models_v A \Rightarrow \langle a, c\rangle \models_v B)\\
\langle a, b\rangle \models_v \Diamond_i A & \textit{iff} & a < b \wedge \exists c, d(aR_ic \wedge bR_id \wedge \langle c, d\rangle \models_v A)\\
\langle a, b\rangle \models_v \Box_i^{\downarrow} A & \textit{iff} & a < b \wedge \forall c, d(cR_ia \wedge dR_ib \wedge c < d \Rightarrow \langle c, d\rangle \models_v A)\\
\langle a, b\rangle \models_v A, X & \textit{iff} & a < b \wedge \exists c(\langle a, c\rangle \models_v A \wedge \langle c, b\rangle \models_v X)\\
\langle a, b\rangle \models_v ({}_iX)_i & \textit{iff} & a < b \wedge \exists c, d(aR_ic \wedge bR_id \wedge \langle c, d\rangle \models_v X)
\end{array}
$$

We say that a sequent $X \Rightarrow A$ is vertically valid ($\models_v X \Rightarrow A$) iff for all models M and worlds a and b: if $M, \langle a, b\rangle \models_v X$, then $M, \langle a, b\rangle \models_v A$.

The second option for a relational interpretation of $\mathbf{L}\Diamond$ is inspired by the embedding from $\mathbf{L}\Diamond$ to $\mathbf{L}$ proposed in Versmissen (1996). Here $\Diamond A$ is translated as $t_0 \bullet A \bullet t_1$, where t_0 and t_1 are two fresh atomic formulas of $\mathbf{L}$. Adapted to relational semantics, this means that there are two distinguished relations R and S (intuitively corresponding to the formulas t_0 and t_1), and a $\Diamond A$-transition can be decomposed into an R-transition, followed by an A-step and an S-step (figure 2). R and S have to be sub-relations of $<$; thus the resulting semantics can be dubbed horizontal semantics.

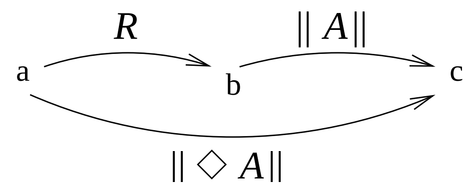

Fig. 2: Horizontal dynamic interpretation of $\diamond A$

To make this precise, a horizontal relational model for $\mathbf{L}\diamond$ is a model for $\mathbf{L}$ which is enriched by a family of pairs of relations R_i and S_i on W such that for all i, $R_i, S_i \subseteq <$.

Definition 5 (Horizontal relational Semantics for $\mathbf{L}\diamond$):

$$
\begin{array}{rcl}
\langle a,b\rangle \models_h p & \textit{iff} & \langle a,b\rangle \in V(p)\\
\langle a,b\rangle \models_h A \bullet B & \textit{iff} & a < b \wedge \exists c(\langle a,c\rangle \models_h A \wedge \langle c,b\rangle \models_h B)\\
\langle a,b\rangle \models_h A \setminus B & \textit{iff} & a < b \wedge \forall c(\langle c,a\rangle \models_h A \Rightarrow \langle c,b\rangle \models_h B)\\
\langle a,b\rangle \models_h B/A & \textit{iff} & a < b \wedge \forall c(\langle b,c\rangle \models_h A \Rightarrow \langle a,c\rangle \models_h B)\\
\langle a,b\rangle \models_h \diamond_i A & \textit{iff} & a < b \wedge \exists c,d(aR_ic \wedge \langle c,d\rangle \models_h A \wedge dS_ib)\\
\langle a,b\rangle \models_h \Box_i^{\downarrow} A & \textit{iff} & a < b \wedge \forall c,d(cR_ia \wedge bS_id \wedge c < d \Rightarrow \langle c,d\rangle \models_h A)\\
\langle a,b\rangle \models_h A, X & \textit{iff} & a < b \wedge \exists c(\langle a,c\rangle \models_h A \wedge \langle c,b\rangle \models_h X)\\
\langle a,b\rangle \models_h ({}_iX)_i & \textit{iff} & a < b \wedge \exists c,d(aR_ic \wedge \langle c,d\rangle \models_h X \wedge dS_ib)
\end{array}
$$

Horizontal validity is defined analogously to vertical validity.

4 Weak completeness of vertical relational semantics

Both notions of validity for $\mathbf{L}\diamond$ given in the previous section are adequate in the sense that they characterize precisely the derivable sequents. These facts are proved in this and the next section, starting with vertical interpretation.

Theorem 1 (Weak Completeness): For every sequent $X \Rightarrow A$:

$$\vdash_{L\diamond} X \Rightarrow A \text{ iff } \models_v X \Rightarrow B$$

Soundness can easily be checked by induction on the length of derivations. The completeness proof follows largely the strategy of Kurtonina (1995) in her completeness proof for **L1** in its relational interpretation. In a first step, we augment the formulas in the sequent system with labels which reflect the truth conditions of formulas. Each formula in a sequent is labeled with a pair of labels, representing the input state and the output state of the corresponding transition. Matters are somewhat complicated by the fact that we have to distinguish horizontal and vertical transitions. To do so, we assume that labels are structured objects themselves: they consist of a *state parameter* $(u, v, w \ldots)$ and a *color index* $(r, s, t, \ldots)$. The color index is written as a subscript to the state parameter. We use letters $a, b, c, \ldots$ as metavariables over labels. The idea is that horizontal transitions only change the state parameter, while vertical transitions change both components. Brackets are treated like formulas; they are labeled with input label and output label as well. For better readability, we use "0_i" and "1_i" instead of opening and closing brackets.

Definition 6 (Labeled Sequent Calculus):

$$\frac{}{u_r v_r : A \Rightarrow u_r v_r : A}\,[id]$$

$$\frac{X \Rightarrow ab : A \qquad Y, ab : A, Z \Rightarrow cd : B}{Y, X, Z \Rightarrow cd : B}\,[Cut]$$

$$\frac{X \Rightarrow ab : A \qquad Y, ac : B, Z \Rightarrow de : C}{Y, X, bc : A \setminus B, Z \Rightarrow de : C}\,[\setminus L]$$

$$\frac{\underline{u}_r v_r : A, X \Rightarrow \underline{u}_r w_r : B}{X \Rightarrow v_r w_r : A \setminus B}\,[\setminus R]$$

$$\frac{X \Rightarrow ab : A \qquad Y, cb : B, Z \Rightarrow de : C}{Y, ac : B/A, X, Z \Rightarrow de : C}\,[/L]$$

$$\frac{X, u_r \underline{v_r} : A \Rightarrow w_r \underline{v_r} : B}{X \Rightarrow w_r u_r : B/A}\,[/R]$$

$$\frac{X, u_r \underline{v_r} : A, \underline{v_r} w_r : B, Y \Rightarrow de : C}{X, u_r w_r : A \bullet B, Y \Rightarrow de : C}\,[\bullet L]$$

$$\frac{X \Rightarrow ab : A \qquad Y \Rightarrow bc : B}{X, Y \Rightarrow ac : A \bullet B}\,[\bullet R]$$

$$\frac{X, u_r \underline{v_s} : 0_i, \underline{v_s} \underline{w_s} : A, \underline{w_s} x_r : 1_i, Y \Rightarrow ef : B}{X, u_r x_r : \Diamond_i A, Y \Rightarrow ef : B}\,[\Diamond_i L]$$

$$\frac{X \Rightarrow u_r v_r : A}{w_s u_r : 0_i, X, v_r x_s : 1_i \Rightarrow w_s x_s : \Diamond_i A}\,[\Diamond_i R]$$

$$\frac{X, u_r v_r : A, Y \Rightarrow ab : B}{X, u_r w_{\underline{s}} : 0_i, w_{\underline{s}} x_{\underline{s}} : \Box_i^{\downarrow} A, x_{\underline{s}} v_r : 1_i, Y \Rightarrow ab : B}\,[\Box_i^{\downarrow} L]$$

$$\frac{\underline{u_r} v_s : 0_i, X, w_s \underline{x_r} : 1_i \Rightarrow \underline{u_r}\,\underline{x_r} : A}{X \Rightarrow v_s w_s : \Box^{\downarrow} A}\,[\Box_i^{\downarrow} R]$$

The underlined labels have to be fresh, i.e. they must not occur elsewhere in the sequent.

Definition 7 (Proper and canonical labeling): *A sequent $a_1 b_1 : A_1, \ldots, a_n b_n : A_n \Rightarrow ab : A$ is properly labeled iff*

- $a_1 = a, b_n = b$
- $\forall i (1 \leq i < n \rightarrow b_i = a_{i+1})$.
- *If $A_i = 0$ or $A_i = 1$, a_i and b_i have different colors.*
- *Otherwise, a_i and b_i have the same color.*
- *If $A_i = 0$, then there is a $j > i$ with $A_j = 1$ and the input color of A_i equals the output color of A_j and vice versa.*
- *If $A_i = 1$, then there is a $j < i$ with $A_j = 0$ and the input color of A_i equals the output color of A_j and vice versa.*

It is canonically labeled iff

- *it is properly labeled.*
- *Each label occurs exactly twice.*

Lemma 1: *If a sequent is derivable, it is properly labeled.*

Proof:
By induction over the length of derivations. $\dashv$

Lemma 2 (Renaming Lemma): *If $a_0a_1 : A_1, \ldots, a_{n-1}a_n : A_n \Rightarrow a_0a_n : B$ is derivable, then the result of renaming all occurrences of an arbitrary a_i with a label of the same color is also derivable.*

Proof:
By induction on the length of derivations. $\dashv$

The idea of the completeness proof can be sketched as follows. Suppose a given sequent $A \Rightarrow B$ is underivable.[2] Then the labeled sequent $ab : A \Rightarrow ab : B$ (a and b being distinct and having the same color) is underivable as well (otherwise we could transform every proof of the latter into a proof of the former simply by dropping the labels). We will construct a falsifying model whose domain is the set of labels and which has the property that $\langle a, b\rangle \models A, \langle a, b\rangle \not\models B$. To this end, we mark labeled formulas with their intended truth value. This gives us the set $\{Tab : A, Fab : B\}$. Let us call such sets T–F sets. We show that every consistent T–F set can be extended to a maximally consistent T–F set, and furthermore that each maximally consistent T–F set corresponds to a model which verifies all T-marked and falsifies all F-marked formulas in it. Hence for each underivable sequent we can construct a falsifying model, which means that every valid sequent is derivable.

To simplify the model construction, we reify the ordering relation and treat $<$ as a formula too.

Definition 8 (T–F set): *A T–F formula is either a formula of $\boldsymbol{L}\Diamond$, "0", "1", or "$<$", which is labeled with a pair of labels and marked either with "T" or with "F". A T–F set is a set of T–F formulas.*

By $\sqsubset_\Delta$ we refer to the transitive closure of the relation $\{\langle a, b\rangle | Tab :<\in \Delta\}$.

Definition 9 (Maxiconsistency): *A T–F set Δ is called* maxiconsistent *if it obeys the following constraints:*

- *For any labeled formula $ab : A$ ($A \neq 0, 1, <$), either $Tab : A$ or $Fab : A$ is in Δ, but not both.*
- *If $Tab : A \in \Delta$ and $a \neq 0, 1$, then $Tab :<\in \Delta$.*
- Δ *is* saturated, *i.e.*
  - *(i) If $Fab : A \setminus B \in \Delta$ and $a \sqsubset_\Delta b$, then there is a c such that $Tca : A, Fcb : B \in \Delta$.*
  - *(ii) If $Fab : A/B \in \Delta$ and $a \sqsubset_\Delta b$, then there is a c such that $Tbc : B, Fac : A \in \Delta$.*
  - *(iii) If $Tab : A \bullet B \in \Delta$, then there is a c such that $Tac : A, Tcb : B \in \Delta$.*
  - *(iv) If $Tab : \Diamond A \in \Delta$, then there are c and d such that $Tac : 0, Tcd : A, Tdb : 1 \in \Delta$.*
  - *(v) If $Fab : \Box^{\downarrow} A \in \Delta$ and $a \sqsubset_\Delta b$, then there are c and d such that $Tca : 0, Fcd : A, Tbd : 1, Tcd :<\in \Delta$.*
  - *(vi) $Tab : 0 \in \Delta$ iff $Tba : 1 \in \Delta$.*

[2] It is sufficient to show completeness for sequents with a single formula as premise, since any proper sequent can be transformed into a formula with the same truth conditions by replacing commas with products and bracket pairs with diamonds.

- Δ *is deductively closed, i.e. if a sequent* $\alpha_1, \dots, \alpha_n \Rightarrow \beta$ *derivable, and for all* $1 \leq i \leq n$: $T\alpha_i \in \Delta$*, then* $T\beta \in \Delta$.

From a maxiconsistent set we can construct a model in the following way:

Definition 10 (Canonical Model): *Let* Δ *be a maxiconsistent set. The canonical model for* Δ *is* $M_\Delta = \langle W, <, I, \{R_i | i \in I\}, V\rangle$*, where*

1. W *is the set of labels occurring in* Δ.
2. $a < b$ *iff* $a \sqsubset_\Delta b$
3. $aR_i b$ *iff* $Tab : 0_i \in \Delta$
4. $\langle a, b\rangle \in V(p)$ *iff* $Tab : p \in \Delta$.

Fact 1: *If* Δ *is maxiconsistent,* M_Δ *is a vertical relational model for* $\boldsymbol{L}\Diamond$

Proof:
Transitivity of $<$ follows immediately from the model construction. The requirement that Δ is maxiconsistent ensures that $V(p) \subseteq <$ for arbitrary atoms p. $\dashv$

Lemma 3 (Truth Lemma): *For all maxiconsistent sets* Δ*, formulas* A *and labels* a, b*:*

$$Tab : A \in \Delta \text{ iff } M_\Delta, ab \models_h A$$

Proof:
By induction on the complexity of A. For the base case, the conclusion follows from the definition of M_Δ.

1. $A = B \bullet C, \Rightarrow$ Since Δ is saturated, there is a c such that $Tac : B, Tcb : C \in \Delta$. By induction hypothesis, $ac \models B, cb \models C$, and furthermore $a < b$, hence $ab \models B \bullet C$.
2. $\Leftarrow$ By the semantics of $\bullet$, there is a c such that $ac \models B, cb \models C$. By induction hypothesis $Tac : B, Tcb : C \in \Delta$. Since $ac : B, cb : C \Rightarrow ab : B \bullet C$, deductive closure of Δ gives us $Tab : B \bullet C \in \Delta$.
3. $A = B \setminus C, \Rightarrow$ Suppose $ab \not\models B \setminus C$. Since $a < b$ by maxiconsistency, there is a c such that $ca \models B, cb \not\models C$. By induction hypothesis, $Tca : B, Fcb : C \in \Delta$. Since $ca : B, ab : B \setminus C \Rightarrow cb : C$, $Tcb : C \in \Delta$, which violates consistency of Δ.
4. $\Leftarrow$ Suppose $Tab : B \setminus C \notin \Delta$. By completeness of Δ, $Fab : B \setminus C \in \Delta$. Since $a < b$ by the semantics of "\", $a \sqsubset_\Delta b$ and therefore saturation entails that there is a c such that $Tca : B, Fcb : C \in \Delta$. By induction hypothesis, $ca \models B, cb \not\models C$, which is impossible.
5. $A = B/C$ Likewise.
6. $A = \Diamond B, \Rightarrow$ By saturation, $Tab :<\in \Delta$, and there are c and d such that $Tac : 0, Tcd : B, Tdb : 1 \in \Delta$. By induction hypothesis, $cd \models B$. The construction of M_Δ ensures that aRc, bRd, and $a < b$. Hence $ab \models \Diamond B$.
7. $\Leftarrow$ By the semantics of $\Diamond$, there are c and d such that aRc, bRd, and $cd \models B$. By induction hypothesis, $Tcd : B \in \Delta$. By the construction of M_Δ and maxiconsistency, $Tac : 0, Tdb : 1 \in \Delta$. Since $\vdash ac : 0, cd : B, db : 1 \Rightarrow ab : \Diamond B$ and Δ is deductively closed, $Tab : \Diamond B \in \Delta$.

8. $A = \Box^{\downarrow}B, \Rightarrow$ Suppose $ab \not\models \Box^{\downarrow}B$. Then there are c and d such that cRa, dRb, $c < d$, and $cd \not\models B$. By induction hypothesis, $Fcd : B \in \Delta$, and the construction of M_Δ ensures that $Tca : 0, Tbd : 1 \in \Delta$. Since $\vdash ca : 0, ab : \Box^{\downarrow}B, bd : 1 \Rightarrow cd : B$, $Tcd : b \in \Delta$, which violates consistency.

9. $\Leftarrow$ Suppose $Tab : \Box^{\downarrow}B \notin \Delta$. By completeness, $Fab : \Box^{\downarrow}B \in \Delta$. By saturation, there are c and d such that $Tca : 0, Tbd : 1, c \sqsubset_\Delta d, Fcd : B \in \Delta$. Hence $cRa, dRb, c < d$ and $cd \not\models B$, which is impossible according to the truth conditions for "$\Box^{\downarrow}$". ⊣

To extend the initial T–F set to a saturated one, we constructively enforce saturation by adding "Henkin witnesses":

Assume an ordering of the set of labels.

Definition 11 (Henkin witnesses): *Let Δ be a T–F set and α be a T–F labeled formula. a and b are always assumed to be distinct.*

(i) *If $\alpha = Tab : A \bullet B$, then $H(\Delta, \alpha) = \Delta \cup \{\alpha, Tac : A, Tac :<, Tcb : B, Tcb :<\}$, where c is the first label having the same color as a which does not occur in Δ.*

(ii) *If $\alpha = Fab : A \setminus B$ and $a \sqsubset_\Delta b$, then $H(\Delta, \alpha) = \Delta \cup \{\alpha, Tca : A, Tca :<, Fcb : B\}$, where c is the first label of a's color not occurring in Δ.*

(iii) *If $\alpha = Fab : A/B$ and $a \sqsubset_\Delta b$, then $H(\Delta, \alpha) = \Delta \cup \{\alpha, Tbc : B, Tbc :<, Fac : A\}$, where c is the first label of a's color not occurring in Δ.*

(iv) *If $\alpha = Tab : \Diamond A$, then $H(\Delta, \alpha) = \Delta \cup \{\alpha, Taw_r : 0, Tw_r a : 1, Tw_r u_r : A, Tw_r u_r :<, Tu_r b : 1, Tbu_r : 0\}$, where w and u are the first distinct state parameters and r is the first color index not occurring in Δ.*

(v) *If $\alpha = Fab : \Box^{\downarrow}A$ and $a \sqsubset_\Delta b$, then $H(\Delta, \alpha) = \Delta \cup \{\alpha, Tw_r a : 0, Taw_r : 1, Fw_r u_r : A, Tbu_r : 1, Tu_r b : 0, Tw_r u_r :<\}$ where w and u are the first distinct state parameters and r is the first color index not occurring in Δ.*

(vi) *Else $H(\Delta, \alpha) = \Delta$.*

Adding Henkin witnesses preserves three properties of T–F sets that are essential to prove maxi-consistency.

Definition 12 (Deep Consistency): *A set Δ is called* deeply consistent *iff it has the properties that if $\vdash \alpha_1, \ldots, \alpha_n \Rightarrow \beta$ and $T\alpha_i \in \Delta$ for all $1 \leq i \leq n$, then $F\beta \notin \Delta$.*

Definition 13 (Acyclicity): *A T–F set Δ is called* acyclic *iff there is no sequence of labels $a_1, \ldots, a_n$ such that $Ta_{i-1}a_i :<, Ta_n a_1 :<\in \Delta$.*

Definition 14 (Well-Coloredness): *A T–F set Δ is well-colored iff the following conditions hold:*

- *If $Tab :<\in \Delta$, then a and b have the same color.*
- *If $Tab : 0 \in \Delta$ or $Tab : 1 \in \Delta$, then a and b have different colors.*

Lemma 4: *If $\alpha \in \Delta$ and Δ is deeply consistent, acyclic and well-colored, then $H(\Delta, \alpha)$ is also deeply consistent, acyclic and well-colored.*

Proof:
As for acyclicity, observe that addition of $Tac :<$ cannot destroy it provided c is fresh and $a \neq c$. This covers cases (ii) trough (v). In the first cases, assume that adding $Tac :<$, $Tcb :<$ destroys acyclicity. This means that there is a sequence $a_1, \ldots, a_n$ such that $Ta_{i-1}a_i :<$, $Ta_na_1 :<\in \Delta \cup \{Tac :<, Tcb :<\}$. In this sequence, all occurrences of c have to occur between a and b. Since the fact that $Tab : A \bullet B \in \Delta$ entails that $Tab :<\in \Delta$, removing all occurrences of c would yield a closed cycle for Δ, contra assumption.

Preservation of well-coloredness is immediate from the definition of Henkin witnesses.

To prove preservation of deep consistency, we assume the contrary and derive a contradiction in each case.

(i) Since in every derivable sequent each label occurs an even number of times, the sequent that violates deep consistency must have the form $X_1, ac : A, cb : B, \ldots, X_n, ac : A, cb : B, Y \Rightarrow \alpha$ where all formulas occurring in $X_1, \ldots, X_n, Y, \alpha$ already occur in Δ. By the renaming lemma, thence the following sequent is also valid: $X_1, ac_1 : A, c_1b : B, \ldots, X_n, ac_n : A, c_nb : B, Y \Rightarrow \alpha$, from which we can derive $X_1, ab : A \bullet B, \ldots, X_n, ab : A \bullet B, Y \Rightarrow \alpha$ Since all formulas involved are already in Δ and Δ is deeply consistent, $F\alpha$ cannot be in Δ, which is a contradiction.

(ii) By the same reasoning as above, both new formulas must occur in the sequent that causes violation of deep consistency. Hence its conclusion is $cb : B$. The only place where the other occurrence of c can possibly occur is the first premise, hence the sequent has the form $ca : A, X \Rightarrow cb : B$ with X consisting only of old T-marked formulas. Since $a \sqsubset_\Delta b$ and Δ is acyclic and hence irreflexive, $a \neq b$ which ensures that X is non-empty. Therefore from this sequent we can derive $X \Rightarrow ab : A \setminus B$, which is excluded by the deep consistency of Δ.

(iii) Likewise.

(iv) Suppose $w_ra : 1$ occurs in the sequent that destroys deep consistency. Since w_r is fresh, there is no F-formula with w_r as input label, and the only T-formula with w_r as output label is $Taw_r : 0$. Hence the sequent in question would have the form $X, aw_r : 0, w_ra : 1, Y \Rightarrow \alpha$, which is impossible since there are no valid sequents where a closing bracket immediately follows an opening bracket. In the same way it can be shown that $Tu_rb : 0$ cannot be involved in the destruction of deep consistency. Thus by familiar reasoning, the guilty sequent has the form $X_1, aw_r : 0, w_ru_r : A, u_rb : 1, \ldots, X_n, aw_r : 0, w_ru_r : A, u_rb : 1, Y \Rightarrow \alpha$. By the renaming lemma, $X_1, aw_{r,1} : 0, w_{r,1}u_{r,1} : A, u_{r,1}b : 1, \ldots, X_n, aw_{r,n} : 0, w_{r,n}u_{r,n} : A, u_{r,n}b : 1, Y \Rightarrow \alpha$ with $w_{r,i}$ and $u_{r,i}$ fresh is also valid. From this we derive the validity of $X_1, ab : \Diamond A, \ldots, ab : \Diamond A, Y \Rightarrow \alpha$ which is incompatible with the assumption of the deep consistency of Δ.

(v) Suppose $aw_r : 1$ would occur in the sequent that undermines deep consistency. Since every valid sequent is properly labeled and w_r is a new label, this sequent has to take the form $A_1, \ldots, aw_r : 1, w_ra : 0, \ldots A_n \Rightarrow \alpha$, where all premises are T-marked and the conclusion is F-marked in $H(\Delta, \alpha)$. By proper labeling we know that $aw_r : 1$ has to be preceded by $cu_r : 0$ for some c, u. But this is impossible since r is a new color. Thus $Taw_r : 1$ cannot destroy deep consistency. The same case can be made for $Tu_rb : 0$. Therefore destruction of deep consistency entails that there is a valid sequent $w_ra : 0, X, bu_r : 1 \Rightarrow w_ru_r : A$ such that all formulas in X are T-marked in Δ. Since $Tab :<\in \Delta$, $a \neq b$ due to acyclicity and hence X is non-empty. Therefore the sequent $x \Rightarrow ab : \Box^{\downarrow} A$ is also valid, which contradicts deep consistency of Δ.

(vi) Immediate. ⊣

It remains to be shown that any deeply consistent, acyclic and well-colored T–F set can be extended to a maxiconsistent T–F set.

Lemma 5: *If Δ is deeply consistent, acyclic, and well-colored, $A \neq 0, 1$ and a and b have the same color, then either $\Delta \cup \{Tab : A, Tab :<\}$ or $\Delta \cup \{Fab : A\}$ is deeply consistent, acyclic, and well-colored.*

Proof:
Suppose adding $Fab : A$ destroys deep consistency, acyclicity, or well-coloredness. Adding an F-marked formula cannot destroy acyclicity or well-coloredness, hence $\Delta \cup \{Fab : A\}$ is not deeply consistent. This means that there is a set of formulas $Tac_1 : A_1, \ldots, Tc_{n-1}b : A_n \in \Delta$ such that $ac_1 : A_1, \ldots, c_{n-1}b : A_n \Rightarrow ab : A$ is valid. Now suppose adding $Tab : A$ would destroy deep consistency, too. Then there would be a valid sequent $X_1, ab : A, \ldots, X_m, ab : A, Y \Rightarrow cd : C$ such that $Fcd : C \in \Delta$ and $X_1, \ldots, X_m$ consist of T-marked formulas from Δ. By repeated application of Cut we would obtain the valid sequent $X_1, ac_1 : A_1, \ldots, c_{n-1}b : A_n, \ldots, X_n, ac_1 : A_1, \ldots, c_{n-1}b : A_n, Y \Rightarrow cd : C$, where the premise consists only of T-marked formulas and the conclusion is F-marked, which is excluded by the deep consistency of Δ. Adding $Tab :<$ cannot destroy acyclicity since $Tac_1 :<, \ldots, Tc_{n-1}b :<$ are in Δ and Δ is acyclic. Preservation of well-coloredness is obvious. ⊣

This allows us to construct a maxiconsistent set by the following procedure:

Definition 15: *Let Δ be a deeply consistent set and φ be an enumeration of labeled formulas (excluding 0, 1, and <).*

1. $\Delta_0 = \Delta$
2. *If $\varphi_n = ab : A$, and $\Delta_n \cup \{T\varphi_n, Tab :<\}$ is deeply consistent, acyclic, and well-colored, then* $\Delta_{n+1} = H(\Delta_n \cup \{T\varphi_n, Tab :<\}, T\varphi_n)$.
3. *Otherwise* $\Delta_{n+1} = H(\Delta_n \cup \{F\varphi_n\}, F\varphi_n)$.
4. $\Delta_\omega = \bigcup_{n\in\omega} \Delta_n$.

Lemma 6: *If $n < m$, a and b are labels occurring in Δ_n, and $\neg a \sqsubset_{\Delta_n} b$, then $\neg a \sqsubset_{\Delta_m} b$.*

Proof:
Induction over n and m. ⊣

Lemma 7: *If Δ is deeply consistent, acyclic, and well-colored, and $\forall a, b(Tab : 0 \in \Delta \leftrightarrow Tba : 1 \in \Delta)$, then Δ_ω is maxiconsistent.*

Proof:
By the construction, either $T\alpha$ or $F\alpha$ is in Δ_ω for all labeled formulas α. Lemmas 4 and 5 ensure that each Δ_n is deeply consistent. If both $T\alpha$ and $F\alpha$ were in Δ_ω, they would be in some Δ_n too, which is impossible since these are deeply consistent. An inspection of the clauses for Henkin witnesses shows that each addition of a formula $Tab : A$ is accompanied by addition of $Tab :<$. Clauses (i) – (v) of saturation are ensured by closure under Henkin witnesses together with lemma 6. By assumption, clause (vi) of the definition of saturation hold of Δ_0, and it is easy to see that it is preserved under every step from Δ_n to Δ_{n+1}. Thus it also holds of Δ_ω since otherwise it would already fail for some Δ_n. Since Δ_ω is complete, failure of deductive closure would entail failure of deep consistency for some Δ_n. ⊣

Lemma 8: *If $a_1b_1 : A_1, \ldots, a_nb_n : A_n \Rightarrow \alpha$ is canonically labeled and underivable, then $\{Ta_ib_i : A_i, F\alpha\} \cup \{Ta_ib_i :< | 0 \neq A_i \neq 1\}$ is deeply consistent, acyclic, and well-colored.*

Proof:
Since the sequent is canonically labeled, the only properly labeled sequent made from its components is the original sequent itself. Hence there is no valid sequent consisting only of formulas from the set in question. Acyclicity and well-coloredness follow from the definition of canonical labeling. ⊣

Lemma 9: *If $ab : A \Rightarrow ab : B$ is derivable in the labeled calculus, $A \Rightarrow B$ is derivable in the unlabeled calculus.*

Proof:
Simply drop the labels in the proof, and replace "0" by "(" and "1" by ")". ⊣

Now suppose $A \Rightarrow B$ is underivable in the unlabeled calculus. By the last lemma, $w_r u_r : A \Rightarrow w_r u_r : B$ (w and u distinct) is canonically labeled and underivable in the labeled calculus. Hence in the canonical model constructed from $\{Tw_r u_r : A, Tw_r u_r :<, Fw_r u_r : B\}_\omega$, $\langle w_r, u_r\rangle$ verifies A and falsifies B. This completes the proof of Theorem 1. ⊣

5 Weak Completeness of horizontal relational semantics

Theorem 2 (Weak Completeness): For every sequent $X \Rightarrow A$:

$$\vdash_{L\diamond} X \Rightarrow A \text{ iff } \models_h X \Rightarrow B$$

The soundness proof is again a straightforward induction over the length of derivations. The completeness proof is very similar to the proof in the previous section, so I will content myself with pointing out the differences.

Definition 16:
Let Δ be a T–F set. We say that $a \sqsubset_\Delta b$ iff there are labels $c_1, \ldots, c_n$ such that $a = c_1, b = c_n, Ta_{i-1}a_i :<\in \Delta \vee Ta_{i-1}a_i : 0 \in \Delta \vee Ta_{i-1}a_i : 1 \in \Delta$ for all $1 \leq i \leq n$.

The definition of a maxiconsistent set now runs as follows:

Definition 17 (Maxiconsistency): *A T–F set Δ is called* maxiconsistent *iff it obeys the following constraints:*

- *For any labeled formula $ab : A$ ($A \neq 0, 1, <$), either $Tab : A$ or $Fab : A$ is in Δ, but not both.*
- *If $Tab : A \in \Delta$ and $A \neq 0, 1$, then $Tab :<\in \Delta$.*
- *Δ is* saturated, *i.e.*
  - *(i) If $Fab : A \setminus B \in \Delta$ and $a \sqsubset_\Delta b$, then there is a c such that $Tca : A, Fcb : B \in \Delta$.*
  - *(ii) If $Fab : A/B \in \Delta$ and $a \sqsubset_\Delta b$, then there is a c such that $Tbc : B, Fac : A \in \Delta$.*
  - *(iii) If $Tab : A \bullet B \in \Delta$, then there is a c such that $Tac : A, Tcb : B \in \Delta$.*
  - *(iv) If $Tab : \diamond A \in \Delta$, then there are c and d such that $Tac : 0, Tcd : A, Tdb : 1 \in \Delta$.*
  - *(v) If $Fab : \Box^\downarrow A \in \Delta$, then there are c and d such that $Tca : 0, Fcd : A, Tbd : 1, Tcd :<\in \Delta$.*
  - *(vi) If $Tab : A \in \Delta$, $A, B \neq 0, 1$, then $Tab :<\in \Delta$.*
- *Δ is deductively closed, i.e. if a sequent $\alpha_1, \ldots, \alpha_n \Rightarrow \beta$ derivable, and for all $1 \leq i \leq n$: $T\alpha_i \in \Delta$, then $T\beta \in \Delta$.*

From a maxiconsistent set we can construct a canonical model for horizontal semantics:

Definition 18 (Canonical Model): *Let Δ be a maxiconsistent set. The canonical model for Δ is $M_\Delta = \langle W, <, I, \{R_i | i \in I\}, \{S_i | i \in I\}, V\rangle$, where*

1. *W is the set of labels occurring in Δ.*
2. *$a < b$ iff $a \sqsubset_\Delta b$*
3. *aR_ib iff $Tab : 0_i \in \Delta$*
4. *aS_ib iff $Tab : 1_i \in \Delta$*
5. *$\langle a, b\rangle \in V(p)$ iff $Tab : p \in \Delta$.*

Fact 2: *If Δ is maxiconsistent, M_Δ is a horizontal relational model for $\boldsymbol{L}\Diamond$*

Proof:
By the definition of $\sqsubset_\Delta$, $<$ is transitive and $R_i, S_i \subseteq <$. The requirement that Δ is maxiconsistent ensures that $V(p) \subseteq <$ for arbitrary atoms p. ⊣

Lemma 10 (Truth Lemma): *For all maxiconsistent sets Δ, formulas A and labels a, b:*

$$Tab : A \in \Delta \text{ iff } M_\Delta, ab \models A$$

Proof:
By induction over the complexity of A. Cases 1–5 are identical to the proof for vertical semantics.

7. $A = \Diamond B, \Rightarrow$ By saturation, $Tab :<\in \Delta$, and there are c and d such that $Tac : 0, Tcd : B, Tdb : 1 \in \Delta$. By induction hypothesis, $cd \models B$. The construction of M_Δ ensures that aRc, dSb, and $a < b$. Hence $ab \models \Diamond B$.
8. $\Leftarrow$ By the semantics of $\Diamond$, there are c and d such that aRc, dSb, and $cd \models B$. By induction hypothesis, $Tcd : B \in \Delta$. By the construction of M_Δ, $Tac : 0, Tdb : 1 \in \Delta$. Since $\vdash ac : 0, cd : B, db : 1 \Rightarrow ab : \Diamond B$ and Δ is deductively closed, $Tab : \Diamond B \in \Delta$.
9. $A = \Box^{\downarrow} B, \Rightarrow$ Suppose $ab \not\models \Box^{\downarrow} B$. Then there are c and d such that cRa, bSd, $c < d$, and $cd \not\models B$. By induction hypothesis, $Fcd : B \in \Delta$, and the construction of M_Δ ensures that $Tca : 0, Tbd : 1 \in \Delta$. Since $\vdash ca : 0, ab : \Box^{\downarrow} B, bd : 1 \Rightarrow cd : B$, $Tcd : B \in \Delta$, which violates consistency.
10. $\Leftarrow$ Suppose $Tab : \Box^{\downarrow} B \notin \Delta$. By completeness, $Fab : \Box^{\downarrow} B \in \Delta$. By saturation, there are c and d such that $Tca : 0, Tbd : 1, Tcd :<, Fcd : B \in \Delta$. Hence $cRa, bSd, c < d$ and $cd \not\models B$, which is impossible due to the truth conditions for "$\Box^{\downarrow}$". ⊣

In the definition of Henkin witnesses, the clauses for the modal formulas are modified:

Definition 19 (Henkin witnesses):

(v) *If $\alpha = Tab : \Diamond A$, then $H(\Delta, \alpha) = \Delta \cup \{\alpha, Tac : 0, Tcd : A, Tcd :<, Tdb : 1\}$, where c and d are the first distinct labels not occurring in Δ.*

(vi) *If $\alpha = Fab : \Box^{\downarrow} A$ and $a \sqsubset_\Delta b$, then $H(\Delta, \alpha) = \Delta \cup \{\alpha, Tca : 0, Fcd : A, Tbd : 1, Tcd :<\}$, where c and d are the first distinct labels not occurring in Δ.*

For horizontal semantics, we can ignore well-coloredness.

Lemma 11: *If $\alpha \in \Delta$ and Δ is deeply consistent and acyclic, then $H(\Delta, \alpha)$ is also deeply consistent and acyclic.*

Proof:
Preservation of acyclicity is as above. As for deep consistency, the proof runs basically as above too. For the Lambek connectives, it is just identical, and for the modal operators, it is even simpler since fewer formulas are added at each step of adding Henkin witnesses.

Lemma 12: *If Δ is deeply consistent and acyclic, and $A \neq 0, 1$, then either $\Delta \cup \{Tab : A, Tab :<\}$ or $\Delta \cup \{Fab : A\}$ is deeply consistent and acyclic.*

Proof:
As above.

The construction of a maxiconsistent T–F set doesn't differ from the vertical case.

Lemma 13: *If Δ is deeply consistent and acyclic, then Δ_ω is maxiconsistent.*

Proof:
See above.

Lemma 14: *If $a_1b_1 : A_1, \ldots, a_nb_n : A_n \Rightarrow \alpha$ is canonically labeled and underivable, then $\{Ta_ib_i : A_i, F\alpha\} \cup \{Ta_ib_i :< | 0 \neq A_i \neq 1\}$ is deeply consistent and acyclic.*

Proof:
See above.

As in the horizontal case, the last lemma ensures that for each underivable sequent, we can construct a model that falsifies it. ⊣

6 Strong completeness

Kurtonina (1995) shows that **L1** is also complete in its relational interpretation if conceived as an "axiomatic-sequent" calculus. Under this perspective, derivability and entailment are relations between (sets of) sequents and not formulas.

Definition 20 (Derivability): *A sequent φ is $\boldsymbol{L}\Diamond$-derivable from a set of sequents Γ iff there is a sequence of sequents $\delta_1, \ldots, \delta_n$ with $\delta_n = \varphi$ such that each δ_i is either an axiom of $\boldsymbol{L}\Diamond$, an element of Γ, or it can be obtained from $\delta_1, \ldots, \delta_{i-1}$ by inference rules of $\boldsymbol{L}\Diamond$.*

A sequent $X \Rightarrow A$ is said to be true in a model M iff $\|X\|_M \subseteq \|A\|_M$. This leads immediately to a notion of entailments between sequents.

Definition 21 (Entailment): *A sequent φ is (horizontally/vertically) entailed by a set of sequents Γ iff in all models where all elements of Γ are (horizontally/vertically) true, φ is true as well.*

Theorem 3 (Strong Completeness): *A sequent φ is $\boldsymbol{L}\Diamond$-derivable from a set of sequents Γ iff it is vertically entailed by Γ iff it is horizontally entailed by Γ.*

Proof:
Soundness is straightforward by induction on the length of derivations. As for completeness,

Kurtonina's (1995) proof for the corresponding theorem for **L1** immediately carries over to **L**$\Diamond$. We assume that φ is not derivable from Γ and show that it cannot be entailed. First we define the set Γ_l as the set of all canonically labeled instances of elements of Γ. The notion of derivability of sequents above (definition 20) is extended to labeled sequents by replacing **L**$\Diamond$ with its labeled version. A set Δ of labeled $T - F$ formulas is called (vertically/horizontally) Γ-maxiconsistent iff it is (vertically/horizontally) maxiconsistent and furthermore it is Γ-closed, i.e. if a sequent $\alpha_1, \ldots, \alpha_n \Rightarrow \beta$ is derivable from Γ_l, and for all $1 \leq i \leq n : T\alpha_i \in \Delta$, then $T\beta \in \Delta$. Since Γ-maxiconsistency is a stronger notion than maxiconsistency, fact 1/2 and lemma 3/10 also hold if we replace the latter by the former. In a similar fashion, we strengthen the notion of deep consistency to Γ-consistency by replacing derivability with derivability from Γ_l. The lemmas 4–7/11–13 remain valid if we replace deep consistency with Γ-consistency. Now suppose $\Gamma \not\vdash_{\mathbf{L}\Diamond} \varphi = A_1, \ldots, A_n \Rightarrow B$. Since this sequent is not derivable from Γ, neither is any of its canonically labeled versions $ab_1 : A_1, \ldots, b_{n-1}c : A_n \Rightarrow ac : B$ derivable from Γ_l. Hence $\{Tab_1 : A_1, \ldots T : b_{n-1}c : A_n, Fac : B\}$ is Γ-consistent, i.e. it can be extended to a Γ-maxiconsistent set which gives rise to a canonical model. By the truth lemma, this model falsifies φ. On the other hand, Γ-closure guarantees that all elements of Γ are true in this model. Hence φ cannot be entailed by Γ. $\dashv$

7 Translation **L**$\Diamond \Rightarrow$ **L**

Versmissen (1996) proves soundness and completeness of the following translation from **L**$\Diamond$ to **L**:

Definition 22:

$$
\begin{aligned}
[p] &= p && \textit{(p atomic)} && (1)\\
[A \bullet B] &= [A] \bullet [B] && && (2)\\
[A \setminus B] &= [A] \setminus [B] && && (3)\\
[A/B] &= [A]/[B] && && (4)\\
[\Diamond_i A] &= t_{i,0} \bullet [A] \bullet t_{i,1} && && (5)\\
[\Box_i^{\downarrow} A] &= t_{i,0} \setminus [A]/t_{i,1} && && (6)\\
[(_i X)_i] &= t_{i,1}, [X], t_{i,1} && && (7)
\end{aligned}
$$

where $t_{i,0}$ and $t_{i,1}$ are fresh atomic formulas.

Versmissen's proof is purely syntactic. Completeness of **L**$\Diamond$ in horizontal relational interpretation lends itself naturally for a semantic proof, following the strategy of Kurtonina and Moortgat (1997). First we show that every horizontal model for **L**$\Diamond$ can be transformed into a model for **L** which verifies the same formulas modulo translation.

Lemma 15: *Let $M = \langle W, <, I, \{R_i | i \in I\}, \{S_i | i \in I\}, V\rangle$ be an arbitrary model for **L**$\Diamond$ and M' be the **L**-model $\langle W, <, V'\rangle$, where V' extends V by mapping $t_{i,0}$ to R_i and $t_{i,1}$ to S_i. Then it holds that for all **L**$\Diamond$-formulas and bracketed sequences of **L**$\Diamond$-formulas X that*

$$M, \langle a, b\rangle \models X \textit{ iff } M', \langle a, b\rangle \models [X]$$

Proof: By induction on the complexity of X. The induction base and the induction step for "$\bullet$", "\", "/" and sequencing are straightforward.

1. $X = \Diamond B, \Rightarrow$ Suppose $M, ab \models \Diamond A$. Then there are c, d such that $aR_0c, M, cd \models B$, and dR_1b. By induction hypothesis, $M', cd \models [B]$. By the construction of M', $M', ac \models t_0, M', db \models t_1$. Hence $ac \models t_0 \bullet [B]$ and $ab \models t_0 \bullet [B] \bullet t_1 = [\Diamond B]$.

2. $\Leftarrow$ Suppose $M', ab \models t_0 \bullet [B] \bullet t_1$. Then there are c, d with $M', ac \models t_0, M', cd \models [B], M', db \models t_1$. By hypothesis, $M, cd \models B$, and by the construction of M', aR_0c, dR_1b. Hence $M, ab \models \Diamond B$.

3. $X = \Box^{\downarrow} B, \Rightarrow$ Suppose $M, ab \models \Box^{\downarrow} B$. This entails that $a < b$. Now assume that $M', ab \not\models t_0 \setminus [B]/t_1$. Then there are c, d such that $M', ca \models t_0, M', bd \models t_1, M', cd \not\models [B]$. By hypothesis $M, cd \not\models B$, and by the construction of M', cR_0a, bR_1d. By transitivity of $<$, $c < d$, which contradicts the assumption.

4. $\Leftarrow$. Suppose $M', ab \models t_0 \setminus [B]/t_1$, and $M, ab \not\models \Box^{\downarrow} B$. Then there are c, d such that cR_0a (i.e. $M', ca \models t_0$) and bR_1d (i.e. $M', bd \models t_1$). By transitivity, $c < d$, and $M, cd \models B$. By induction hypothesis, $M', cd \models [B]$, which leads to a contradiction.

5. $X = (Y)$ Analogous to $\Diamond$.

$\dashv$

Theorem 4:

$$\vdash_{\mathbf{L}\Diamond} X \Rightarrow A \text{ iff } \vdash_{\mathbf{L}} [X] \Rightarrow [A]$$

Left to right is an easy induction on the length of derivations. For the other direction, assume that $\not\vdash_{\mathbf{L}\Diamond} X \Rightarrow A$. By completeness, there is a model M such that $M \models X, M \not\models A$. By the truth lemma, $M' \models [X], M' \not\models [A]$. By soundness, $\not\vdash_{\mathbf{L}} [X] \Rightarrow [A]$.

$\dashv$

8 Conclusion

In this paper we proposed to extend the relational semantics for **L** that was developed in Pankrat'ev (1994) and Andréka and Mikulás (1994) to Moorgat's (1996) multimodal extension **L**$\Diamond$ of **L**. We investigated two such extension, one being inspired by Versmissen's (1996) translation from **L**$\Diamond$ to **L**, and one by the standard Kripke semantics of unary modal operators. We established soundness, weak completeness and strong completeness for both interpretations, thereby following Kurtonina's (1995) strategy of using labeled deduction to construct canonical models. Finally we showed that one of these relational interpretations can be employed to give a semantic proof for the completeness of Versmissen's translation.

These results raise several issues for further research. First, we restricted attention to multimodal Lambek calculi which comprise just one binary mode. It seems worth exploring whether natural relational interpretations are possible for a multimodal system where several binary modes coexist[3] Second, the unary modes that we considered are plain residuation modalities; they neither interact with each other nor with the binary mode. Many applications of multimodal Lambek calculi assume interaction postulates between the modes. It remains to be seen which of these postulates have semantic counterparts under a relational interpretation. Finally, our models where entirely abstract, perhaps it is possible to relate the semantics developed here to more concrete instantiations of relational interpretation that are specific to the intended linguistic applications of the logics under discussion.

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[3] This was justly pointed out by an anonymous referee.

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What is the real logic of games after all?

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Abstract

Over the past several decades many versions of game semantics for logic were introduced, none of which, however, has been perceived as a refinement of or a reasonable alternative to classical semantics. Apparently there are three major reasons:

1) Those semantics are often not very natural and artificially adjusted to some non-semantical considerations, such as justifying certain existing axiomatic systems (intuitionism, linear logic).

2) Non-artificial game semantics either do not yield any new logic but classical logic, or have to totally depart from the original meaning of the language of logic as a formal counterpart of natural language that allows us to talk about the surrounding world.

3) The technical results (axiomatization, decidability/enumerability etc.) proven so far regarding the logics generated by relatively natural game semantics have not been sufficiently comprehensive and have been usually limited to only some rather restricted fragments of the language.

The present paper introduces and studies a game semantics which, as the author believes, does not suffer from the above shortcomings. The corresponding logic is fully axiomatized in a language that can be considered the union of the languages of classical and linear logics. This logic is a conservative extension of classical logic and of a certain version of linear logic, thus unifying these two logics (in a broad sense of the term "linear logic") within the framework of one general semantical approach.

Key words: game logics, game semantics, linear logic.

1 Introduction

The goal of the present paper is to introduce a mathematically elegant and intuitively convincing semantics for parallel game combinations and announce some main results regarding the corresponding logic, such as a full axiomatization of it at both the propositional and the predicate levels. These results are stated without proofs as their high degree of technical involvement, in conjunction with the space limitations, did not allow the author to include any

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serious proofs here. A lengthy extended paper with detailed proofs for the main technical results claimed in this article is currently in the process of preparation.

Beginning from Lorenzen [8], a variety of game semantics have been studied by different authors. With some technical differences in game-playing rules, common to most of those semantics is the idea of considering sentences as games between two players — *proponent*, who is trying to “defend” the sentence, and *opponent*, who is “attacking” the sentence, where disjunction or existential quantifier signifies proponent’s move/choice, and conjunction or universal quantifier is opponent’s move/choice. The main motivation for Lorenzen was to find a semantical justification of Heyting’s intuitionistic logic. After trial and error, the goal of obtaining completeness of that logic with respect to Lorenzen’s semantics was achieved [5]. The value of this achievement, however, seems dubious, as it was obtained by artificially tuning the semantics to adjust it to the goal, at the expense of losing its naturality and modeling potential. After all, virtually every formal system can be “justified” with some sort of specially designed technical semantics, and the whole point is how natural and usable the semantics is with respect to which completeness is proven. Among the non-convincing features of Lorenzen’s game semantics is treating atoms as “black boxes” ([2]) that cannot be opened by proponent, so that an atom can be defended only if opponent has previously claimed its truth, as well as a series of other supplementary rules governing the conditions under which statements can be attacked or re-attacked, yielding somewhat confusing asymmetries between the two players.

Among the most natural game semantics known so far is the one introduced by Blass [4]. The idea of understanding negation as role switch and disjunction, conjunction and quantifiers as choice operations, employed in [4], was not new and had been used, in an essentially the same way, by virtually all the authors who tackled game semantics. Blass, however, was apparently the first to add to considerations another, parallel way of combining games, thus opening the possibility of a game-semantical justification for the distinction between the two (additive and multiplicative) sorts of operators in substructural logics. Blass’s semantics no longer treats atoms as “black boxes” and it enjoys perfect symmetry between the two players. As a result of this symmetry, however, the corresponding game logic collapses to classical logic as long as atoms are interpreted as elementary (no-move) games or even finite-depth games, — the natural requirement that, by the way, was present in Lorenzen’s approach. To avoid trivialization of the logic of games, Blass insisted that atoms be interpreted as infinite-depth, indetermined (and thus highly non-constructive) games. This precluded his semantics from being considered a reasonable and appealing alternative to classical semantics (what infinite-depth game should be the interpretation of 0=0 after all?).

With atoms interpreted as infinite-depth games, Blass’s semantics yields a substructural logic that is strictly between Affine logic (linear logic plus weakening) and classical logic. Unfortunately, however, this logic has not been axiomatized, and no other reasonable syntactic characterization (such as a decision algorithm) of it has been found, either. So far we only have two partial completeness results. In particular, it was shown in [4] that:

1. The multiplicative propositional fragment of Blass's logic (the propositional fragment where only multiplicative operators are allowed) is decidable, and it is a proper extension of the corresponding fragment of Affine logic.

2. The fragment whose formulas can be represented with purely additive (multiplicative-free) sequents is also decidable and it is exactly the corresponding fragment of Affine logic.

The question on decidability or recursive enumerability of the full (additive-multiplicative) logic, or even the full propositional fragment of it, however, remains open.

The fact that Blass's logic takes us far beyond Girard's original linear logic was perceived as its shortcoming by some authors who believed that linear logic must be the "right logic". Abramsky and Jagadeesan [1] revised Blass's semantics by considering a version of games that are always by opponent to start. This way, at the expense of losing symmetry between players and somewhat "denaturalizing" Blass's semantics, the goal of disvalidating the weakening rule has been achieved, even though the resulting logic still remains stronger than linear logic. In particular, in [1] it was shown that the multiplicative propositional fragment of the logic induced by this semantics is exactly the corresponding fragment of linear logic plus the mix rule (the latter being inadmissible in linear logic). Just as in [4], however, the question on decidability or enumerability of the full additive-multiplicative logic, or the propositional fragment of it, was not answered in [1].

Thus, neither linear logic, nor affine logic or even linear logic with mix can be considered the logic of games. What is then the "real logic of games"? The present paper contains an answer, with a full axiomatization of such a logic without any restrictions on its language. The language of the logic of games I consider here is a proper extension of the language of additive-multiplicative linear logic. There are two features that make the former stronger than the latter:

1) My language contains an additional sort of operators that allow us to capture games with imperfect information.

2) It also has two sorts of atoms: one for any games, and one for elementary (zero-depth) games. Elementary games are, in fact, nothing but predicates in the classical sense, which makes my semantics a generalization/refinement of classical semantics and makes my logic a conservative extension of classical logic, with classical operators understood as their multiplicative counterparts.

Besides considering a more expressive language of the logic of games, some of the features that distinguish my approach from that of Blass include:

1) I allow both of the players to have legal moves in the same position. This way we get a more general concept of games, as well as a more natural semantics for multiplicatives as parallel game combination operators. For example, my understanding of the multiplicative conjunction $A \wedge B$ is a game playing which

means playing the two games A and B in parallel, where both players are free to make a move in either component as long as that move is legal for that component. On the other hand, in Blass's games, only one of the players may have a legal move in any position. To comply to this requirement, Blass would not allow opponent to move in A as long as proponent has a legal move in B. This somewhat disqualifies plays over $A \wedge B$ as really parallel and independent plays of A and B: what can be done in one of the conjuncts depends on what is happening in the other conjunct.

2) I, as in [6], require that proponent's strategies be effective. This eliminates the necessity to appeal to infinite-depth, indetermined games in completeness proofs. This also yields a more interesting semantics as it makes games an adequate formalization of the intuitive notion of interactive computational problems, thus making the concept of winnability a generalization of computability/decidability. The requirement of effectiveness of proponent's strategies was absent in [4]. In fact, it would be redundant within the framework of Blass's approach as it would not affect the concept of validity in any way.

3) My games can contain variables. This makes games a generalization of classical predicates and enables us to define quantifiers as natural unary operations on games.

2 The propositional logic of racefree games

From now on I will refer to opponent and proponent with the more technical names P_0 and P_1, respectively.

Let us fix three infinite sets of expressions: *Mov* (*moves*), *Var* (*variables*) and *Dom* (*domain*). Without loss of generality we may assume that $Dom = \{0, 1, 2, \ldots\}$ and *Mov* is the set of all strings over the alphabet $\{0\text{-}9, a\text{-}z, .\}$. A move suffixed with "$P_0$" or "$P_1$" is called a *labeled move*, where the label P_i indicates which player has made the move. Sequences of labeled moves we call *runs*, and finite runs call *positions*.

The letters $\alpha, \beta, ...$ will be used as metavariables for moves, $\Gamma, \Delta, ...$ for runs, $x, y, ...$ for variables and $X, Y, ...$ for finite sets of variables. An *instance* of such an X is a function $f: X \to Dom$. If $X_1 \subseteq X_2$, f_1 is an instance of X_1 and f_2 is an instance of X_2 that agrees with f_1 on X_1, then f_1 is said to be the *restriction* of f_2 to X_1 and f_2 is an *extension* of f_1 to X_2.

Definition 2.1 Where n is a natural number, an n-ary *game* is a tuple

$$[M_0, M_1, X, F]$$

such that:

- M_0 and M_1 are functions that send positions to subsets of *Mov*. We call the elements of $M_i\langle\Gamma\rangle$ P_i's *legal moves* in position Γ. A *legal run* is a run where, for any initial segment $\langle\Gamma, \alpha P_i\rangle$, we have $\alpha \in M_i\langle\Gamma\rangle$. We assume that if Γ is an illegal run, then $M_0\langle\Gamma\rangle = M_0\langle\Gamma\rangle = \emptyset$.

- X is a set of n (distinct) variables.
- F is a function that returns one of the values P_0 or P_1 for every pair (Γ, f), where Γ is a legal run and f is an instance of X. F tells us who the winner is in a given run for a given instance of X. This function is extended to all runs by stipulating that an illegal run is lost by the player who has made the first illegal move.

A *strict game* is a game where, in every position, at most one of the players has legal moves. Games that only have a finite number of legal runs we call *finite*. A game with no infinitely long legal runs is said to be *finite-depth*. A finite-depth game is *bounded-depth* if there is an upper bound for the lengths of its legal runs; this bound is called the *depth* of the game. Games without any legal moves we call *elementary*. Every predicate $R(X)$ on Dom can be understood as the elementary game $(\emptyset, \emptyset, X, F)$ such that, for any instance f of X, $F(\langle\rangle, f) = P_1$ iff $R(f)$ is true (here $\emptyset$ stands for the function that returns $\emptyset$ for every position, and $\langle\rangle$ is the empty run). This makes predicates special cases of games, and classical logic a special case of the logic of games.

Here are definitions of some basic propositional operations on games:

Definition 2.2 (*Negation*)

$$\neg[M_0, M_1, X, F] = [M_0{}', M_1{}', X, F'],$$

defined as follows. Let $\overline{\Gamma}$ mean the result of interchanging the labels P_0 and P_1 in run Γ. Then, for every Γ, we have:

- $M_0'\langle\Gamma\rangle = M_1\langle\overline{\Gamma}\rangle$ and $M_1'\langle\Gamma\rangle = M_0\langle\overline{\Gamma}\rangle$.
- For every instance f of X, $F'(\Gamma, f) = P_1$ iff $F(\overline{\Gamma}, f) = P_0$.

Thus, $\neg G$ is nothing but G with the roles of the players switched.

Definition 2.3 (*Multiplicative conjunction*)

$$[M_0^l, M_1^l, X^l, F^l] \wedge [M_0^r, M_1^r, X^r, F^r] = [M_0, M_1, X^l \cup X^r, F],$$

defined as follows. For $d \in \{l, r\}$, let Γ^d stand for the result of removing from Γ all labeled moves except those that start with $d.$, and then deleting the prefix $d.$ in the remaining labeled moves. Then:

- For every position Γ, $i \in \{0, 1\}$ and $d \in \{l, r\}$, we have $M_i\langle\Gamma\rangle = \{d.\alpha \mid d \in \{l, r\},\ \alpha \in M_i^d\langle\Gamma^d\rangle\}$.
- For every instance f of $X^l \cup X^r$, whenever Γ is a legal run, we have $F(\langle\Gamma\rangle, f) = P_1$ iff $F^l(\langle\Gamma^l\rangle, f^l) = F^r(\langle\Gamma^r\rangle, f^r) = P_1$, where f^l and f^r are the restrictions of f to X^l and X^r, respectively.

Playing $G^l \wedge G^r$ means playing, in parallel, the two games G^l and G^r, where P_1 has to win in both components to be the winner. A move $l.\alpha$ (resp. $r.\alpha$) means that the move α was made in the left (resp. right) component. Obviously, when applied to elementary games, the operations $\neg$ and $\wedge$ again produce elementary games and, remembering that the latters are nothing but predicates, these two operators act exactly as the classical negation and conjunction. However, when not restricted to elementary games, their behavior rather resembles that of the corresponding *multiplicative* operators of linear logic.

Definition 2.4 (*Additive conjunction*)

$$[M_0^l, M_1^l, X^l, F^l] \sqcap [M_0^r, M_1^r, X^r, F^r] = [M_0, M_1, X^l \cup X^r, F],$$

where:

- $M_0\langle\rangle = \{l, r\}$, $M_1\langle\rangle = \emptyset$ and, for any run Γ, $i \in \{0,1\}$ and $d \in \{l, r\}$, we have $M_i\langle dP_0, \Gamma\rangle = M_i^d\langle\Gamma\rangle$.
- For every instance f of $X^l \cup X^r$, we have: $F(\langle\rangle, f) = P_1$ and, for any run Γ and $d \in \{l, r\}$, we have $F(\langle dP_0, \Gamma\rangle, f) = F^d(\langle\Gamma\rangle, f^d)$, where f^d is the restriction of f to X^d.

As its name indicates, $\sqcap$ corresponds to the additive conjunction of linear logic. In $G^l \sqcap G^r$ the opening move is by P_0, who has to choose between l (*left*) and r (*right*), after which the play continues according to the rules of the chosen subgame. If no initial choice was made, this game is considered won by P_1.

Definition 2.5 (*Trivially lost game*) $\bot$ is the elementary 0-ary game whose only possible run (empty run) is lost by P_1.

The remaining propositional game operations can be defined in terms of the above four operations as follows:

Definition 2.6

Additive disjunction	$A \sqcup B = \neg(\neg A \sqcap \neg B)$
Multiplicative disjunction	$A \vee B = \neg(\neg A \wedge \neg B)$
Multiplicative implication	$A \rightarrow B = \neg A \vee B$
Trivially won game	$\top = \neg\bot$

One can see that the only difference between $\sqcup$ and $\sqcap$ is that, in the former, it is P_1 rather than P_0 who makes the initial choice between *left* and *right*. $A \vee B$, just like $A \wedge B$, means a parallel combination of A and B, with the difference that, in order to win in $A \vee B$, for P_1 it is sufficient to win in one of the components. $\top$ is an elementary game automatically won by P_1. As for $\rightarrow$, it corresponds to the intuition of *reducing* game B to game A. To win this game, for P_1, means to win B as long as P_0 wins game A against P_1.

Let Γ and Δ be runs and $i \in \{0,1\}$. We say that Δ is a *P_i-delay* of Γ iff

- for each $j \in \{0,1\}$, the subsequence of P_j-labeled moves of Γ is the same as that of Δ, and
- for any $n, m \geq 1$, if the nth P_i-labeled move is made later than (is to the right of) the mth P_{1-i}-labeled move in Γ, then so is it in Δ;

in other words, in Δ each player has made the same sequence of moves as in Γ, only, in Δ, P_i might have been acting with some delay. Then we say that game $[M_0, M_1, X, F]$ is *racefree* if, for each $i \in \{0,1\}$, whenever f is an instance of X, Γ is a run with $F(\Gamma, f) = P_i$ and Δ is a P_i-delay of Γ, we have $F(\Delta, f) = P_i$.

Intuitively, a racefree game is a game where speed does not give any advantage to a player: if it has a "good" move in a given position, then that move will as well remain "good" after the other player has made several moves, so that no opportunities can be missed by not acting fast enough (see Example 2.8

below). Notice that all strict games are racefree. One can verify that $\neg$, $\wedge$ and $\sqcap$ preserve the racefree property of games. The closure of elementary games under these three operations forms a natural class of finite racefree games. And the closure of elementary games under $\neg$ and $\sqcap$ forms a class of finite strict games.

My philosophy is to allow P_0 follow any strategies while require that P_1 only follows effective strategies. A way to materialize this approach is to assume that P_1 is a (deterministic) Turing machine M with a special additional state called $MOVE$ and an additional read-only tape (along with the usual read/write work tape) called the *run tape*. The latter serves as a dynamic input for the machine, spelling, at any moment, the current position of the game, i.e. the sequence of moves made by the players so far. For simplicity we assume that M has no halt state, — the effect of halting, if we wish, can be achieved by infinitely looping without reading and writing.

A *configuration* is a tuple (S, W, w, R, r), where S is a state of M, W is contents of the work tape, w is a position of the read/write head on the work tape, R is contents of the run tape, and r is a position of the read head on the run tape. In the *initial configuration*, the run tape is blanc and the work tape contains an instance of the variables of the game played by the machine; in that configuration, as this can be understood, S is the start state of the machine and w, r are the leftmost positions on the corresponding tapes.

A configuration $C' = (S', W', w', R', r')$ that can legally follow the current configuration $C = (S, W, w, R, r)$ is said to be a *successor* of C. The conditions that S', W', w', r' should here satisfy must be clear: these four parameters are uniquely determined by the previous configuration and the transition function of the machine in the standard way. The only nondetermined and nonstandard parameter is R', which will be affected by the possible moves that P_0can make during the transition from C to C'. The condition that we require R' to satisfy is the following:

- If $S \neq MOVE$, then R' is the result of appending to R any (possibly empty) finite sequence of P_0-labeled moves.
- If $S = MOVE$, then R' is the result of appending to R any finite sequence of P_0-labeled moves followed by αP_1, where α is (the string spelled by) the initial segment of W up to the first blanc symbol.

Thus, M makes a move by entering the $MOVE$ state, in which case the contents of the work tape from its beginning up to the first blanc cell of the tape will be interpreted as the move it has made. While M is making this move (or just going to the next step without a move), any finite number of any moves by P_0 may be appended to the contents of the run tape, corresponding to the intuition that not only the strategy of P_0 can be arbitrary, but its speed as well, as long as P_0 can only make a finite number of moves per time unit. This sort of nondeterministic transition from R to R' creates different branches of computation that M may have to follow, each branch being a sequence of configurations where the first configuration is the initial configuration and every other configuration is a successor of the previous one. Each branch incrementally spells a (possibly infinite) run on the run tape. We call such a run a *run with* M. Then we say that

Definition 2.7 Machine M *wins* game $[M_0, M_1, X, F]$ iff for every instance f of X and every run Γ with M (with f on its work tape in the initial configuration) we have $F(\Gamma, f) = P_1$.

A game for which such a winning Turing machine M exists will be said to be *effectively winnable*.

The logic GM_{prop} that I am going to introduce has the following connectives: $\neg, \wedge, \vee, \rightarrow, \bot, \top, \sqcap, \sqcup$ and two sorts of infinitely many propositional atoms: *elementary atoms* and *general atoms*. An *interpretation* is a function * which assigns, to each elementary (resp. general) atom P, an elementary (resp. racefree) game P^*. This assignment naturally extends to all formulas by commuting with all operators: $\bot^* = \bot$; $(\neg Q)^* = \neg(Q^*)$; $(Q \wedge R)^* = Q^* \wedge R^*$; $(Q \sqcap R)^* = Q^* \sqcap R^*$; etc. A formula Q is said to be *valid* iff the game Q^* is effectively winnable for every interpretation *.

Example 2.8 The formula $A \vee \neg A$ is valid, whether A be an elementary atom, a general atom or a compound formula. Using the notation Γ^d from Definition 2.3, any legal run Γ of $A^* \vee \neg A^*$ will have two disjoint subruns: Γ^l and Γ^r that are legal runs of A^* and $\neg A^*$, respectively, with Γ won by P_1 if and only if either Γ^l or Γ^r is won. This game will be won by a machine that uses the following strategy: Ignore the input (an instance of the variables of the game); keep polling the run tape; every time you see that P_0 has made a move $l.\alpha$ (resp. $r.\alpha$), make the move $r.\alpha$ (resp. $l.\alpha$). In other words, keep mimicking, in A^*, P_0's moves in $\neg A^*$, and vice versa.

Of course, unless A^* is a strict game, the runs Γ^l and Γ^r may not be exactly symmetric. For example, if both of the players have a legal move in A^*, a fast P_0 can make moves in both A^* and $\neg A^*$ before the machine gets a chance to copy P_0's first move. Generally this could make winning the game impossible. But what saves the day is that the definition of interpretation requires A^* to be racefree. Even though the machine cannot ensure that $\Gamma^r = \overline{\Gamma^l}$ (see Definition 2.2 for notation $\overline{\Gamma}$), a little analysis of the situation can convince us that the above mimicking strategy, however, guarantees that Γ^r is a P_1-delay of $\overline{\Gamma^l}$. So that, if Γ^l is lost, which implies that $\overline{\Gamma^l}$ is won, then Γ^r, as a P_1-delay of $\overline{\Gamma^l}$, will be won, which makes the mimicking strategy a winning strategy for $A^* \vee \neg A^*$.

On the other hand, $A \sqcup \neg A$ is not valid, even when A is an elementary letter. To see this, it is sufficient to consider an interpretation that interprets A as an undecidable predicate.

By an *elementary formula* we mean a formula that does not contain general atoms, $\sqcap$ or $\sqcup$. A *surface occurrence* of a subformula is an occurrence that is not in the scope of $\sqcap$ or $\sqcup$. Understanding $A \rightarrow B$ as an abbreviation for $\neg A \vee B$, an occurrence is *negative*, if it is in the scope of an odd number of occurrences of $\neg$; otherwise it is *positive*. The *elementarization* of a formula Q is the result of replacing in Q all positive (resp. negative) surface occurrences of general atoms by $\bot$ (resp. $\top$), all (positive or negative) surface occurrences of the form $R_1 \sqcap R_2$ by $\top$ and all (positive or negative) surface occurrences of the form $R_1 \sqcup R_2$ by $\bot$.

Definition 2.9 The *axioms* of **logic** GM_{prop} are all the elementary formulas that are classical tautologies, and the *rules* of inference are:

1. $Q' \vdash Q$, where Q' has at most two occurrences of a certain elementary atom a, and Q is the result of replacing in Q' (each occurrence of) this atom by a general atom A.
2. $Q' \vdash Q$, where Q' is the result of replacing in Q a negative (resp. positive) surface occurrence of a subformula $R_1 \sqcap R_2$ (resp. $R_1 \sqcup R_2$) by R_1 or R_2.
3. $Q', \vec{S} \vdash Q$, where Q' is the elementarization of Q and $\vec{S}$ is a (the minimal) set of formulas satisfying the following condition: If Q has a positive (resp. negative) surface occurrence of a subformula $R_1 \sqcap R_2$ (resp. $R_1 \sqcup R_2$), then, for each $i \in \{1, 2\}$, $\vec{S}$ contains the result of replacing this occurrence in Q by R_i.

Theorem 2.10 $GM_{prop} \vdash Q$ iff Q is valid. Moreover:

a) There is an effective procedure that takes a GM_{prop}-proof of a formula Q and constructs a Turing machine that wins Q^* for every interpretation *.

b) If $GM_{prop} \not\vdash Q$, then Q^* is not effectively winnable for some interpretation * that interprets general atoms as finite strict games of depth ≤ 2.

One can show that GM_{prop} is decidable in polynomial space. How does GM_{prop} compare with other logics? Obviously its restriction to elementary formulas is classical propositional logic. And its *general-base fragment*, — the fragment where elementary atoms are forbidden, — is a proper extension of propositional Affine logic. Here are two examples of principles underivable in the latter but provable in GM_{prop}:

$$(A \wedge B) \vee (C \wedge D) \rightarrow (A \vee C) \wedge (B \vee D)$$

$$\big(A \wedge (C \sqcap D)\big) \sqcap \big(B \wedge (C \sqcap D)\big) \sqcap \big((A \sqcap B) \wedge C\big) \sqcap \big((A \sqcap B) \wedge D\big) \rightarrow (A \sqcap B) \wedge (C \sqcap D)$$

As these are apparently the shortest possible formulas that separate GM_{prop} from Affine logic, we can say that the latter is "almost complete" with respect to our semantics. As for the distance between the general-base fragment of GM_{prop} and classical logic, it is huge as GM_{prop} does not prove $A \rightarrow A \wedge A$.

3 The predicate logic of racefree games

Here come two quantifier-style operations on games.

Definition 3.1 (*Additive universal quantifier*)

$$\sqcap x[M_0, M_1, X, F] = [{M_0}', {M_1}', X\text{-}\{x\}, F'],$$

where:

- ${M_0}'\langle\rangle = Dom$, ${M_1}'\langle\rangle = \emptyset$ and, for any run Γ, $i \in \{0,1\}$ and $d \in Dom$, we have ${M_i}'\langle dP_0, \Gamma\rangle = M_i\langle\Gamma\rangle$.
- For every instance f of X-$\{x\}$, we have: $F'(\langle\rangle, f) = P_1$ and, for any run Γ and $d \in Dom$, we have $F'(\langle dP_0, \Gamma\rangle, f) = F(\langle\Gamma\rangle, f^d)$, where f^d is the extension of f to X such that $f^d(x) = d$.

The above operation is similar to $\sqcap$ and its meaning can be described by $\sqcap xG(x) = G(0)\sqcap G(1)\sqcap G(2)\sqcap\ldots$. At the beginning of the play over $\sqcap xG$, which can be written as $\sqcap xG(x)$, P_0 has to choose one particular element d of *Dom*. Then the play continues according to the rules of $G(d)$, where the latter is $G(x)$ with the possible instances of the variables restricted to those that return d for x. Note that effective winnability of a game G containing variables can be redefined in terms of effective winnability of 0-ary games by stipulating that G is effectively winnable iff its $\sqcap$-closure is so.

Definition 3.2 (*Blind universal quantifier*)

$$\forall x[M_0, M_1, X, F] = [M_0, M_1, X\text{-}\{x\}, F'],$$

where for every instance f of X-$\{x\}$ and any run Γ, we have $F'(\langle\Gamma\rangle, f) = P_1$ iff for every extension f' of f to X, $F(\langle\Gamma\rangle, f') = P_1$.

The operation $\forall x$, which obviously acts as the classical universal quantifier for elementary games, is not exactly similar to $\wedge$ when not restricted to elementary games, and has no counterpart in linear logic (see Section 4 for a definition of the "multiplicative universal quantifier"). This operation in fact produces what is called *games with imperfect information* ([2]). Intuitively, $\forall xG(x)$ is the same as $\sqcap xG(x)$, with the difference that, in $\forall xG(x)$, the particular value d that P_0 "selects" and for which the game $G(d)$ should be won, is invisible to P_1, so that it has to play blindly in a way that guarantees success no matter what this value d is.

For each of the above two sorts of universal quantifier we have a corresponding existential quantifier:

Definition 3.3 *Additive existential quantifier* $\sqcup xA(x) = \neg\sqcap x\neg A(x)$
Blind existential quantifier $\exists xA(x) = \neg\forall x\neg A(x)$

We define one more, rather technical, game operation:

Definition 3.4 (*Substitution of variables*) Suppose $G = [M_0, M_1, X, F]$ is a game, $\{y_1, \ldots, y_n\} = Y$ are (distinct) variables, and $z_1, \ldots, z_n$ is a sequence of n (not necessarily distinct) variables. We may assume that $X \cap Y = \{y_1, \ldots, y_k\}$ $(0 \leq k \leq n)$ and X-$Y = \{x_1, \ldots, x_m\}$ $(m \geq 0)$, so that $X = \{x_1, \ldots, x_m, y_1, \ldots, y_k\}$. Then

$$G(y_1/z_1, \ldots, y_n/z_n) = [M_0, M_1, X', F'],$$

where:

- $X' = \{x_1, \ldots, x_m, z_1, \ldots, z_k\}$.
- For any position Γ and instance f of X', $F'(\Gamma, f) = F(\Gamma, g)$, where g is the instance of X such that $g(x_i) = f(x_i)$ and $g(y_j) = f(z_j)$, for each i, j with $1 \leq i \leq m$, $1 \leq j \leq k$.

The language of logic GM_{pred} that I am going to introduce, in addition to the propositional connectives of GM_{prop}, has four quantifiers: $\forall, \exists, \bigsqcap, \bigsqcup$. The set *Var* that we fixed at the beginning of Section 2 serves as its set of individual variables. The language has two sorts of predicate letters: *elementary letters* and *general letters*. With each (elementary or general) letter P is associated an n-tuple of distinct variables, called its *canonical tuple*, where the number n is called the *arity* of P. We assume that for each n, there are infinitely many n-ary elementary and general letters. The expression $P(x_1, \ldots, x_n)$, where P is an n-ary elementary (resp. general) letter and $x_1, \ldots, x_n$ are (not necessarily distinct) variables, is called an elementary (resp. general) *atom*. If here $(x_1, \ldots, x_n)$ is the canonical tuple of P, then $P(x_1, \ldots, x_n)$ is an (elementary or general) *canonical atom*. *Formulas* are built in the standard way, only, for simplicity of analysis, we assume that no formula can contain both a free and a bound occurrence of the same variable.

An *interpretation* for a formula Q is a function * which assigns an elementary (resp. racefree) game $P^* = [M_0, M_1, X, F]$ to each elementary (resp. general) canonical atom $P(x_1, \ldots, x_n)$, such that, if a variable x is in X but not among $x_1, \ldots, x_n$, then x does not occur in Q. This assignment extends to all formulas as follows: For an atom $P(y_1, \ldots, y_n)$, $(P(y_1, \ldots, y_n))^* = P^*(x_1/y_1, \ldots, x_n/y_n)$, where $(x_1, \ldots, x_n)$ is the canonical tuple of P; $(Q \wedge R)^* = Q^* \wedge R^*$; $(\bigsqcap x Q)^* = \bigsqcap x(Q^*)$; etc.

A formula Q is said to be *valid* iff the game Q^* is effectively winnable for every interpretation * for Q. Redefining the terms "elementary formula", "surface occurrence" and "elementarization" for the predicate language, an *elementary formula* now is a formula which does not contain general letters, $\sqcap, \sqcup, \bigsqcap$ or $\bigsqcup$; a *surface occurrence* of a subformula is an occurrence that is not in the scope of one of these four additive operators; and the *elementarization* of a formula Q is the result of replacing in Q all positive (resp. negative) surface occurrences of general atoms by $\perp$ (resp. $\top$), all (positive or negative) surface occurrences of the form $R_1 \sqcap R_2$ or $\bigsqcap x R$ by $\top$, and all (positive or negative) surface occurrences of the form $R_1 \sqcup R_2$ or $\bigsqcup x R$ by $\perp$.

Definition 3.5 The *axioms* of **logic** GM_{pred} are all the elementary formulas provable in classical predicate calculus, and the *rules* of inference are:

1. $Q' \vdash Q$, where Q' has at most two occurrences of a certain elementary n-ary letter a, and Q is the result of replacing in Q' this letter by a general n-ary letter A.

2a. $Q' \vdash Q$, where Q' is the result of replacing in Q a negative (resp. positive) surface occurrence of a subformula $R_1 \sqcap R_2$ (resp. $R_1 \sqcup R_2$) by R_1 or R_2.

2b. $Q' \vdash Q$, where Q' is the result of replacing in Q a negative (resp. positive) surface occurrence of a subformula $\bigsqcap x R(x)$ (resp. $\bigsqcup x R(x)$) by $R(y)$ for some y (y not bound in Q').

3. $Q', \vec{S} \vdash Q$, where Q' is the elementarization of Q and $\vec{S}$ is a (minimal, finite) set of formulas satisfying the following conditions:

a) If Q has a positive (resp. negative) surface occurrence of a subformula $R_1 \sqcap R_2$ (resp. $R_1 \sqcup R_2$), then, for each $i \in \{1, 2\}$, $\vec{S}$ contains the result of replacing this occurrence in Q by R_i;

b) If Q has a positive (resp. negative) surface occurrence of a subformula $\bigsqcap x R(x)$ (resp. $\bigsqcup x R(x)$), then $\vec{S}$ contains the result of replacing this occurrence in Q by $R(y)$ for some y not occurring in Q.

Theorem 3.6 $GM_{pred} \vdash Q$ iff Q is valid. Moreover:

a) There is an effective procedure that takes a GM_{pred}-proof of a formula Q and constructs a Turing machine that wins Q^* for every interpretation * for Q.

b) If $GM_{pred} \not\vdash Q$, then Q^* is not effectively winnable for some interpretation * for Q that interprets general atoms as finite strict games of depth ≤ 2.

Remember the definitions of the game properties "racefree", "bounded-depth", "finite-depth", "finite", "elementary, "strict" from Section 2. These definitions can be extended to game operations by stipulating that an operation is racefree (bounded-depth, etc.) if it preserves the racefree (bounded-depth, etc.) property of games. One can show that all of the game operations that we introduced so far are racefree, bounded-depth and finite-depth. So that, in view of the clause (b) of Theorem 3.6, GM_{pred} can be called not only the logic of racefree games, but also the logic of bounded-depth racefree games or the logic of finite-depth racefree games.

Because of space limitations, I did not have a chance to give a sufficiently detailed explanation of the (very natural) intuition behind the racefree property of games. To the reader who has difficulty visualizing this concept, I would suggest to understand "racefree games" as elements of the closure of elementary games under $\neg, \wedge, \vee, \rightarrow, \sqcap, \sqcup, \bigsqcap, \bigsqcup, \forall$ and $\exists$, which forms a natural subclass of racefree bounded-depth games, and for which GM_{pred} remains complete.

Obviously GM_{pred} is recursively enumerable. One can also show that the $\forall, \exists$-free fragment of GM_{pred} is decidable (in polynomial space), even though it contains the quantifier-type operators $\bigsqcap$ and $\bigsqcup$.

It is easy to see that the elementary fragment of GM_{pred} (the fragment where general letters and additives are forbidden) is exactly classical predicate logic, so that GM_{pred} is a conservative extension of classical logic. This is so because classical semantics is, in fact, nothing but the projection of our game semantics on the elementary fragment of the language. To appreciate the technical strength of Theorem 3.6, note that the language of GM_{pred} is by an order of magnitude more expressive than the classical language as it includes the language of additive-multiplicative linear logic as well.

The latter (the language of linear logic) is the fragment of the language of GM_{pred} where elementary letters and $\forall, \exists$ are forbidden. One can show that the corresponding fragment of GM_{pred} is a proper extension of predicate Affine logic; in particular, it is exactly logic ET that I introduced in [6]. This is a decidable logic that was defined nonaxiomatically, through a decision algorithm. The semantics with respect to which ET was shown to be sound and complete in [6], was similar to, but not the same as, the present game semantics. In particular, the games considered in [6], just as Blass's [4] games, were strict, and hence the multiplicative operations there had to be strict, while the multiplicative operations defined in the present paper are not strict.

Finally, the fragment of GM_{pred} where general letters are forbidden, — the *elementary-base fragment*, — is exactly logic L introduced in [7], where I axiomatically defined that logic with a (yet another) semantics in mind, with respect to which L was proven to be sound and complete. That semantics understood sentences as *tasks* that are to be carried out by an agent (machine, robot) working as a slave for its master (user, environment).

Thus, there are at least four technically quite different semantics that yield the same (relevant fragments of) logic GM_{pred}: our present game semantics, classical semantics and the two different semantics employed in [6] and [7]. Most likely Blass's game semantics can also be added to this list, — at least, the two syntactically defined fragments of the logic generated by that semantics (the propositional multiplicative fragment and the purely additive fragment) for which Blass had found completeness proofs, coincide with the corresponding fragments of GM_{pred}. This remarkably high degree of robustness of GM_{pred} with respect to technical variations of the notion of validity is a strong indication of its being natural, and a justification of my claim that GM_{pred} is "the real logic of games". After all, the Church-Turing thesis is based on a similar argument of robustness with respect to broad variations of how algorithms are formalized.

4 Beyond finite-depth operations

The class of natural racefree operations is not limited to those studied so far. Here are some more operations of this kind:

Definition 4.1 (*Multiplicative universal quantifier*)

$$\bigwedge x[M_0, M_1, X, F] = [{M_0}', {M_1}', X\text{-}\{x\}, F'],$$

defined as follows. For $d \in Dom$, let Γ^d mean the result of removing from Γ all labeled moves except those that start with $d.$, and then deleting the prefix $d.$ in the remaining labeled moves. Then:

- For every position Γ, and $i \in \{0,1\}$, we have $M_i{}'\langle\Gamma\rangle = \{d.\alpha \mid d \in Dom,\ \alpha \in M_i\langle\Gamma^d\rangle\}$.
- For every instance f of X-$\{x\}$, whenever Γ is a legal run, we have $F'(\langle\Gamma\rangle, f) = P_1$ iff $F(\langle\Gamma^d\rangle, f^d) = P_1$ for every d with $d \in Dom$, where f^d is the extension of f to X such that $f^d(x) = d$.

Thus, $\bigwedge x G(x)$ can be thought of as the infinite multiplicative conjunction $G(0) \wedge G(1) \wedge G(2) \wedge \ldots$.

Another natural racefree operation is $!$, a counterpart of what is called *storage* (*exponential*) in linear logic, for which I am only giving a semiformal definition:

Definition 4.2 (*Storage*) A legal run of $!G$ can be thought of as a tree whose arcs are labeled with labeled moves. Branches of this tree spell legal runs of G, and thus to each node of the tree corresponds a legal position of G spelled by the (sub)branch that ends in that node. In particular, the play starts with the root as the only node of the tree; at any time, P_1 can make a move α in (add an αP_1-arc to) any leaf, as long as α is a legal move for P_1 in the corresponding position of G; so can P_0, with the difference that the node where the move is made may as well be internal, in which case a new branch is created. Then, P_1 is considered the winner iff it is the winner, in the sense of G, in all branches of the tree.

Thus, in $!G$, P_0 is allowed to restart the game G an unlimited number of times without terminating the already-in-progress plays of G, creating, this way, more and more parallel plays of G with the possibility to try different strategies in each of them and become the winner as long as one of those strategies succeeds. Moreover, P_0 does not have to really restart G from the very beginning every time it "restarts" it, but rather it can select to continue it from any of the previous positions, thus depriving P_1 of the possibility to reconsider its previously made moves. Notice that $!$ is not a strict operation. A strict (and hence slightly less natural) version of $!$, called the *repetition operator*, was introduced by Blass in [3].

The operations $\bigwedge$ and $!$ have their duals:

Definition 4.3 *Multiplicative existential quantifier* $\bigvee x A(x) = \neg \bigwedge x \neg A(x)$
Costorage $?A = \neg ! \neg A$

Unlike all the other operations studied in this paper, $\bigwedge x, \bigvee x, !$ and $?$ are no longer bounded-depth or finite-depth. This is my natural excuse for not including them in the present analysis of the logic of (bounded- or finite-depth) racefree games. Exploring the game logic whose language would include these four operators is an interesting and presumably challenging task for the future.

The following list summarizes the properties of our game operations.

- Racefree: All
- Bounded-depth, Finite-depth: All except $\bigwedge, \bigvee, !, ?$
- Elementary: All except $\sqcap, \sqcup, \bigsqcap, \bigsqcup$
- Finite: All except $\bigsqcap, \bigsqcup, \bigwedge, \bigvee, !, ?$
- Strict: All except $\wedge, \vee, \rightarrow, \bigwedge, \bigvee, !, ?$

Note that the obviously elementary operations $\bigwedge$ and $!$ (together with their duals $\bigvee$ and $?$) are hardly of any interest in the context of elementary games, for we have $\bigwedge xG = \forall xG$ and $!G = G$ as long as G is elementary.

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A continuum of incomplete intermediate logics

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Abstract

Although in 1977 V.B. Shehtman constructed the first Kripke incomplete intermediate logic, no-one in known literature has completed his work by constructing a continuum of such logics. After a substantial reminder on how an incomplete logic can be obtained, I will construct a sequence of frames similar to those used by Jankov and Fine. None of these frames can be reduced by p-morphism to another; simultaneously, there are no p-morphisms from generated subframes of Fine-Shehtman frame onto any frame from the considered sequence. All of the frames satisfy all Shehtman's axioms. Therefore, by using characteristic formulae of the frames from the sequence it is possible to obtain the desired conclusion.

In 1970s, a number of important, deep and technically complicated results concerning relational semantics for modal logics was obtained by such authors as S. Thomason, K. Fine, M.S. Gerson, R.I. Goldblatt, J. F. A. K. van Benthem and W. Blok; it was the Golden Age of the subject, see [Bu82], [Bu83] and [ChZ97] for references and summaries of the most important works. The main goal of my paper is to attract attention to the fact that many important results lack superintuitionistic analogues, although the task of transferring them is highly nontrivial. This gap may be partially due to the fact that Kripke semantics never became so popular in the realm of intermediate logics as they are in the realm of modal logics, which are more suitable and flexible tools to deal with frames. There were fewer experts working on relational semantics for intuitionistic logics. One of the most distinguished persons in the field, V. B. Shehtman constructed in 1977 the first Kripke incomplete intermediate propositional logic. His construction was based mainly on the frame from [Fi74b], but he has very ingeniously used the formula introduced in [GdJ74]. Nevertheless, he did not follow

Fine's suggestion that it seems to be possible to construct a continuum of incomplete logics. Such a continuum of $S4$ logics was presented in [Ry77] in the same year as Shehtman's construction; it is known, however, that incompleteness of a modal logic does not imply incompleteness of its intuitionistic equivalent. In [On72] one may find information that there exists a continuum of incomplete predicate superintuitionistic logics. Unfortunately, the information is given without a proof; besides, it is far easier to construct an incomplete predicate superintuitionistic logic than to construct an incomplete propositional superintuitionistic logic. It is really surprising but up to this day no-one has published a proof that there exists a continuum of such logics. I am going to fill this gap.

In this paper I shall try to conform to the standard definitions and symbols which may be found, for example, in a monograph by Chagrov & Zakharyaschev [ChZ97]. Nevertheless, for the sake of convenience, let me remind the most standard ones. Unless otherwise stated, by a logic I shall mean a superintuitionistic (intermediate) logic.

Definition 1 A (Kripke) structure/frame *consists of a set and a relation of partial order* $\mathcal{F} = \langle W, \leqslant \rangle$.

Definition 2 A substructure/subframe *of a structure* $\mathcal{F} = \langle W, \leqslant \rangle$ *is a frame* $\mathcal{G} = \langle V, \leqslant_1 \rangle$ *where* $V \subseteq W$ *and* $\leqslant_1 = V^2 \cap \leqslant$.

Definition 3 A (Kripke) model *is an ordered pair* $\mathcal{M} = \langle \mathcal{F}, \mathcal{B} \rangle$ *consisting of a frame* $\mathcal{F} = \langle W, \leqslant \rangle$ *and a function* $\mathcal{B}$ *from a set of propositional variables to the set of upward closed subsets of* W*. Valuation is extended to all formulae in the usual way.*

Now I want to introduce two technical notions, weaker than *finite approximability* (*finite model property*) and stronger than *completeness*

Definition 4 *A logic is* fa-approximable *iff the set of its theorems coincides with the set of all formulae true in some class of rooted frames with no infinite antichains.*

Definition 5 *A logic is* ac-approximable *iff the set of its theorems coincides with the set of all formulae true in some class of frames with no infinite ascending chains — Chagrov & Zakharayaschev call such orders* Noetherian.

Professor A. Wroński suggested to me that fa-approximability implies ac-approximability. It would give rise to a following picture:

finite approximability $\Rightarrow$ fa-approximability $\Rightarrow$ ac-approximability $\Rightarrow$ completeness.

In my paper, I shall prove that there exists a continuum of propositional logics even outside the broadest class, i.e. class of all complete logics. Nevertheless, first let me describe how an incomplete logic can be obtained — it is an easy generalization of Shehtman's method [Sh77].

Theorem 1 *A logic L lacks ac-approximability iff its modal companion above* **Grz** τL *is incomplete.*

Proof It is enough to recall that **Grz** is complete with respect to all partial orders without infinite ascending chains. $\dashv$

Theorem 2 *If there exists any rule of the form*

$$\frac{(\psi \ \vee \ (\psi \ \rightarrow \ e(\chi))) \ \rightarrow \ \chi}{\chi}$$

(e is any uniform substitution) which is not admissible in some intermediate logic, then this logic lacks ac-approximability and thus lacks finite model property.

Proof (sketch) In any family of frames adequate for the logic (if there exists such) there must be a frame validating

$$(\psi \ \vee \ (\psi \ \rightarrow \ e(\chi))) \ \rightarrow \ \chi$$

with all substitutions (because the formula belongs to the logic) and refuting χ under some valuation. It can be easily seen that such a frame must contain an infinite ascending chain — see figure 1. $\dashv$

Corollary 3 *If an intermediate logic satisfies the assumptions of theorem 2, then its companion above* **Grz** *is incomplete.*

Proof A consequence of theorems 1 and 2. $\dashv$

In fact far more can be proven about such a logic — see my forthcoming paper [Li02].

Theorem 4 *If there exists any rule of the form*

$$\frac{\begin{array}{c}(\psi \ \vee \ (\psi \ \rightarrow \ e(\chi))) \ \rightarrow \ \chi \\ \psi \ \leftrightarrow \ \varsigma \ \rightarrow \ \tau \\ \tau \ \rightarrow \ e(\tau)\end{array}}{\chi}$$

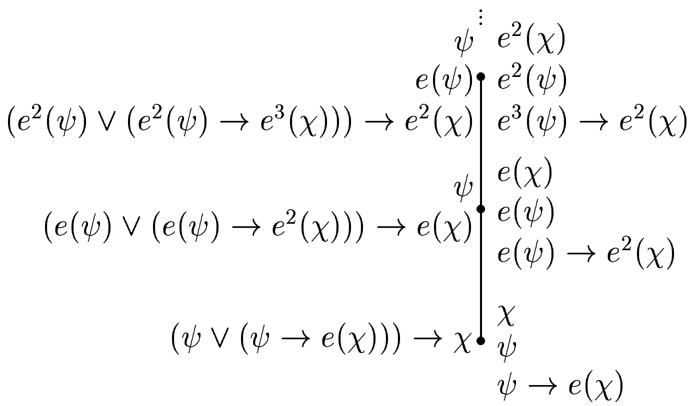

Figure 1: A model refuting χ but verifying $\psi \vee (\psi \to e(\chi)) \to \chi$

which is not admissible in a logic L, then in any class of frames adequate for L (if there exists any) there must be a structure containing an infinite comb *or* a willow *(see fig. 2) as a substructure; thus L must lack both ac-approximability and fa-approximability.*

Proof Similar to the proof of theorem 2 — see fig. 4. ⊣

Let me recall the celebrated Gabbay-de Jongh axioms [GdJ74]

$$\boldsymbol{bb_n} = \bigwedge_{i=0}^{n}((p_i \to \bigvee_{j \neq i} p_j) \to \bigvee_{j \neq i} p_j) \to \bigvee_{i=0}^{n} p_i \qquad (n \geq 1)$$

which are complete with respect to the class of all finite frames of branching n. It is well known that they can be refuted in the infinite comb. Nevertheless, not every frame containing the infinite comb as a substructure refutes these axioms — see figure 3. Therefore the following theorem is nontrivial:

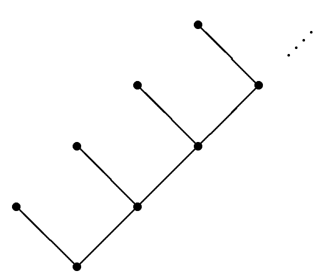

Figure 2: Infinite comb

Theorem 5 *If there exists any rule of the form*

$$\frac{\begin{array}{c}(\psi \ \lor \ (\psi \ \to \ e(\chi))) \ \to \ \chi \\ \psi \ \leftrightarrow \ \varsigma \ \to \ \tau \\ \varsigma \ \lor \ \tau \ \to e(\varsigma) \ \land \ e(\tau) \\ \chi \ \leftrightarrow \ \psi \ \lor \ e(\tau)\end{array}}{\chi} \tag{1}$$

which is not admissible in some intermediate logic L, then in any class of frames adequate for L (if there exists any) there must exist a structure refuting $\boldsymbol{bb_n}(n \geq 2)$*. Thus if L contains any of Gabbay-de Jongh axioms, it must be incomplete.*

Proof may be carried out in a manner similar to the one of Shehtman [Sh77], but it is needlessly complicated, e.g. with a superfluous use of transfinite induction. Therefore I would like to sketch more elegant and intuitive proof. Assume then that there is a frame $\mathcal{F}$ for L, a valuation $\mathcal{V}$ and a point x in $\mathcal{F}$ such that $x \not\models_{\mathcal{V}} \chi$. It is easy to check that x must be the root of submodel of $\langle \mathcal{F}, \mathcal{V} \rangle$ depicted on a picture 4. Now let me define a new valuation $\mathcal{B}$ based on $\mathcal{V}$ and inspired by the figure 4:

$$\mathcal{B}(p_0) \ := \ \bigcup_{n \,=\, 3m} \mathcal{V}(e^n(\varsigma) \ \land \ \bigwedge_{k \,=\, 0}^{n \,-\, 1} e^k(\chi)) \ \cup \ \bigcap_{n \,\in\, \omega} \mathcal{V}(e^n(\psi)),$$

$$\mathcal{B}(p_1) \ := \ \bigcup_{n \,=\, 3m+1} \mathcal{V}(e^n(\varsigma) \ \land \ \bigwedge_{k \,=\, 0}^{n \,-\, 1} e^k(\chi)) \ \cup \ \bigcap_{n \,\in\, \omega} \mathcal{V}(e^n(\psi)),$$

$$\mathcal{B}(p_2) \ := \ \bigcup_{n \,=\, 3m+2} \mathcal{V}(e^n(\varsigma) \ \land \ \bigwedge_{k \,=\, 0}^{n \,-\, 1} e^k(\chi)) \ \cup \ \bigcap_{n \,\in\, \omega} \mathcal{V}(e^n(\psi)).$$

Axioms of L and the figure 4 assure us that sets $\mathcal{B}(p_0)$, $\mathcal{B}(p_1)$ and $\mathcal{B}(p_2)$ are distinct and nonempty. It is easily seen that the consequent of $\boldsymbol{bb_2}$ is

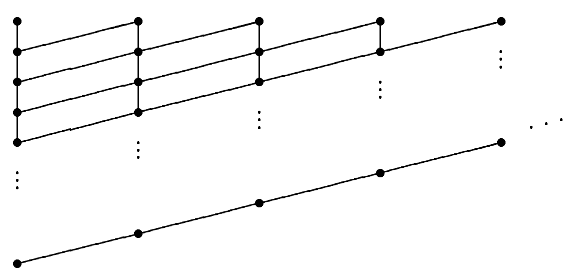

Figure 3: A structure containing the infinite comb as a substructure where Gabbay-de Jongh axiom $\boldsymbol{bb_2}$ is true

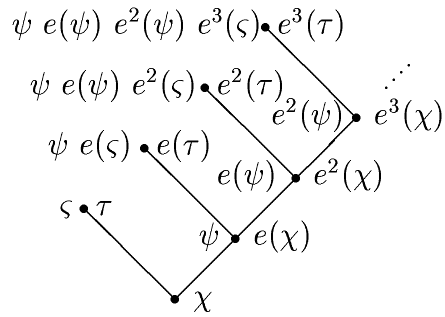

Figure 4: A submodel of $\langle \mathcal{F}, \mathcal{V} \rangle$ whose root refutes χ.

refuted at x under the valuation $\mathcal{B}$. Now suppose that there is some $y \geqslant x$ such that some conjunct of the premise of $\boldsymbol{bb_2}$ is *classically* refuted at y, e.g. $y \vDash_{\mathcal{B}} p_0 \rightarrow p_1 \vee p_2$ and $y \nvDash_{\mathcal{B}} p_1 \vee p_2$. If there is some $n \in \omega$ such that $y \vDash_{\mathcal{V}} e^n(\varsigma)$ then by the axioms of L $y \vDash_{\mathcal{V}} e^m(\chi)$ for any $m \leq n$ and $y \vDash_{\mathcal{B}} p_0 \vee p_1 \vee p_2$, which leads to an immediate contradiction. Hence for no $n \in \omega$, $y \vDash_{\mathcal{V}} e^n(\varsigma \vee \tau)$. But now there must be some $n \in \omega$ such that $y \nvDash_{\mathcal{V}} e^n(\psi)$ (otherwise $y \in \bigcap_{n \in \omega} \mathcal{V}(e^n(\psi))$). Hence $y \nvDash_{\mathcal{V}} e^n(\chi)$ and it is possible to construct an infinite comb similar to the one from the figure 4 whose root is y. But it is easy to find some point from this comb which belongs to $\mathcal{B}(p_0)$ and does not belong to $\mathcal{B}(p_1 \vee p_2)$. ⊣

It may be worth mentioning that the rule 1 is as a matter of fact inspired by the form of axioms in later Shehtman's paper [Sh80]. In his paper from 1977 [Sh77] the axioms were more complicated and to make Shehtman's 1977 theorem a consequence of theorem 5 — as I am going to do — the rule 1 should be replaced by the following one:

$$\frac{\begin{array}{c}(\psi \vee (\psi \rightarrow e(\chi))) \rightarrow \chi \\ \psi \leftrightarrow \varsigma \rightarrow \tau \\ \tau \rightarrow e(\tau) \\ \chi \leftrightarrow \psi \vee e(\psi) \\ e(\psi) \rightarrow \psi \vee e(\tau)\end{array}}{\chi} \tag{2}$$

Now let me consider a family of formulae introduced by Shehtman:

$$\beta_{-1} = p, \qquad \gamma_{-1} = q, \qquad \beta_0 = q \rightarrow p, \qquad \gamma_0 = p \rightarrow q,$$

$$\beta_{n+1} = \gamma_n \rightarrow \beta_n \vee \gamma_{n-1}, \qquad \gamma_{n+1} = \beta_n \rightarrow \gamma_n \vee \beta_{n-1},$$
$$\alpha_n = \beta_{n+2} \wedge \gamma_{n+2} \rightarrow \beta_{n+1} \vee \gamma_{n+1} \qquad (n \in \omega),$$

$$\eta = \alpha_0 \to \alpha_1 \vee \alpha_2, \qquad \epsilon = \alpha_0 \vee \alpha_1,$$
$$\delta = \eta \to \epsilon, \qquad \kappa = \alpha_1 \to \alpha_0 \vee \beta_2$$

If ς stands for $\beta_2 \wedge \gamma_2$, τ stands for $\beta_1 \vee \gamma_1$ and e is defined as follows:

$$e(p) := q \vee (q \to p), \qquad e(q) := p \vee (p \to q),$$

then the following observation allows me to use the variant of theorem 5 concerning the rule 2

α_0	is of the form ψ, i.e.	$\varsigma \to \tau$,
ϵ	is of the form χ, i.e.	$\psi \vee e(\psi)$,
δ	is equivalent to	$(\psi \vee (\psi \to e(\chi))) \to \chi$,
κ	intuitionistically implies	$e(\psi) \to \psi \vee e(\tau)$,
$\tau \to e(\tau)$	is an **Int**-tautology.	

Of course, it would be also possible to use the theorem 5 without any modification. In this case one should define ϵ as $\alpha_0 \vee \beta_2 \vee \gamma_2$ or even $\alpha_0 \vee \beta_2$, δ as $(\alpha_0 \to \alpha_1 \vee \beta_3) \to \alpha_0 \vee \beta_2$ and no κ is needed at all. Nevertheless, I am going to stick to the first paper of Shehtman to make references easier; the paper from 1980 [Sh80] is less known.

Lemma 6 *Axioms δ and κ are true in a structure known as the* Fine frame *(see figure 5). The axiom $\boldsymbol{bb_n}$ is true in a general frame based on the Fine frame and generated by the two upward closed singletons. The same general frame refutes the axiom ϵ.*

Proof is quite easy and may be found, for example, in [Sh77]. ⊣

Corollary 7 (Shehtman) *An intermediate logic L determined by the axioms δ, κ, and $\boldsymbol{bb_2}$ is incomplete.*

Proof A consequence or theorem 5 and lemma 6. ⊣

Now I may construct a continuum of incomplete logics inspired by the ideas from classical Kit Fine's papers [Fi74a], [Fi74b]. I will construct a sequence of frames $\mathcal{F}_n$ (see fig. 6) very similar to the sequence from [Fi74a].

Lemma 8 *For any $n \in \omega$, $\mathcal{F}_n \models \delta \wedge \kappa \wedge \boldsymbol{bb_2}$. Besides, $\mathcal{F}_n \models \epsilon$.*

Proof The fact that Gabbay-de Jongh axioms are true in all of those frames is obvious. It is impossible to refute simultaneously α_0 and α_1 in any of the frames which implies that $\mathcal{F}_n \models \delta \wedge \epsilon$. Validity of κ is checked similarly as in case of the Fine frame. ⊣

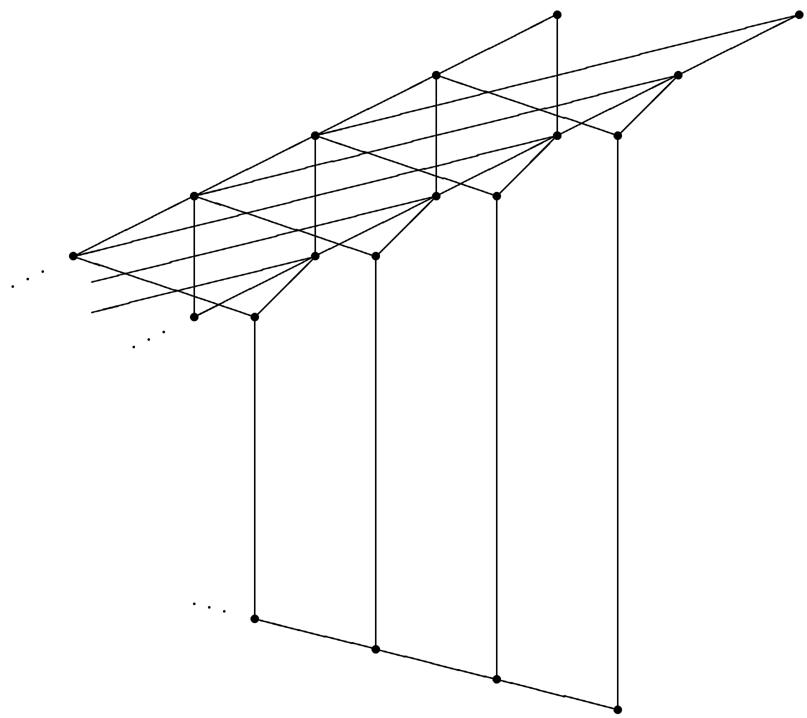

Figure 5: The Fine frame

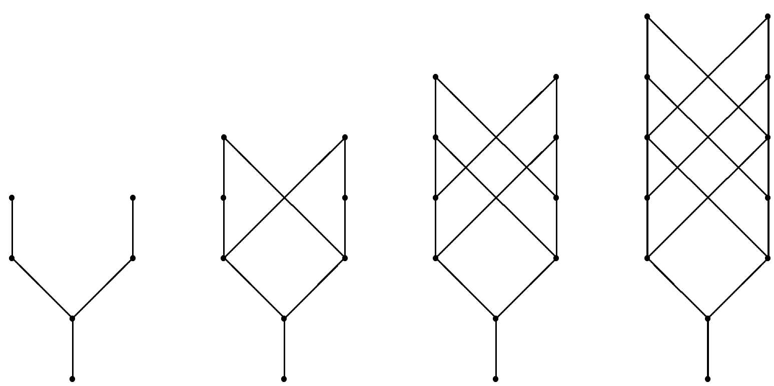

Figure 6: Frames $\mathcal{F}_0$, $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$

Lemma 9 *For any $n \in \omega$, there exists no p-morphism from any generated subframe of $\mathcal{F}_n$ onto $\mathcal{F}_m$ $(m \neq n)$. In other words,*

$$\mathcal{F}_n \models \beta^{\#}(\mathcal{F}_m, \perp)(m \neq n),$$

where $\beta^{\#}(\mathcal{F}_m, \perp)$ is a Jankov formula for $\mathcal{F}_m$.

Proof is similar to the one in [Fi74a] (by induction). ⊣

Lemma 10 *For any $n \in \omega$, there exists no p-morphism from any generated subframe of the Fine frame onto $\mathcal{F}_n$. In other words, Jankov formulae for all the sequence are satisfied in the Fine frame.*

Proof as above. ⊣

Theorem 11 *Distinct subsets of natural numbers generate distinct intermediate logics whose axioms are δ, κ, $\boldsymbol{bb_2}$ and Jankov formulae of those frames from the sequence whose indices belong to a given subset of ω. All of those logics are incomplete.*

Proof The fact that those logic are all distinct is a consequence of lemmata 8 and 9. The fact that these logics are incomplete follows from the theorems 5 and lemmata 6, 10 — a suitable inference rule is not admissible in any of the logics. ⊣

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Linguistic Processors and Their Application to a GeorgianText to Speech System

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Linguistic processors for the Georgian language and the problem of their use in the system of machine synthesis of speech are discussed. In realizing a spoken dialogue we can pursue the task to bring the machine "voice" closer to the human voice. This may be achieved by means of text intonation and giving to the sounding of "voice" the inflection of human voice. For creating one kind of intonation contour we suggest the use of a morphological analytical processor, and for achieving some approximation to the natural sounding of voice - a phonetic processor.

Key-words: processor, morphologic, phonetic, speech, intonation.

Linguistic processors already have a great theoretical (linguistic) importance, but with the passage of time also obtain an increasing technological. They become reliable auxiliary means in various spheres of human activity (machine translation, automated teaching languages, carrying out a dialogue with a computer in natural language, etc.). It seems very important to proceed from already obtained results to develop the research which represents an attempt to grasp the essence of the realization (we would say "verbal innervation") of a verbal act (utterance/understanding - synthesis/analysis) (see Chikoidze, 1997). It is true that the problem falls outside the limits of pure linguistics (cf. "black box" theory), but the research (and the findings) was determined by the work on linguistic processors and the implementation of processors (this makes the role of linguistic processors more important).

Below we will touch upon some linguistic processors (for the Georgian language) and their use in machine synthesis of speech, in particular, processors for the system of compilative synthesis in Georgian.

Speech synthesis or dialogue with computer are important problems (for the details see Ramishvili, Japaridze, 1997). These problems can be better solved if linguistic processors are included in speech synthesis. Obviously, that would draw machine speech closer to human speaking: it would make it resemble natural intonation and approximate the sound of the machine voice to natural voices. From point of view of language modeling, the necessity of this addition is also justified by the fact that the system will be incomplete (from a theoretical point of view), if it fails to provide some approximation and resemblance to the "output" of natural systems (particularly, to human speech).

For the creation of one kind of natural intonation contour (in particular, the intonation of general questions) one may use a morphological (namely, analytical) linguistic processor, and for the achievement of natural sound we suggest a phonetic processor.

For the definition of an intonation stress position (in a sentence expressing a general question) by the system of compilative synthesis of speech, the information about the predicate is of decisive importance - the intonation stress falls on the predicate. This means that it is necessary to include in a verbal dialogue system a morphological analytical processor that will find a predicate (which may be expressed by a verb personal form or by a group-word predicate) automatically. The search and fixing of predicates will happen by analyzing automatically each word-form of a text, viz. by identification of all affixes included in any word-form and by obtaining correct (unambiguous) information about them.

The personal form of the verb is distinguished among all other Georgian word forms by the complexity of its structure. Accordingly, its analyzing algorithm is also complicated. Besides the fact, that complexity is due to homonyms (that is a common phenomenon in any language), it is also made worse by the prefixes and suffixes of Georgian word-forms. For this reason, there was even made an attempt to carry out analysis not from the left side to the right (the normal procedure), but from the right side to the left. The preference was given to the first alternative, for the following reasons:

1). Beginning the word-form analysis from the end (from the right side to the left) is as problematic as (if not more problematic) as it is from the left to the right direction. We have got convinced about this after attempting to analyze verb infinitives (which have a relatively simpler composition than other verb-forms) starting from the end.

2). It is a very strong factor that we read and write from left to right. Therefore we believe that carrying out automatic analysis of Georgian verb forms from the left to the right is as natural a procedure as reading and writing from the left to the right (the more so, since the other alternative - from the right to the left - has no advantages).

Almost all affixes (be it a prefix or suffix) are homonymous and need "deciphering". The problem of homonyms is solved mainly on the morphological level, but often the interpretation of a homonym is possible only on higher (syntactic, semantic) levels (e.g. the form "amesenebine" is a direct contact active voice plusquamperfect form, as well as a causative contact passive voice aorist form and this can only be decided at the syntactic level).

When interpreting those homonyms that can be "deciphered" at the morphological level, we are using stem information (the stem contains appropriate information - characteristic for the given type only) and the rules of affix distribution and arrangement in a word form which have a very complex structure: one affix in a word form depends on the presence of another affix including the root. The process of connecting is controlled by the regularity of structure formation that is based on the principle of language economy. As there is no sentence which would comprise all types of sentences and all parts of a sentence, so there is no word-form containing simultaneously all units of the morphological level. We cannot help to call attention here to the complex category skreeve structure, typical for the Georgian verb. In the formation of a skreeve seven element-categories take part. But instead of 2^7=128 skreeves, expected theoretically, we have 11 ones. The regularity of structure formation makes that as a result, structural elements (affixes in this case) are distributed in a very rational and economical way, and the language keeps to this regularity very closely. By using the ways for connecting elements in a masterly way; one and the same element can have several different functions. Automated "deciphering" (necessary for a computer) is highly possible (native speakers of the language do this unconsciously. For example, *a* is a neutral version marker (a-gebs), prefix (a-geba), "sazedao" ("on the surface") situation marker (a-khatavs). According to the processor, in the forms: a-gebs/a-geba the difference between two *a*- is "guessed" by distribution of suffixes: with eb+s the complex prefix *a*- is a neutral version marker, and with eb+a complex *a* is a "sazedao"-prefix. The neutral version is distinguished from the "sazedao" situation by the stem type (-g- and –khat- are different kinds of stems, the relevant information belongs to the stem). One more interesting fact: the verbs, that have the "sazedao" situation category and use a prefix for this purpose, for the formation of the neutral version use a zero marker (0-khatavs, cf. a-gebs) and the verbs of neutral version with the marker *a*- have no "sazedao" situation category (and they do not need it either, because of their semantics). Taking all of this into account, we cannot help thinking that homonyms of morphological level and homonyms of the other levels must not be considered on one and the same plane. Homonyms of morphological level can be evaluated coming out of language economy principle and be considered as a positive phenomenon of a language - the language uses the same element successfully for different purposes. These units may be called "homonoma" or in general "decipherable" affixes. An analytical processor has a rather complicated composition. It is composed of a stem vocabulary (we designate it by L which is divided in zones – l_i) and a table-algorithm. Table-algorithms are constructed mainly according to the rows of affix attachment to the stem (we have tables for three ranges of prefixes, and six ranges of suffixes), but often (especially in suffix tables) affixes of different ranges (homonyms) are presented together. The operation of the processor is illustrated by the scheme 1 (see appendix): it represents one of the branches of the general scheme which reflects the way of analysis of verbs with the *eb*- stem marker. Dots at the end of an arrow denote numbers of word finishing rules.

The phonetic processor (it was created in accordance with G. Akhvlediani's works, see Akhvlediani, 1949) implies the transformation of an orthographic text into a phonetic record immediately transformable into sound. It is well known fact that orthographic text and its phonetic record do not exactly correspond with each other: the spoken language differs from the written one. Georgian letter-sounds are no exception in this respect (so the saying "in Georgian a word is spoken as it is written and vice versa" is not entirely true). Speaking persons unconsciously change speech-sounds and the listening person equally unconsciously interprets them correctly. Moreover it seems even artificial if we try to say with emphasis (clearly) the following morphological units "masshtabi, vpikrob, gtsers, schirs, gkonda, vazhkatsi" instead of the phonetic units: "mashtabi, fpikrob, ktser, shchirs, kkonda, vashkatsi."

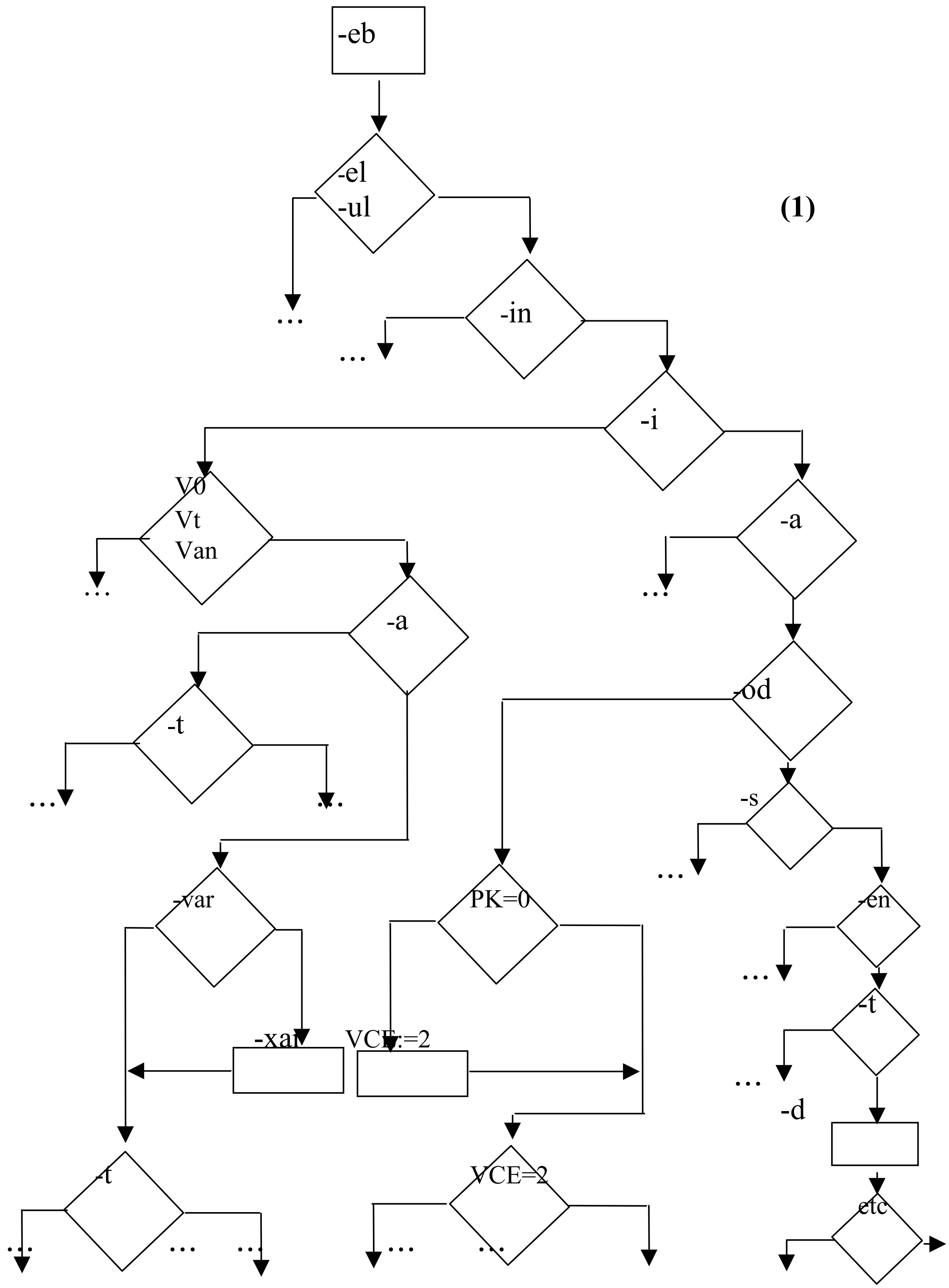
From the above said it follows by no means that changes in speech-sounds should be reflected in Georgian orthography, but only that this is appropriate in synthetic computer systems. We think that in those systems, together with other phenomena (such as intonation, stress, pause, etc.) the changes in the speech-sounds must be taken into account. It is true that in this respect Georgian language differs considerably from many other languages (particularly, phonetic processes that take place in Georgian speech do not have phonological value), but if during the formation of synthetic speech the changes in speech-sounds are not ignored, the quality of the artificial speech (which due to some objective causes is artificial in its sounding) will improve considerably.

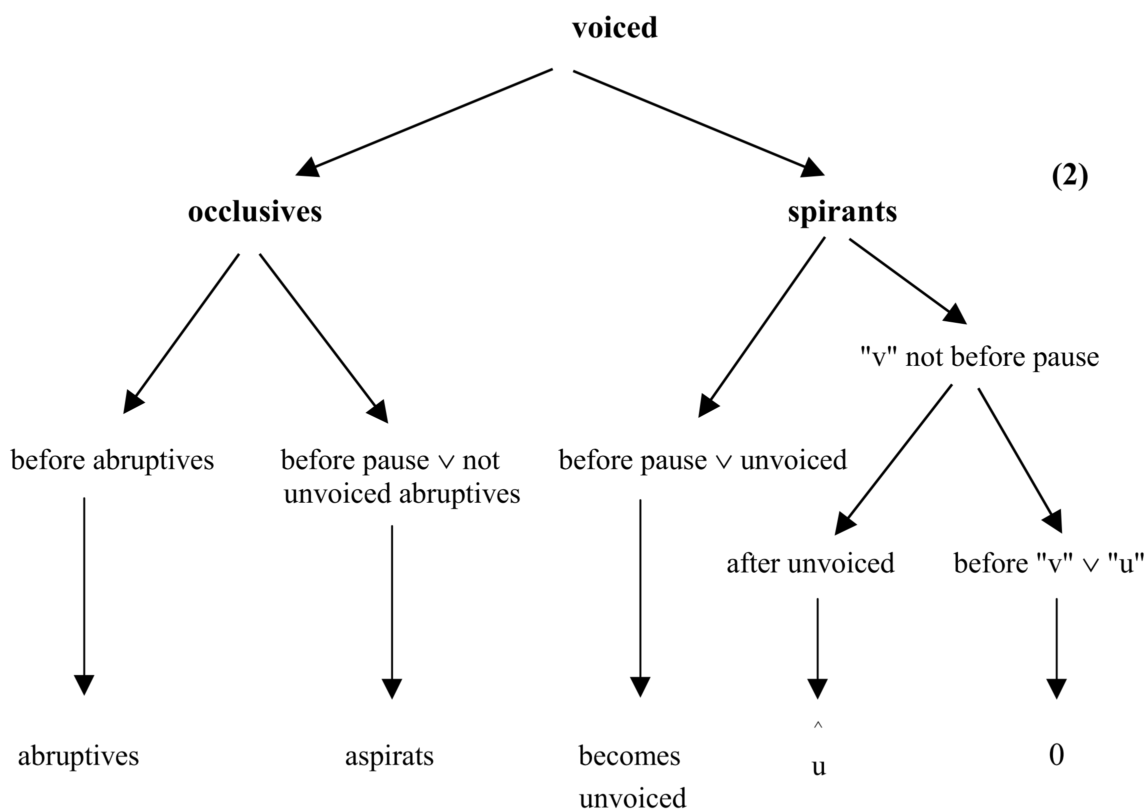
The changes in speech-sounds occur within words and at the places where words meet: this is due to the fact that in fluent speech the boundaries between words are lost and several words or several orthographic units become a single phonetic unit. As a result the problem of transformation of an orthographic text into a phonetic record, together with other aspects, includes two following transformations: placement of pauses in phrases, and bringing of the phonetic units closer to the machine sounds. Solving this problem we sometimes use technical parameters of a text (there is also a text input for compilative synthesis). These parameters are punctuation marks and gaps. Some rules for fixing pauses positions depend on the gap and punctuation marks. Fixing of spaces is also needed for the isolation of words - information about word boundaries (as well as pauses) is essential for many phonetic changes. While discussing changes in the speech-sounds we are considering not the changes caused by co-articulation of a sound, but the changes stipulated phonetically (loss of a sound, its becoming voiceless/voiced, this is followed in most cases by replacing a sound with a homorganic sound of different function, etc.)

The phonetic processor is based on a formalized (systematic) description of the changes in sounds and physiological characteristics of sounds (see Margvelani, 1988) and represents an algorithm - totality of rules (productions), schemes (trees), matrix-tables. It is very easy to include in a synthetic system of speech.

As an example in appendix is given one of the schemes (Fig.2) of the system which deals with the changes in voiced consonants.

Appendix





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A Structured Functional Programming Language

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Abstract. Declarative languages provide means of specifying problem solutions, without giving much attention to how the proposed solution is going to be effectively implemented. When only specification is provided, the system attempts execution using search techniques which may compromise efficiency. In this paper, we propose a new programming language that integrates functional and logic features together with procedural ones. In normal cases, this hybrid language should provide the programmer with advantages of both i.e., *efficiency* and *clarity*. Using this language, very rapid prototyping is possible.

1 Introduction

Purely functional and logic languages are ideal for developers of large-scale packages as they allow the programmers to easily prove the soundness of the built software. Declarative languages have many advantages, which include [7]: the soundness of declarative programs can be easily proven; the exploitation of parallelism in declarative programs is greatly simplified; the clarity of declarative programs is re-enforced. However, as purely functional and logic programs only specify the solution of a given problem without providing any details of the corresponding implementation, compilers of these programming languages are normally slow compared to compilers of imperative languages. Furthermore, when actually trying to write programs, it is often highly inconvenient to be limited to declarative features only. This is mainly due to efficiency of evaluation.

Programs are bound to have an algorithmic details and specifications. In all main line programming languages these are inextricably linked. This is as much true of logic and functional programming languages like Lisp, Prolog, Miranda [18], Haskell [5, 16], etc. as of procedural and object-oriented ones like Pascal, C, Java, C++ etc. By separating the two, it is possible to have different algorithms implementing the same function definition with an automatic guarantee of partial correctness, rather than two obscure function implementations that still have to be verified with respect to a specification in a different language. In reality, this claim is a slight exaggeration, but it does point to an overall (unattainable) objective in a programming language, which is as far away in procedural languages as in declarative ones. The specifications, which a program uses cannot be truly abstract since functions defined using unbounded quantification may not terminate and so need not be executable. One needs to insist on computable definitions.

In this paper, we propose a programming language which is hybrid. It offers declarative and imperative features and thus yields the advantages of both: *clarity* and *efficiency*. The proposed language attempts to cover most of the life cycle of a program which is required to be both *verifiable* and *efficient*. It enables the systematic development of a program starting with the construction of a formal specification to which more and more details can be added and results in a correct and relatively fast implementation. With less effort, one can write programs which consist of executable code and neglect the abstract specification level by the inclusion of specific algorithmic information. In such a case, very rapid prototyping is possible. We will call our language *πlog*.

We first describe the general characteristics of the proposed language as well as the overall structure of *πlog* programs. As a new programming language is best conveyed through examples, we will do it through an illustrative example. After that, we detail the most essential features of *πlog* while giving a glance at the syntax of the language. We then give some implementation issues of the language compiler. Finally, we conclude the paper.

2 The Language General Structure

A *πlog* program is an extension of C to a functional programming language in which C is the command language. Thus a C program is a *πlog* program. More specifically, a *πlog* program is a C program in which various functions are defined not using C, but using a functional programming style to calculate their returned values. It is assumed that the reader has knowledge of C and that therefore the main interest is in the definition of the functions.

The operational semantics behind the functional programming paradigm is now well-known [3, 4, 7, 14, 17]. It consists of using a set of equations considered as left-to-right rewrite rules to simplify a given term, called the *subject term*. Starting from this term, the evaluation process produces a sequence of expressions by repeatedly replacing instances of left sides of rules with their corresponding right sides until no further replacements are possible. An instance of a left side in the subject expression is called a *redex*, and an expression with no redex is said to be in *normal form*. The *pattern-matching* process provides a rule whose left side matches the expression considered [8-13]. When various redexes can be identified by the pattern-matching process, one redex has to be chosen to be reduced. This choice is made based on the reduction strategy in use. There exist several reduction strategies such as *eager* vs. *lazy* and *bottom-up* vs. *top-down* strategies [15]. The most commonly used strategy is called the *outermost-leftmost strategy* also called *normal order*.

A *πlog* program is an environment composed of a set of possibly nested *modules* followed by the *main* body of the *πlog* program. Nesting of modules is achieved by importing them through the directives *uses* and *refines*. Each module is the implementation of an abstract data type and is composed of two main parts: a *specification* of the abstract data type and the *implementation* of the corresponding functions.

- The specification part of a module is mainly a *declaration* part. The first part of a module specification includes a *type name*, *variables*, *constructors* and *functions* declarations. A constructor is a function that does not have any implementation. It is used to construct an object of a given abstract data type. For instance, *empty* and *:* in Example 2.1 are constructors. An object stack that contains the integers 1, 2 and 3 that were pushed in this order, would be represented as 3:2:1:empty. A function is declared by its name, the list of its parameter types and the type of the value returned.
- The implementation part of a module is composed of the *procedures* that implement the functions declared before. Each procedure can declare new objects i.e., types and variables. The body of the procedure is composed of a list of rewriting rules followed by a set of *directives* allowing the compiler to decide which rule to use next. To make the language more expressive conditional rules are allowed. Each rule of a given procedure is associated with a *rule strategy* that helps to reduce the work needed to the select the next redex. The directives may specify the reduction strategy to reduce terms or choose one of the known reduction [15].

A module can be parameterised as is illustrated in the module description of Example 2.1. This raises the degree of expressiveness of the proposed language.

Example 2.1: Consider a data type that describes the use of a stack. In Figure 2.2, we show the header of the module as well as its constructors and functions.

```
MODULE StackType(itemType) IS
      TYPE stack;
      CONSTRUCTORS
      stack empty;
      stack itemType:stack;
      FUNCTIONS
      stack      push(ItemType);
                 itemType   pop(stack);
                 Boolean    emptyStack(stack);
```

Figure 2.2. *Header and specification part of a module.*

Figures 2.3(a) and 2.3(b) describes the *StackType* module indicated in Figure 2.2 functions. For the operator, the user can suggest a reduction strategy. For instance, the strategy [1, 2, 0] in procedure *push* suggests the reduction process of a term *push*(*x*, *y*), where *x* and *y* are stacks should first reduce the subterm which is substituted for *x* then that which is substituted for *y* and then attempt to reduce the whole term. The implementation of the reduction strategy suggested by the programmer is described in the Section 3.

```
      PROCEDURE emptyStack;
      VARIABALES
            stack      s;
            itemType   i;
      RULES
            1: emptyStack(empty)  = true;;
            2: emptyStack(i:s))   = false;;
      {
            BottomUp;
      };
      PROCEDURE push [1,2,0];
      VARIABLE
            stack      s;
            itemType   i;
      RULES
            3:push(empty,i)= i:empty;
            4:push(s, i)    = i:s;;
      {
            Lazy;
      }
```

Figure 2.3(a). *Module function implementation - procedures emptyStack and push.*

```
PROCEDURE pop  [1,0];
      VARIABALES
            stack s, s1;
            itemType i, j;
      RULES
            5: pop(empty)= null;
            6: pop(i:s)  = i;;
            7: pop(i:empty)  = i;  [1,0];
            8: pop(push(s, i)= i; [0];
            9: pop(s)=IF(s == empty) null
                       ELSE j
                       WHERE (s== j:s1);;
      {};
 };
```

Figure 2.3(b). *Module function implementation - procedure pop.*

The main body of a *πlog* program is a set of C instructions which include assignments, conditional instructions like an *if-then-else* and *switch*, constructs allowing iterations as *while*, *do*, and *for* and functions calls. The values returned by a function call should be computed using the rewrite rules specified in the body of the procedure implementing that function. For instance, Figure 2.4 describes how a C program could use the functions of module *StackType* described in Figure 2.2.

```
void main() {
      stackOfFloat  = newStackType(float);
      stackOfStacks = newStackType(StackOfFloat);
      stackOfFloat s1, s2, s3 = empty;
      stackOfStacks ss = empty;
      float x, y, z;
      for (int i=0; i<10; i++) {
            scanf("%f %f %f", x, y, z);
            s1 = push(s1, x);
            s2 = push(s2, y);
            s3 = push(s3, z);
            ss = push(s1:s2, s3);
      }
}
```

Figure 2.4. *Program main body*.

3 Implementation Issues

The philosophy behind the *πlog* language is to clearly separate algorithmic details, memory management and specification so that each of these aspects of program generation may be tackled independently. The execution of programs in this language is performed with greater and greater efficiency according as information is added. If only the specification is given then the system attempts execution using search techniques, but termination is not guaranteed to the same extent as when specific algorithmic information is given to the runtime system in some form. The semantics of the language guarantees how it is executed given this additional information and also defines the model of computation. However, in the absence of any control whatsoever, i.e. when only a specification is provided, as in OBJ [2] for example, the algorithm which searches for solutions is not determined. This is left to the implementer of the language compiler as there is no algorithm which will in general solve any problem that can be defined in the language. So, various heuristics are required which may well depend on the use to which the system is put.

One of the great advantages of separating control, specification and data structures is that they may be implemented one at the time in the program enabling rapid prototyping and alternative choices of control to be tested.

The runtime system [8-13] is a theorem prover, which builds up a number of theorems that are held available as long as is necessary [7-9]. It is these theorems, deduced from the original rules, which form the system's memory. They have the form of additional rewrite rules which give values for functions needed in future computations. Thus, the memory holds natural values with very clear meaning: there are no obscure variables with a complex relationship between them.

The algorithmic control in the form of rules which are added to the program may disturb the logic of the functional programming language *πlog*. This then requires a theorem-proving capability to verify the rules against the specification. Any implementation of the language is required to do such theorem-proving as it is necessary to ensure that the program meets its specification or report what still needs to be proved to enable verification. Normally, such theorem-proving capabilities will be interactive since proving theorems is rather a hard and undecidable process [6]. However, the compiler does have a switch by which the necessary verification, which is costly in time, may be switched off. Totally verified sections of code are marked as such and the proofs filed for future reference. For efficiency, code is date stamped so that future changes to code will only invalidate a minimal part of an already existent proof. Previously verified code which is thus invalidated uses the recorded proof in an attempt to re-verify the code. The need for verification can almost be avoided by using rewrite rules for the algorithmic information and copying them for the specification. Only termination and consistency properties then have to be checked.

Operationally, the evaluation of a given term using the suggested reduction strategy can be described by the following rewriting rules:

$$\begin{aligned}
&evaluate(t) && = rewrite(t, strategy(t)) \\
&rewrite(t,\varnothing) && = t \\
&rewrite(t,cons(0,l) && = \text{if}(match(t), evaluate(template(t)), rewrite(t, l)) \\
&rewrite(t,cons(x,l) && = rewrite(substitute(t, x, evaluate(argument(t, x))), l)
\end{aligned}$$

where function *evaluate* rewrites a term to its normal form, function *rewrite* reduces the given term using its top operator strategy, function *strategy* returns the given term's top operator strategy, function *match* determines whether or not the given term is a reducible expression, function *template* returns the contractum of the given term, function *substitute* replaces a given subterm with another, and finally, function *argument* returns a given argument.

This straightforward implementation can be improved using the technique of *memoisation*, which consists of bookkeeping already evaluated terms. Marking evaluated terms improves the evaluation time by a great deal (see [7] for more details).

The language *πlog* has the structure of a program written in a *C*-like language. Functions are defined using a functional programming style to calculate their values. A *πlog* function can be implemented by one or more procedures using rewrite rules. With this functional programming language, the user has the same algorithmic control as with imperative languages. This allows him to write programs which are as efficient in terms of memory space and execution time as if he were using an imperative programming language. The added control should increase efficiency by avoiding the process of pattern-matching that selects the rule to use and the process of redex selection that chooses the next subterm to rewrite. These two properties make the proposed language provably as efficient as an imperative language. In *πlog*, functions are defined in an environment made up of a number of abstract data types. This means that the types needed for the function arguments and values are not defined explicitly by constructing them from the basic built-in types such as Boolean and integer. They are constructed implicitly using the functions which operate on them. This includes the use of constant functions, i.e. functions which have no parameters.

4 Conclusion

In this paper, we proposed a new programming language that integrates functional and logic features together with procedural ones without detailing all the features available to the programmer. Normally, this hybrid language should provide the programmer with two advantages: efficiency due to the imperative constructs and clarity and provability due to the declarative features.

The compiler of the *πlog* language is being implemented. We intend to evaluate the performance of such a compiler in comparison with purely declarative systems like the OBJ3 system [2] and a purely imperative language as Pascal.

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Reasoning about Updates

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Abstract

The aims of this paper are (i) to show advantages of using belief networks for natural language interpretation, and (ii) to argue that two layers of reasoning - ***in*** **and** ***with*** **the network - are needed in order to evaluate different ways of updating the network. The argument is empirically based on the phenomenon of model conversation failures which occur when a speaker is perceived to be inconsistent by the interpreter.**

1 Introduction

In the process of interpretation, human agents make use of their world knowledge together with the beliefs they hold of other agents. In particular, they evaluate the reliability of the information they receive (a) with respect to their previous beliefs about the topic, and (b) with respect to the reliability of **the source**: *Is (s)he well-informed? - Can I trust her/him?* Information update is influenced by the nature of the answer to the latter two questions. Therefore, from the point of view of interpretation modelling, it appears desirable to represent the impact of the degree of credibility and well-informedness ascribed by the interpreter to other agents. As argued by [7], it is in fact necessary to do so when representing the effect of *model conversation failures* on the interpretation process. Model failures (as opposed to *input failures*, viz [5]) occur when the speaker is perceived to be inconsistent by the interpreter (which can be either a human agent or a natural language dialogue system), either by contradicting herself, or by correcting or retracting previously asserted proposition. In previous approaches to information update (e.g., [1] - S. Larsson, p.c., [10]), corrections and retractions either result in bringing on an absurd state, or a "blind" revision (replacing the old information with the new update, with no further consequences). This is problematic not only for theoretical reasons, but also for practical purposes, as the interpretive module of a dialogue system should be able to distinguish between negotiable and non-negotiable information.[1] Instead, [7] propose to represent the belief state of an interpreter as a Bayesian (belief) network ([4]), in which interpreter's previous beliefs (about the world, about sources of information) influence the interpretation process.

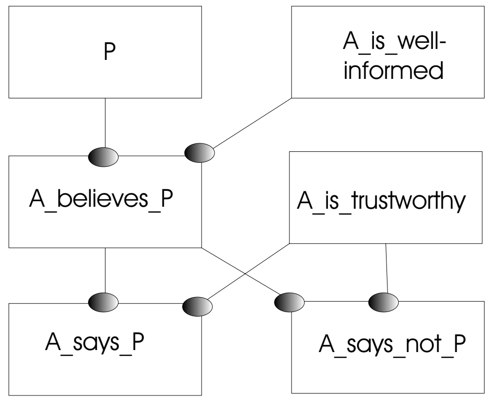

Figure 1.: Example of a belief network for interpretation

A Bayesian network is a directed acyclic graph with probability assignments associated with its nodes. The nodes (propositional variables) stand for beliefs of the agent and arrows in the graph are causal connections between beliefs. Information update is modelled by probability conditionalization locally propagating through the network.[2] Unlike in the previous approaches cited earlier, the process of information update is thus represented *quantitatively* and relations among beliefs directly influence the result of the update.

Although the approach of [7] leads to a simple and quite realistic representation of several cases of conversation failures, it cannot deal with the fact that agents appear to **reason** about ways of updating their beliefs. Especially if repeatedly communicating with the same agent, the interpreter can decide to disregard inconsistencies and still consider the agent trustworthy (the conversation failure can be ascribed to misperception instead). On the other hand, if the interpreter believes with a high probability in a proposition p and the speaker asserts $\neg p$, rather than revising her beliefs with respect to the proposition in question, the interpreter can instead decide that the speaker is not well-informed. (We know from experience that we often refuse to believe surprising news at first. The chance that we accept the news to be genuine increases if we acquire it iteratively from additional sources - a process which can, in fact, be captured in the belief network in [7].) Finally, reasoning about different kinds of update comes handy in cases of semantic ambiguities, when the interpreter decides for

[1] E.g., the information about user's travel destination is negotiable, while the price of the airplane ticket to the destination is not. See [2] for discussion.

[2] For an example of a network for one proposition and one agent, see Figure 1.

an update that conflicts the least with her previous beliefs (yet still has some information value). For example, if in a restaurant we see the sign "Please wait for the waiter to be seated", we would probably not look forward expectantly to the waiter taking his seat (and fastening his seat belt) but rather leading us to a free table.

In the present article, we will build on the previous work of [7]. In order to improve their model, we will explore three possible ways of representing reasoning about updates in Bayesian networks. Using the decision-theoretic terminology of, a.o., [9], we make the following general assumptions:

- agents are *utility maximizers* - they attempt to obtain the most relevant information with minimum effort;
- *utility* in the interpretation process is correlated with information value;
- *effort* increases if the interpreter has to revise strongly held beliefs.

2 The "Useful" Node

First, we attempt at a naive solution, consisting simply in adding a "useful" node into the original network. The table below shows (probabilistic) values associated with the node "Useful" given all possible combinations of the values of its parents. E.g., if the interpreter believes that q is true, p is true and the speaker A is credible, the positive value of the node will be 0.9. In general, the belief that a certain update is useful is high (0.9) when an agent is considered well-informed (the value TRUE), but goes down (0.6) if the agent is not considered well-informed, (the value FALSE). The network simulates situations where information received from the agent A is in contradiction with several of the interpreter's previous beliefs.

q	p	**A cred.**	**TRUE**	**FALSE**
TRUE	TRUE	TRUE	0.9	0.1
TRUE	TRUE	FALSE	0.4	0.6
TRUE	FALSE	TRUE	0.9	0.1
TRUE	FALSE	FALSE	0.4	0.6
FALSE	TRUE	TRUE	0.9	0.1
FALSE	TRUE	FALSE	0.4	0.6
FALSE	FALSE	TRUE	0.9	0.1
FALSE	FALSE	FALSE	0.4	0.6

The major objection to this approach is that no reasoning about an update is actually taking place. In cases where results of two kinds of updates have to be compared to each other (as for ambiguities), there is still no available mechanism to do so (e.g., one that would compare two different values of the node *Useful*). In other words, the network does not reflect the strife between *utility* and *effort* which we want to represent.

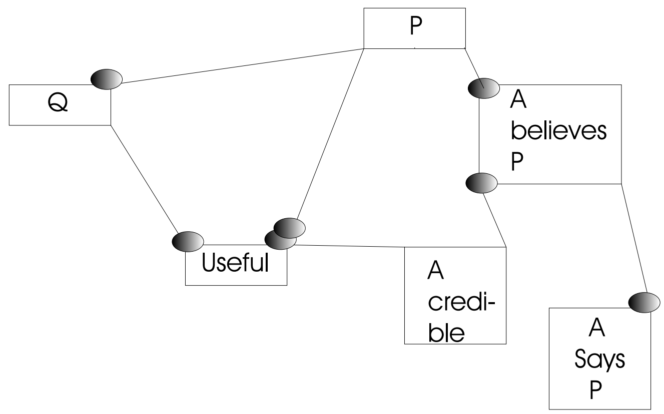

Figure 2.: The naïve approach.

As we will see in the next section, the difficulty with the second possible representation, via **decision networks**, will turn out to be on a par with the one just raised. We will explore the option nevertheless, because decision networks have been proposed as a general mechanism for rational decisions in the belief network literature ([6], [3]).

3 Decision Networks

"In its most general form, a decision network represents information about the agent's current state, its possible actions, the state that will result from the agent's action, and the utility of that state" ([6], p.484). Decision networks (sometimes also called *influence diagrams*) are belief networks enriched with nodes for decisions (rectangles) and utilities (diamonds). The usual chance nodes are represented as ovals.

Decisions about future actions are based on a utility evaluation of each possible setting of the decision node. In other words, for each possible action, the decision node is first set to that action and subsequent posterior probabilities of the oval nodes and the resulting value of the utility node are calculated. The selected action is the action with the highest expected utility. Obviously, a level of reasoning above the decision network is thus needed in order to compare the utility values of different states of the network.

In Figure 3., $< \alpha >$ and $< \beta >$ are possible actions the interpreter can take. By taking the action $< \alpha >$, she would decrease the probability that the speaker is credible and retain her previous beliefs concerning the actual world. By action $< \beta >$, she would maintain the belief that the speaker is credible and rather revise her beliefs about the world, based on the information obtained from the speaker.However, the network cannot update in such a manner that would maximize information value of the update, while minimizing its effort.

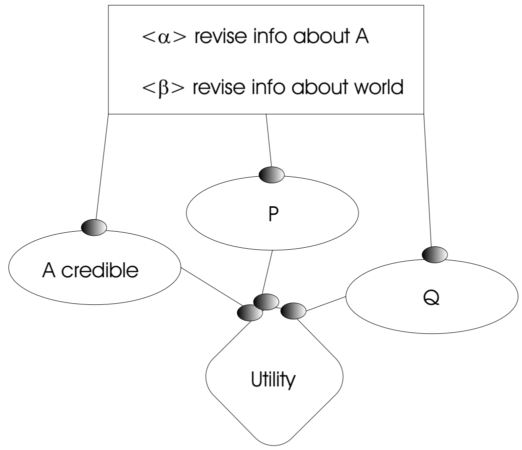

Figure 3.: Decision Network

For example, if the interpreter strongly believes that p and q are true but the information she receives from the speaker A is to the contrary, in the very limited model considered here it would be more advantageous for her to perform action $< \alpha >$ (by the reasoning that it requires less effort to revise beliefs in one proposition than in two), especially if her belief in speaker A's credibility is quite low to begin with. On the other hand, the interpreter is after information maximization, so she also does not disregard the information value resulting from the update. It is quite obvious that the *Utility* node cannot reflect the expected utility of the interpretation act, because it is itself a part of the network from which the utility values should be derived (by comparison of the effects of different types of updates). In consideration of this fact, we will propose that the expected utility be calculated in a process superimposed over the original belief network.

4 A Refined Architecture

The idea of representing the interpretation process on two levels appeared in the work of [8] on belief networks for purposes of medical diagnosis. There, a *control layer* containing non-probabilistic knowledge is used to control different types of problem solving, performed by the *probabilistic layer*. In our proposal, the control layer queries the probabilistic layer about (i) the probability change associated with the proposition under discussion, and (ii) the probability distribution, both prior and after an update. With the assumption that both positive and negative information is useful,[3] we propose to calculate the *utility* of the update as the difference in probability assigned to the proposition under discussion before an information update and after. Thus,

$$U = |P(X_n) - P'(X_n)|$$

where U is the utility of the update, $P(X_n)$ is the original probability of the node X_n and $P'(X_n)$ is the probability associated with the node after the update. On the other hand, there is effort associated with changes of probabilities at nodes influenced by the change of probability at X_n (non-d-separated nodes). Thus, for the part of the network of i nodes, locally affected by the update, we can calculate the effort of the update as

$$E = |P(X_1) - P'(X_1)| + ... + |P(X_{n-1}) - P'(X_{n-1})| + |P(X_{n+1}) - P'(X_{n+1})| + ... + |P(X_i) - P'(X_i)|$$

In order to compare the utility of two different updates (in the previous section referred to as actions $< \alpha >$ and $< \beta >$, respectively), the control layer calculates the utility and effort for (i) X_n = well-informedness (or credibility) of the speaker (A), i.e., for the case when the network would be updated with the proposition "The speaker is not credible", and (ii) X_n = an update with the proposition under discussion p. Of the nine possible outcomes, the first six (indicated in the table below) can be decided straightforwardly:

	Utility	**Effort**	**Decision**
1.	$U_i > U_{ii}$	$E_i = E_{ii}$	$< \alpha >$
2.	$U_i < U_{ii}$	$E_i = E_{ii}$	$< \beta >$
3.	$U_i = U_{ii}$	$E_i < E_{ii}$	$< \alpha >$
4.	$U_i = U_{ii}$	$E_i > E_{ii}$	$< \beta >$
5.	$U_i > U_{ii}$	$E_i < E_{ii}$	$< \alpha >$
6.	$U_i < U_{ii}$	$E_i > E_{ii}$	$< \beta >$
7.	$U_i = U_{ii}$	$E_i = E_{ii}$	arbitrary
8.	$U_i > U_{ii}$	$E_i > E_{ii}$	?
9.	$U_i < U_{ii}$	$E_i < E_{ii}$	?

In case of the seventh outcome, either action can be taken (though preference for $< \beta >$ could be assumed). To choose an action in case of the last two outcomes, we can foresee two possible strategies:

- give superiority to either *utility* or *effort*; [4]
- give preference to either action $< \alpha >$ or action $< \beta >$.

[3] For example, if the interpreter thinks with equal probability (0.5) that it is going to rain tomorrow, or that it is not going to rain, change in the probability in either direction is useful information, e.g., with respect to her decision problem *Should I take an umbrella?*

[4] This strategy could be relativized with respect to a constant α (e.g., for Outcome 8, if $E_i > \alpha$, then perform action $< \beta >$.)

It is not quite obvious which (if any) of the two options is in fact selected for by the interpreter in actual conversations.

5 Conclusion and Discussion

We have argued for a potential use of belief networks in one area of natural language interpretation. We indicated several difficulties regarding reasoning about updates in the approach of [7] and discussed three possible ways of modelling the process with belief networks. Of the three suggested options, a *Utility* node, decision networks and an additional *control layer*, the third appears to be most suitable for evaluating the utility versus effort of the update. We proposed to calculate the utility of an update as the absolute difference in probability value of the proposition under discussion before and after an update. As for effort, we have construed it as the sum of changes in probabilities at nodes affected by the update. By comparing the two values, the interpreter maximizes the general utility of the information update by either updating her beliefs about the source of information, or by updating the proposition under discussion. From a theoretical point of view, it is important that in our architecture, we have merged together the Gricean maxim of quality (here spelled-out as credibility and well-informedness) with the semantic module. This was crucial both for representing model conversation failures, but also for deciding about the general utility of different types of updates.

♢
The author would like to thank Balder ten Cate, Robert van Rooy and Henk Zeevat for helpful comments regarding the content of this article.
♢

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New Analytical Perfects in Modern Georgian

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Abstract

The paper describes diachronic and synchronic aspects of two analytical perfects that can be found in Modern Georgian.

Analytical (periphrastical) perfects are well attested in human languages (Benveniste 1960, Maslow 1989, Schmalstieg, Sackokia 1985, 1998 and references there). Analytical perfects may be functioning together with synthetic (inflectional) forms or not. They may have oblique (ergative) or nominative morphosyntactic structures.

In Modern Georgian several analytical morphosyntactic sequences may be singled out. Among them, the different types of analytical perfects are the most important (for details, see Sackokia 1985, 1998, 1999). Some of the taxemes are described by Ak. Shanidze in his grammar as: "absolutive compound predicate" like the ones in (1).

(1) cerili gagzavnilia
the letter is sent
puli gadaxdilia
the money is paid
saxli asenebulia
the house is constructed

(cf. below the agentless perfect A), and "relative compound predicate" as in (2)

(2) my brother has sent the money
cems dzmas gagzavnili akvs puli
S-dat-Agens part-past-passive copula V-habere - O

(cf. below, the perfect - B) (Shanidze 1973, 295, paragr. 365). These perfective Georgian sentences were also discussed by different authors at different times (Boeder 1980, Macavariani 1983, Canishvili 1981).

The classification of these sentence forms was especially studied by Sackokia 1985, 1998, 1999, 2000 in the light of typological comparison and diachronical analysis of possessivity, ergativity and transitivity in Georgian (Kartvelian) and Indo-European languages (All conclusions mentioned below may be useful and relevant for logical and formal structural or computational analyses of natural languages. I distinguished two new analytical perfects in modern Georgian: A and B (Sackokia 1985: 141-186) with models:
A: S-gen+Part.Pass.+V-esse+O-nom and
B: S-dat+Part.Pass.+V-habere+O-nom (*habet* as *estalicui*, so the agent appears in dative possessive). It may be expressed by the formulae: A: alicuius factum est aliquid (factum est aliquid); B: alicui factum habet aliquid (*habet* as *est alicui*).

(3) A:
Misi gamomcxvaria torti.
The cake is baked by her.
Cemi gaketebulia sadili.
The dinner is cooked by me.

B:
Mas nanaxi akvs es pilmi.
He has seen this film.
Mas naqidi akvs puri.
He has bought some bread.
Mas nanaxi hqavs bavsvi.
She has seen the children.

As I have claimed before, these morphosyntactic surface structures are essentially *ergative*. All periphrastic constructions mentioned above have morphosyntactic ergative subject structures. The agent is expressed by an oblique subject, that is in an other than nominative case. The copula is obligatory in forming the perfect A in Georgian, without the V-esse this utterance is adjectival as in (4).

(4) Cemi gaketebuli sadili
The dinner done by me.

Modern Georgian perfect taxemes include V-esse and V-habere as auxiliaries. The interchange of V-esse and V-habere in different periphrastical perfective taxemes in Georgian seems similar to the pattern in certain Indo-European data. The formal model of Georgian verba-habere and corresponding B-perfective taxemes includes the semantics of "animate/inanimate" for the direct object (cf. m-akv-s, m-qavs). But both semantics are correlated with the same formal model "est mihi" (cf. Russian *umenya*, Estonian, Latvian *man ir*. Lithuanian *man yra*, etc) (cf. Gamkrelidze-Ivanov 1984: 288, 288-293). I.e. formal clusters with models Indirect Obj becoming S-obl (possessor) with V-esse, and corresponding perfective taxemes including V-esse and V-habere are found in both Indo-European and Kartvelian. Observing the Georgian diachronic data throughout Old Georgian- Middle Georgian - Modern Georgian, I suppose, that the Modern Georgian perfect A is derived from the Old Georgian and Middle Georgian constructions in (5).

(5) Ese ustari ars cemgan monagvacebi (Rustaveli)
Agens-gen (+gan-postpos.) +Part.Pass.+V-esse (copula) +O-nom. (postposition *GAN* "from")
This letter is written by me

The combination Gen+"gan" functions as Ablativus Agentis (details in Sackokia 1998).

These Georgian data show typological similarity with postpositional agentivity or prepositional periphrastic participle taxemes (Slavic, Russian, Latvian, etc), as in Old Russian

(6) Ubien ot Jaroslava
He is killed by Jaroslaw

(see Schmalstieg 1985, Orr 1989 and others). As to the second, dative perfect B, which may be expressed by the formula *Alicui habet factum aliquid*, it seems to arise in later Middle Georgian, (XVII- XVIII cent.) and is rather frequent from the second part of the XIXth century. (7)gives an example.

(7) Es qoliperi gatvaliscinebuli hkonda mtavrobas.(XIX cent. Cereteli)
All these things were considered by the government.

The cyclic regeneration of morphosyntactic clusterings shows the "great" and the "small" cycles, - the great cycle involving whole phrase forms (taxemes), and the small its separate elements (case forms, pre- and postpositions, the change of copula and so on). The great morphosyntactic cycle includes the small cycle. The universal process of the cyclic renovation of declension seems here to be especially important. The small cycles in Georgian in analytical perfect A involve the form of agens: GEN-, GEN-POSTPOS. - GEN. The cycle of the copula: ars - a-aris ("is" full-short-full), misi- misgan-misi gaketebuli ars - a-aris ("he has done"). One sequence of person markers derives from personal forms of the copula (V-esse) and is added in the first place to intransitive verbs. The Intransitive verbs of the IIIrd series in Georgian show copulative personal markers as in (8).

(8) cavsul-var, cas(r)ul ..ars, a
S-intr-Nom + Part.Intr. +CopulaV-esse
I am gone, he (she) is gone.

The cycle of forms is *cas(r)ul arian casulan*, *casuliarian*. So the perfective sequences show the oblique (ergative) agent forms for transitive verbs, and S-intrans-Nom for intransitives. The agentivity of S-gen and S-dat in the perfects A and B may be attested by the possibility of coordination of agents as in (9).

(9) Mas nanaxi akvs es pilmi da ar mova
He has seen this film and he won't come to see it
Misi gamomcxvaria torti da ar unda (ar secams)
The cake is baked by her and she won't have it
Mas naqidi akvs puri da ar cirdeba
He has bought some bread and he doesn't need it
Cemi gaketebulia sadili da davpatizeb .....ar minda
The dinner is cooked by me and I don't want it...and I shall invite them.

The procedure of dialogic "question- answer" shows the interchange of Agentless and Agentive forms, and the importance of the agent for perfective taxemes.

(10) Visi gamogzavnili xar?
S-agens-gen+ Part. Pass. + Past-V-esse
By who sent are you?
Who has sent you?

Bavsvi gamogzavnilia.
-Visi? - Mascavleblis
"The child is sent -
-By whom (gen.)?- By the teacher (gen).

It is interesting and important to note, that the inflected Modern Georgian perfects tend to have a modal semantics, and that the new analytical perfects tend to have a resultative perfective semantics. They also occur in different styles: A and B are more used in colloquial, scientific, press and office style, while the inflected, "turmeobiti"- is more used in the *belles lettres*. Thus, the A and B perfects are proper perfects without modal semantics. Also, the Modern Georgian dative taxemes with future passive participial nucleus usually express obligation. The action expressed by these taxemes is obligatory for both transitives and intransitives.

(11) S-agens-dative+Part.-Pass.-Fut.+copula- V-Habere+ O-nom-trans:
Saqideli makvs puri.
I have to buy some bread
Sanaxavi mqavs bebia.
I have to visit the grandmother.
Intransitive.
casasvleli var
I have to go.
Sintr.Nom+Part.Fut.Intr.+ copula-esse

The future participial perfective possessive (ergativoide) clusterings in Modern Georgian express especially obligation or imperativity. The communicative semantics is connected with surface morphosyntactic features. The interchange of V-esse and V-habere in different perfective taxemes in Georgian seem similar to certain diachronic Indo-European data.

The A-perfective future models (perfecta futuri) have the same semantics of obligation.

(12) Cemi saqidelia puri
l have to buy some bread
Cemi agsazrdelia bavsvi
I have to educate the children
S-gen-Agent+Part.Fut. + copula-V-esse+O-nom

The problem of the Georgian ergativoid aorist taxemes including the active intransitive verb lexemes is relevant here. I suppose verba intransitiva activa may become ergativoid by analogy to ergativoid transitiva. The semantics defines which verb lexemes may acquire an ergative surface structure. So, S-erg for active intransitive is of secondary nature, depending on their active ("dual") semantics. As a result Georgian shows in the 2nd series so-called "split ergativity" (see Schmalstieg 1986, 1989, Boeder, Harris). Perhaps also, it is partially the semantics of "volitionality" (cf. Boeder for Kartvelian, Sackokia 1999).

In Georgian, the analytical perfects of verbs such as: to dance, to cry, to run, to cough, to sing, to bite, to frisk, to laugh, to smile and many others show the intransitive models.

(13) S-intr-Nom+ Part.act.Intr.+copula-V-esse
nacekvi var
I have danced.

The correlation of different participial forms with lexical and grammatical semantics can be observed in Georgian, e. g. the "pseudotransitive taxemes" with the model (14).

(14) S-nom+ Part.-Past.-Pass. +V-esse

(cf. the same model with S-erg-dat and V-habere). This model functions as transitive, as (15)

(15) S-dat+Part+V-habere.
nacami, nasvami, (slang: nacmi, nasmi), nakitxi var
I have eaten, drunk, read

The interchange of the verbs "esse" and "habere" in different perfective taxemes seems here similar to certain Indo-European data. (esse instead of habere). I(semantical) and II (grammatical) morphosyntactic types which can be dominants or cofunctioners in the language system seem important here. Part.Pass. and Part.Mediopass can function as transitives or pseudo-transitives.

(16) S-nom+Part.+V-esse
nacami var, dakvirvebuli var
I have observed
narbeni var
I have run
nakitxi var
I have read
cemi sarbenia, sasiarulo makvs
I have to run, to go

The use of different participial forms must be precisely studied from a diachronic perspective . Three Georgian verbal series are inflected (see Schanidze), the IVth is analytical, a new, additional series. This is probably a new feature of analytical nature within the dominant morphological range of the Georgian language system. These types of past or future perfective possessive(ergative) clusterings Modern Georgian may be interpreted as a new, additional, IVth series with the principal variants (A S-gen and B S-dat) showing all possible (traditional) Shanidze's "screeves", as full verb conjugation paradigms, with the passive participial in past or future. The new conjugation paradigm operates by the interchange of both copulas V-esse and V-habere. Special morphosyntactic constructions like ergative or constructions of "grammatical possession" also include the cofunctioning of I (semantical) and II (grammatical) morphosyntactic types being dominants or cofunctioners in the language system.

The special cases show both diachronic and synchronic interdependence of semantics on the one and grammatical arguments on the other hand. E.g. the arrangement of animate/inanimate in Georgian verba habendi and corresponding Modern Georgian perfect B. The "special" constructions from the point of view of the morphosyntax show the both diachronic and synchronic interfunctioning of semantics and grammatical arguments, e.g. two possessive Georgian "verba habere"" with dative models: for animate *mqvas* and for inanimate *makus*.

They resemble the two corresponding Modern Georgian perfect taxemes (ergative) as in (17):

(17) Perfect B:
mas nanaxi hqvas avdmqopi
He (she) has seen (visited) the patient (anim)
mas nanaxi akus pilmi
He (she) has seen the film (inan)

The intersections "passivoide/ergativoide" in Old Georgian and Middle Georgian are unique but quite important typologically. More common are contaminations on the basis of the participial predicate nucleus being able to coordinate with both "possessive" or "passive" Agent forms (GEN, DAT, INSTR, etc). So, see the Agent-gen+Postp "gan" from (by) misgan (by him) cf. Russian "ot" (from). That is, the participial nucleus in the diachrony of perfect "A": S-agent-gen+Gan. This participial nucleus is of a *dual* nature: both possessive/passive, as in (18) (gandidebulia and ididebis are morphosyntactic synonyms).

(18) misgan ididebis
"He(she) is exalted of him
inflectional passive verb
misgan (misi) gandidebuli ars(-a)
participial nucleus

The paradigmatical exchange noun-verb in participial predicate nucleus shows the phenomenon and its - phenomenological mechanisms of both formal-structural and deep-semantical connections between the ideas of possessivity and transitivity-ergativity in perfective utterences (clusters). So the nouns or denominative lexemes and verb participial nucleus are exchangable (or interchangable) in perfective predicate paradigms by the scheme: NOUN-verb (N-V) or thing /object - action.

That is the possessive noun sentences on the one and analytical perfective clusters on the other hand among the structures (models) mentioned below are systematically related.

(19) Es cemi naxatia (sacukaria, naceria, targmania, naxelavia, agmocenaa)
This picture is mine, this is my picture (present, script(letter), translation, handcraft, discovery).
Mas naxati (sacukari, naceri, targmani, naxelavi, agmocena) akvs.
He (She) has a picture (a present, a letter, a translation, a handcraft, a discovery).
Perfect A: Es cemi daxatulia (nacukaria, dacerilia, natargmnia, targmnilia gaketebulia, agmocenilia).
I have painted (I have made a present, written, done, translated, discovered) it.
Perfect B: Mas daxatuli akvs (nacukari, dacerili, gaketebuli, targmnili, natargmni, agmocenili).
He has painted (written, done, translated, made a present, discovered)

On the other hand, the Georgian taxemes with actor expressed by active participial forms are used in Modern Georgian discourse on the whole ininterrogation or in negation taxemes. (only rarely in affirmative sentences). They include the semantics of ”possibility” as in (20)

(20) Amis gamketebelia axla es?
Is he the doer of this action?
Can he do it?
He is not the doer of this action”.

The communicative semantics indicates ”possibility” , ”obligation”, ”imperativity”, ”mind”, ”knowledge”, ”modality”, ”interrogation”, ”negation” and so on). As we see, the communicative semantics is also here connected with surface morphosyntactic features.

I mean here the different pragmatic functions expressed, like obligation, negation, interrogation, especially in imperative utterances. In the same way, the different semantical roles may be distinguished (Possessor, Recipient, Location etc.). There is also a correlation between the formal model and the semantics of the participial nucleus.

For example, for different semantical groups the dative possessive morphosyntactic surface structure model can be seen, S-dat with different semantical roles like Recipient, Possessor, Actor, Agent, Direction, Adressee.

In the Perfect B (S-dat) a split of roles can be observed. There are clusters with Subject Orientation (S-orient) and others with Object Orientation (O-Orient); the syntactic role of S-Agent-Obl(Dat) is S-oriented and Indirect Object (Dat) is O-oriented. Predicate nuclei like in (21) have split semantic roles.

(21) S-oriented
Mas nabrdzanebi (commanded), natkvami (said), davalebuli (entrusted), micemuli (given) akvs mistvis.
He has commanded, said, entrusted, given to somebody.
S-Dat-Agent+ Part-Pass +copula-V-habendi+ O-external (preposition tvis (for))
Object-oriented
Misgan nabrdzanebi (commanded), natkvami (said), davalebuli (entrusted), micemuli (given) mas.
It is commanded, said entrusted, given... (to him, her...)O orient (O-DatRecipient/Adressee).
S-agent-prepositional Part-Pass. O-Dat.

Such split roles (semantic or syntactic or pragmatic roles) can only be distinguished by context. Generally the role semantics is dependent on lexical semantics or the predicate nucleus and/or on the semantics of S and O. I mean here semantic features such as: animate, inanimate, human, non-human, person, according to M. Silverstein’s scheme of the hierarchy of noun semantics features.

The morphological forms of more distant actants shows the true role semantics of principal actants S and O too (as e.g. O-external ”for him”, and ”from somebody” mentioned above: mistvis and misgan).

A start has been made in a computational project led by Levan Chxaidze to provide formal descriptions of all the structural semantic morphosyntactic clusters described above indicating the different semantic and syntactic roles.

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A Communicating Process Model of Language

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Abstract

We introduce a communicating model for representing linguistic processes. The framework introduces the basic notion of a linguistic process and the communication between these processes. These linguistic processes can communicate with each other and compete for the limited resources available. We have applied the model for different aspects of grammatical modeling and language modeling. In this work, we report on the application of the model for grammatical modeling and we will also discuss the possible application of the model for modeling gene scrambling. Finally we describe a formal specification of a process grammar for this purpose.

1 Introduction

The idea of brain as a computer or a processing machine has been the topic of research in cognitive science. The traditional approach to modeling brain has focused on brain as a set of dynamic evolving mental states. A different approach to the study of brain is to consider brain activities as a set of dynamic communicating processes. The shift from a fine-grained notion of *mental state* to a more coarse-grained notion of *mental process* is one aspect of such a model, but the notion of communication among mental processes is another aspect of such a dynamic approach.

Language is an area of research for the study of brain processes and psycholinguists have looked at different aspects of processing language by humans. In psycholinguistics, a focus of research on human parsing system has been the discovery of constraints on the human parser [Crocker, 1996]. Most of the basic models have presupposed the existence of traditional tree diagrams in linguistics. The psycholinguistic constraints are defined and imposed on these structures.

In our work we take a different view to language and we model grammar as a set of communicating process structures. Our focus will be the study of constraints on communication. A shift from static tree structures in linguistics to dynamic *linguistic resources* for which the processes can compete. For this we will turn to the dynamic models in computer science and look into the problem of representing linguistic processes and the communication between them.

In this paper we will first look briefly at the current work in formal modeling of communicating processes in computer science, then we will explain a possible approach for modeling linguistic processes, linguistics channels and the linguistic resources. Based on this model a set of resource constraints on word order have been discovered that will be presented. Then we will discuss the application of this model for modeling non-cognitive domain of gene scrambling. Finally we describe a formal specification of a process grammar for this model.

2 Process and Communication

Communicating Sequential Processes (CSP) [Hoare, 1985] provides a formal model for the interaction of processes through communication channels. Communicating processes have also been modelled by Calculus of Communicating Systems (CCS) [Milner, 1989] and its mobile extension π-Calculus [Milner, 1993].

In a process model or algebra, the fundamental notion is *process*. Examples of a basic process are a simple event (e.g. the utterance of a letter, dropping of a coin) and termination (e.g. the end of utterance)[1].

A process can also be more complex: and it can be a sequence of processes or a parallel combination of processes where the order of execution is not specified. The "." operator is used for prefixing or sequence. P.Q specifies the behaviour of a process which behaves as sequence of P and Q. An example is the utterance of two letters or words by a single person e.g. "He slept".

"|" is the parallel operator. P|Q specifies the behaviour of a process which behaves as P and Q in any order. These processes are run concurrently and can start and terminate at any time independently from each other. An example of which might be the utterance of two words by two persons. The order of uttering these words is not specified.

"Choice" is another operator for joining two processes. The choice operator "+" ensures only one action among several candidates is performed and it eliminates the others. P+Q depicts a process that can behave as P or as Q, but not both. A person cannot utter two letters at the same time and he has choice over the letters of the alphabet. Similarly one cannot insert two coins simultaneously into a machine with only one slot.

What makes a "process" more powerful than a "state" in a finite state automata is the fact that processes can communicate with each other and they can send or receive data over communication channels. An illustration of this is when two humans speak to each other. The channel for communication might be a telephone line or open air. One process emits a value or message over a channel and the other process having access to the same channel will receive the message. When the sender sends the message **m** over channel **c** by $\overline{c}\langle m\rangle$, the receiver will receive it by executing the process c(x) which will receive the message and upon receiving the message, will bind its free variable **x** to it, that is $\{m/x\}$. The sender channel is marked with a line

[1] There are other basic processes for *no-action* (analogous to pause in an utterance) and *deadlock*.

over it marking it as negative polarity, that communicates with a positive polarity channel with the same name, but no extra marking.

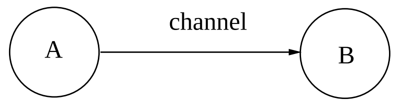

Figure 1: Communication Over Channels

A channel is illustrated in Figure 1.

The channel is the medium through which names can be exported, e.g. c in $\overline{c}(m)$, or imported, e.g. c in c(x).

In this work, we introduce the notion of communication for "linguistic" processes. By *linguistic process*, we have the domain of grammatical representation in mind. Like CSP, the communication channels in linguistic processes connect only two processes together. But what are these processes and what is their internal structure? We will answer these questions and define the notion of channel of communication for these linguistic processes.

3 Communicating Linguistic Processes

In [Rezaei, 1997] the basic notion of a linguistic process and its structure are defined. By using a sequence operator, the linguistic processes are constructed out of smaller units of words which are themselves constructed out of smaller morphological units. The more complex a linguistic process becomes, the more complex its communication can be.

In our model, a linguistic process is defined as a communicating structure that has an associated time-span. Using this notion of time-span, the temporal relation of linear precedence for these processes has been defined.

[Rezaei and Crocker, 1997] introduces the notion of linguistic communication channel by a dynamic representation of grammatical relations. Linguistic constraints on grammatical trees can be re-expressed to introduce constraints on communication. Whether we consider grammatical relations as primitive elements in the theory or we derive them from other principles of the theory, this decision will have further consequences for our future expansion of the theory. Here our main attempt is to add a dynamic aspect to the present approaches for linguistic modeling and we have decided to follow theories like LFG [Bresnan and Kaplan, 1982] in considering grammatical relations as primitive elements in our theory. In our dynamic model we take them as communication channels between process structures (i.e. constituents) which can communicate with other processes. This is depicted in Figure 2.

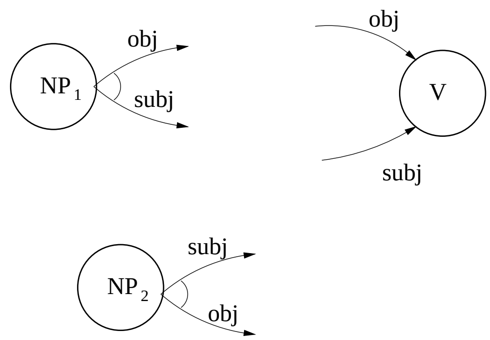

Figure 2: Communicating Linguistic Processes

In this figure, the channels are marked by potential grammatical roles that they require or provide[2]. This is especially important for modeling free word order languages in which different permutations of the sentence constituents are possible. This flexibility in word order is called *scrambling*.

For channels, we have considered two stages: marking and acquisition. The first step is channel marking, when the pre/post positions case mark a bare NP, at this stage the possible channels that an NP-process can compete for are specified. At the final stage, only one of these channels can be acquired by the process and this stage is channel acquisition.

It is at this stage that the actual act of communication happens between two processes over a single channel. In Figure 2 only one of the NPs can communicate with the **obj** channel of the verb.

There are certain constraints on channel marking and acquisition in different languages. For fixed word order languages the location and positions are very important parameters for channel marking.

For cases where there are multiple choices, competition determines which channel is acquired by a process. A choice operator guarantees that only one of the candidates can participate and the constraints on channels guarantee a single winner. Using a *path-set* data structure, a computational model for implementing these linguistic processes can be implemented [Rezaei, 2000b]. Figure 3 shows the result of parsing after competition for resources is done.

[2]The choice operator for channels of the NPs in the figure is highlighted with an arc.

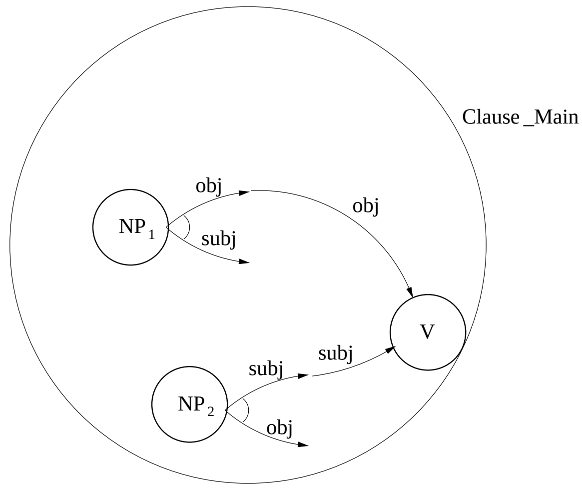

Figure 3: After Communication of Local Resources

To deal with Long Distance Scrambling (LDS)[3] we use a mechanism which is analogous to functional uncertainty in theories like LFG. Under certain conditions (e.g. barriers theory of GB) some channels can be passed or exported from one clause to an embedded one.

Figure 1 compares other aspect of our framework with concepts in Government Binding (GB) theory and Lexical Functional Grammar (LFG).

Table 1: A Comparison with GB and LFG.

GB	LFG	Our CLP
Case	?	Channel marking
θ theory	Functional uncertainty	Channel acquisition
Move$-\alpha$	Functional uncertainty	(Mobile) channels
Barriers	?	Mobility Constraints
Control	Control	Conditional channels
Optimality	probabilistic feature str.	Fuzzy channels

One additional advantage of a communication model of language is the ability to specify resource based constraints.

[3]In LDS a constituent will be moved across the boundary of two clause boundaries.

4 Resource Constraints on Parsing

Over the last few years a different conceptualisation concerning 'resource sensitivity' has emerged in several disciplines connected to the study of language. This idea has been explored within categorial grammar in [Carpenter, 1996] and [Morill, 1994]. More recently [Johnson, 1997a] and [Johnson, 1997b] introduce a resource-based conceptualisation of LFG. In [Johnson, 1996] the approach is illustrated with a view of characterising constructions in terms of 'plugging'. A set of objects are constructed and some of these objects need to combine with other objects to become saturated, and rules determine what can be 'plugged into' what.

Phenomena such as argument attachment in natural languages are inherently resource based and most linguistic theories use some mechanism of resource sensitivity for argument attachment. From a resource-based perspective, other constraints related to performance and parsing can be defined that restrict the possible word orders.

A computational model for implementing these linguistic processes has been elaborated in [Rezaei, 2000b]. In our model we use a dynamic structure to allow a set of paths or threads to progress in parallel and to compete for the limited grammatical resources. This mechanism allows a process to compete for one or more channels at the same time.

Based on this model, we have introduced a notion of limited resources by the Resource Limitation Principle (RLP) for representing local scrambling in some free constituent order languages.

Resource Limitation Principle
No two NPs can exist in a clause with the same grammatical functions.

By using blocking word order rules we blocked the progress of paths which contained two instances of a grammatical resource.

We also discovered another blocking constraint, the Resource Barrier Principle (RBP) for long distance scrambling (LDS) and export of resources. In LDS a constituent will be moved across the boundary of two clause boundaries.

Resource Barrier Principle
If a resource exists in a clause, it acts as a barrier in front of the resources with the same grammatical functions which want to scramble into lower level embedded clauses from higher ones.

The notion of limited resources for scrambling is a grammatical constraint that hasn't been investigated earlier. This aspect of the communication model interacts with the notion of competition among the linguistic processes.

Competition can be modelled in an algebraic way that can be implemented with fewer problems than in other models for competition such as optimality theory [Smolensky et al., 1992]. Based on the notion of "algebraic optimality", the competition aspect of the model has been implemented [Rezaei, 2000b].

5 From Word to Gene Scrambling

The communicating linguistic processes model that we have discussed is a general model and one can specify constraints on the number of channels for each process and come up with different classes of communicating linguistic processes (CLP). In this section we will discuss the CLP-II model where each process has access to two instances of linguistic channels. These binary processes form a string or pipeline of processes, but the processes are further constrained by the word order constraints.

The CLP-II model can be used for modeling the traditional cursive English handwriting system, where each letter or glyph is connected to its immediate right and left neighbour, and for each letter we have initial form, medial form and final form. We have used CLP-II for modeling the connectivity of font graphs for Arabic cursive script [Rezaei, 2000a].

Arabic cursive script is used for writing different languages. Figure 4 shows an example of cursive font graph for Kurdish language which is an an Indo-European language. The script is written and read from right to left. and each letter can have three forms of initial, medial and final. The first column on the right in the figure shows the initial forms for some letters (h,l,y,b,d,z,j,s,sh). The second column shows the corresponding medial forms and the left-most column shows the corresponding final forms.

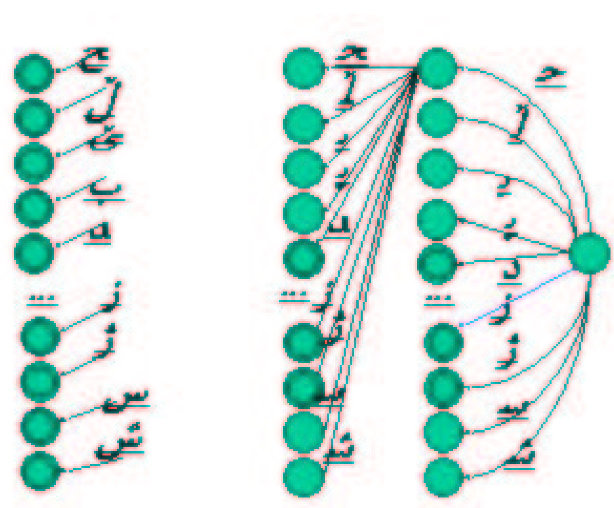

Figure 4: Font Graph for an Arabic cursive Script

In FIGURE 4 we have shown possible medial connections attaching to initial **h** on its left. These connections can be shown for most other initial forms and the number of possible connections increases as more letters are attached. The problem of testing all possible connections for this font graph is a combinatorial problem.

In our work, we have represented each glyph form (initial, medial and final) as a linguistic process that can communicate with its left and right neighbouring glyph. The interface parameters for each specific connection has been considered for matching the connections or glyph channels. Glyph channels are instances of linguistic channels.

By using a CLP representation, a finite state model based on communication channels has been derived and the connectivity of the font has been tested.

The CLP model for cursive font graphs is a CLP-II model where each glyph process has access to its immediate right and left neighbour. Another example of application of CLP-II is for modeling DNA strings and gene scrambling. Figure 5 shows an example of a scrambled gene. The DNA segments should be unscrambled to come up with the actual order for the gene (cf. canonical word order in grammar).

Each gene segment has boundary pointers in its left and right that give clues about the possible canonical order for a gene. An example of these pointers for ordered gene segments (based on [Landweber et al., 2000]) is shown next.

Table 2: DNA Segments (DSs) and boundary pointers

Left boundary	DSs	Right boundary
CAAAA	1	TA
TA	2	ATCTAAGAATGATGA
...	...	...
CTTGAAAATCAA	16	ATGCTTGAaTC
ATGCTTGAAcTC	17	AAGAAAAC
AAGAAAC	18	ATGATCTT
...	...	...
TTcCCAAaAAGGAAA	47	TCATG
TCATG	48	n.d.

We use a notion of gene process as an example of linguistic process that can communicate with its left and right neighbour. We model gene pointers as communication channels.

A gene process like a glyph process has access to two channels, but a gene process

Figure 5: Parsing Gene Scrambling

can scramble in a gene string. This contrasts with the strict ordering constraints on communication for glyphs. A glyph in an string can only communicate with its immediate left and right neighbour in the string. In this regard, the word order constraints for glyph processes are analogous to the word order constraints in fixed word order languages, while the word order constraints for gene processes allow more flexibility, analogous to the free constituent languages.

For parsing scrambled genes we use the notion of path-set. The use of time-span for each process guarantees that only one instance of a specific gene segment in gene string will be used in each path. After parsing, depending on ambiguity in the gene representation, we will obtain one or few paths that correspond to the canonical order for the gene. The parsing program should also be robust to take into account the fuzziness in the gene left or right boundaries that might not match exactly with other boundaries. The notion of *fuzzy communication channel* [Rezaei, 1999] helps us to represent such a notion.

6 From π-calculus to π-grammar

In the previous sections we gave an outline of the basic concepts for a communicating process model of language and some of its non-linguistic applications. Figure 6 puts the present work in a background of the previous work in process algebra.

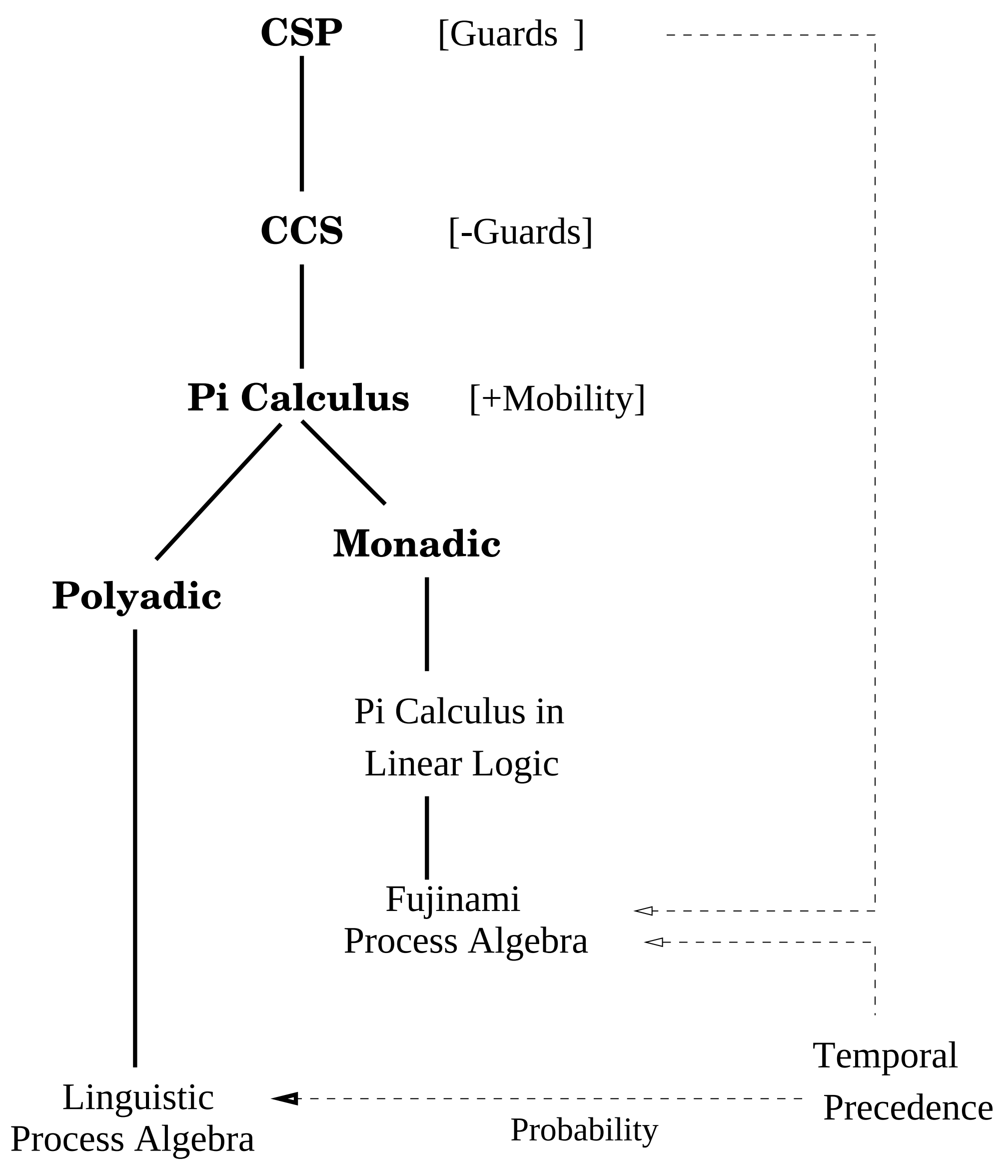

Figure 6: Process Algebras

In this process algebra background we see the evolution of process algebras into more complex algebras which allow the specification of mobility of processes in addition to the communication between processes. The minimal operators needed for our model are listed in the following:

P $\longrightarrow$	0 (termination)	
	$\mid \alpha$	(event)
	$\mid \alpha \cdot Q$	(prefix)
	$\mid$ [x=y]P	(match]
	$\mid$ P $\mid$ Q	(parallel)
	$\mid$ P + Q	(choice)
$\alpha \longrightarrow$	$\overline{x}(y)$	(exporting)
	$\mid$ x(y)	(importing)
	$\mid \tau$	(silent action)

Still these formal algebras lack the flexibility that we require for modeling the linguistic processes. Notions such as *ambients* [Gordon and Cardelli, 1998] need to be incorporated into these algebras to model locality. Stochastic and temporal extensions of these process algebras are also not easy to develop.

Another way to formalize our process model is to develop a grammar of communicating processes by incorporating a minimal set of operators and features for parallelism, communication and competition into the existing formal grammatical models in Computer Science such as Categorial Grammar(CG). The result will be a *process grammar* in which the notion of *process structure* – in contrast to a category – plays a key role. Major aspects of this process grammar are described in the remaining of this section.

- The morphology of a process[4] is described by prefix (.) and CG slash operators (\ and /).
- + and - are used for specifying the communication over a channel (e.g. *obj*). For example NP-obj and V+obj combine to form a communication link V=obj=NP.
- [ ] is used for temporal time stamp for each process. V[1] and S[1-6] depict processes that exist in first time period and first through sixth time periods respectively.
- {} is used to depict multiple communication channels for a process, e.g. V{+obj, +subj}; and multiple communication, e.g. S{V[3]=obj=NP[1],V[3]=subj=NP[2]}. The name before {} is used to specify a locality[5] or ambient.
- | is used for describing alternatives and competition at channel level and interprocess communication level. The competing NP-obj and NP-subj are depicted as NP{-obj | -subj}. The competing V=obj=NP[1] and V=obj=NP[1]

[4]Note that the operators we use in this subset are different from the operators that we used in the previous section based on the work in process algebra research.

[5]|| operator is used for flexible word order inside a locality.

for an ambiguous case marking of an NP is depicted as S{V=obj=NP[1] | V=subj=NP[1]}.

- Process creation or reductions are depicted by $\rightarrow$. S $\rightarrow$ S S_{embed} is an example of process creation. In the case of localities or ambients, the export of mobile channels is possible during the process creation.

- The stochastic measures for activation are specified by # at all levels of specifications (channels, communication, localities, alternatives, ...). These measures are *possibility* values borrowed from fuzzy logic and fuzzy sets[6].

- The precedence operator ($\prec$) and its stochastic version[7] with an associated # value impose *optimal* linear constraints on channels.

The process grammar (π-grammar) that we described provides the notion of *imperfect channel* by the activation measure #. But the specification of π-grammar in its present form does not provide *conditional* channels. This needs further research. But π-grammar is expressive and flexible to be extended. It provides operators for expressing a large set of constraints and principles. For example the Resource Limitation principle that we specified earlier has been implemented by using a stochastic precedence operator [Rezaei, 2000b].

7 Conclusion

Starting from the communicating "sequential" processes in CSP, one can reach a richer notion of communicating "linguistic" processes (CLP). The main task is the extension of syntax and semantics of a formal model such as CSP or Pi-calculus in a minimal way to accommodate the required linguistic counterparts.

A parallel research in computational semantics has argued for the existence of channels of communication for semantic modeling. The introduction of a syntactic process algebra or a syntactic channel algebra will complement the work in semantics and provide a communication model for linguistic modeling [Rezaei, 1999]. Such a formal framework for modeling human communication apparatus is general enough and can be applied to other non-linguistic cognitive-biological domains, such as genetic modeling.

The formal specification of the model in terms of process algebras remains a complex task, but instead we have incorporated the communication notions into an existing grammatical framework. The result is Π-grammar, a process grammar.

[6] In a feature based specification of the grammar, we have for each feature or set, a corresponding # value or # set respectively. This requires the incorporation of fuzzy sets into feature unification.

[7] See [Rezaei, 2000b] for further detail.

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Existential Import

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The present paper deals with the semantics of *all*, *some* and *not* in English and related languages. Since these semantics imply a logic, it also deals with the logic of quantification in natural language. To the extent that the linguistic findings are universally valid, the results are relevant to the study of human cognition in general.

Since standard quantification theory (SQT) replaced Aristotelian Predicate Calculus (APC), philosophers of language and semanticists have struggled with the fact that APC corresponds better with natural semantic intuitions than SQT. Pragmatics was called in to restore APC on non-logical, pragmatic grounds. The replacement of APC by SQT was motivated on the grounds that (a) APC suffers from improper existential import, and (b) SQT is a straightforward application of Boolean algebra, and as such preserves syllogistic reasoning on a mathematical basis.

Logically speaking, SQT differs from APC in that the Aristotelian subaltern entailments are abolished, which makes the Aristotelian Square collapse but saves the equivalences (conversions) between $\neg\forall\neg$ and $\exists$, and $\neg\exists\neg$ and $\forall$. In SQT, the quantifying predicate $\exists$ over pairs of sets $<X, Y>$ yields truth iff $X \cap Y \neq Ø$ and $\forall$ does so iff $X \subseteq Y$. The author recently found that if the condition $X \neq Ø$ is added to the condition for $\forall$ and the conversions are changed into one-way entailments (i.e. $\forall x(Fx,Gx) \vdash \neg\exists x(Fx,\neg Gx)$ and $\forall x(Fx,\neg Gx) \vdash \neg\exists x(Fx,Gx)$, but not vice versa), APC is restored without improper existential import (only the subcontraries are lost). The Boolean basis is unaffected since whenever $\forall$ yields truth, it is still so that $X \subseteq Y$. The resulting revised Aristotelian predicate calculus (RAPC), is represented in the Hexagon of fig. 1 (arrows stand for entailments, 'C' for contraries, 'CD' for contradictories). Note that the traditional (Boethian) letter types **A**,.**I**, **E** and **O** have been replaced with $\forall$, $\exists$, $\forall\neg$ and $\exists\neg$, respectively, since APC contains only the standard quantifiers $\forall$ and $\exists$, plus the standard negation $\neg$. (This answers the question, raised by Horn (1972, 1989:252-67) and Levinson (2000:69-71), of why the **O**-corner in APC is never lexicalised: there ís no **O**-corner to be lexicalized!)

The revision of APC is easily shown by means of a valuation space interpretation (VSI) (Van Fraassen 1971). Let the valuation space (VS) of a sentence *A*, /*A*/, be the set of situations in a universe **U** of possible situations in which *A* is true. To say that *A* is true now amounts to saying that the actual situation $s_a \in /A/$. Clearly, $/\neg A/ = \mathbf{U} - /A/$, $/A \wedge B/ = /A/ \cap /B/$, and $/A \vee B/ = /A/ \cup /B/$, and the whole of standard propositional calculus can be derived. APC can be rendered in VSI terms as in fig. 2, where each ring (circle) is marked for the VS of each Aristotelian sentence type. Given the fact that, as indicated in fig. 2, relations of entailment, contrariety, subcontrariety and contradiction can be read from the VSI diagram, the whole of APC is represented in fig. 2.

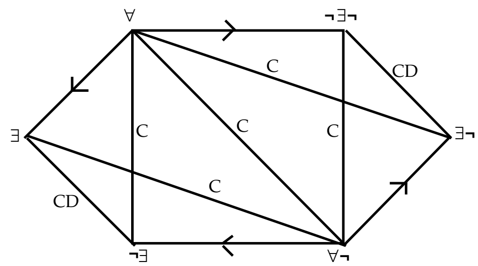

Figure 1

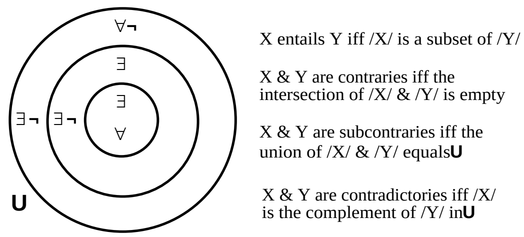

Figure 2

APC, however, fails to take into account the set of situations where the F-class is empty. Therefore, **U** in fig. 2 is incomplete and must be extended with a further ring containing those situations in which there is no representative of the F-predicate, as shown in fig. 3a.

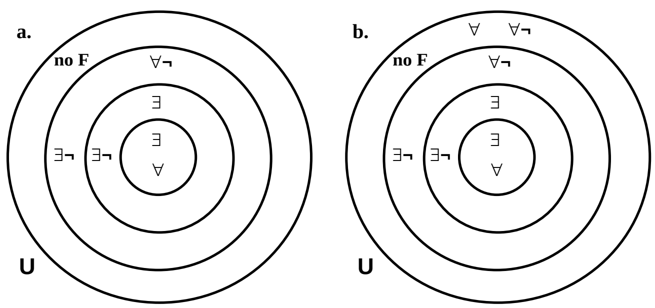

Figure 3

Now, however, the logical relations have changed. The subcontraries have gone and the conversions have been changed into one-way entailments as shown in the Hexagon of fig. 1. One notes that in fig. 3a, which represents RAPC, ∀, ∃, ∀¬ and ∃¬ are all false in cases where there is no F. SQT, on the contrary, declares both ∀ and ∀¬ true in such cases, as shown in fig. 3b. This, in fact, is the only difference between RAPC and SQT.

From the point of view of natural intuitions, the loss of the subcontraries does not appear serious, as it is easy to see the simultaneous falsity of “Some F is G” and “Some F is not-G” for cases where the F-class is non-instantiated. The replacement of the equivalences by one-way entailments from the universal quantifier ∀ likewise looks empirically promising. The inference from “All students did not pass the exam” to “No student passed the exam” seems correct, whereas the inference from “No proof of the man’s guilt has been found” to “All proofs of the man’s guilt have not been found” does not. Similarly, “All students passed” seems to license

"No student did not pass", but not vice versa, since many speakers will judge the former false but the latter true in cases where there were no students. If these judgements are correct, the Hexagon, i.e. RAPC, corresponds even better to natural intuitions than the original APC.

Like SQT, however, RAPC still fails to account for intensional (i.e. imagined) entities and intensional predicates (i.e. predicates that do not require real existence of the argument term referent for truth). For example, a sentence like "Some gods are worshipped in that temple" may well be true without it being necessary to conclude that there exist real gods. For that reason, the quantificational calculus must be modified and generalised to account for intensional phenomena as well.

Since natural language refers to and quantifies over intensional objects in precisely the same way as it does with regard to extensional (really existing) objects, there appears to be a prima facie requirement that single theories of reference and of quantification should account for both the extensional and the intensional cases. This makes it mandatory to accept an ontology containing incompletely defined *intensional objects*, as proposed by the Austrian philosopher Alexius Meinong (1853-1920). In an intensional theory of quantification, the universe of individuals **I** must contain all really existing as well as all imagined entities (objects). The quantifiers are still higher order predicates over pairs of sets (generalized quantifiers). However, the restrictor set (R-set) is no longer the standard extension of the predicate Fx, [[Fx]], comprising the set of entities that satisfy Fx, but the *intensional extension* {Fx} or the set of entities that satisfy Fx plus those that satisfy Π(Fx) (where Π is an intensional predicate/operator). One notes that {Fx} cannot be empty, since whenever Fx is mentioned it has automatically been imagined. This makes it possible to remove the condition $X \neq \emptyset$ from the satisfaction conditions of $\forall$.

At this point the minimal, presupposition-preserving negation "~" must be defined. We take it that the satisfaction conditions of a predicate *P* are divided into two subsets, the *preconditions* and the *update conditions* (cp. Seuren at al. 2001). The former define the *presuppositions* of the proposition *Pa*, i.e. the conditions of contextual coherence ('discourse anchoring') for *Pa*; the latter define the *semantic contribution* made by *Pa* to the discourse at hand. Together they form the truth conditions of *Pa*. Since an unanchored sentence lacks a truth value and does not express a proposition (as when I say to you now "The man was right after all", without any explanation as to the identity of the man or the issue at hand), the preconditions are truth-conditional, not just pragmatic, as is widely held in pragmatic circles. For good functional reasons of coherent discourse, the normal default negation in natural language toggles between satisfaction and non-satisfaction of the update conditions, leaving the preconditions unaffected. This negation is called the *minimal negation*, represented by "~". In VSI terms, we say that for each sentence *A* there is a *subuniverse* of possible situations $\mathbf{U}_A$, where the presuppositions of *A* are true (cp.

Seuren et al. 2001). If *A* has no presuppositions (i.e. the predicate of *A* has no preconditions), $\mathbf{U}_A = \mathbf{U}$. We now say that for all sentences *A*, $/A/ \subseteq \mathbf{U}_A$ and $/{\sim}A/ \subseteq \mathbf{U}_A$, and $/{\sim}A/ = \mathbf{U}_A - /A/$. There is also a *radical negation* $\simeq$, such that $/{\simeq}A/ = \mathbf{U} - \mathbf{U}_A$. This, however, is left out of consideration here. But note that $/{\sim}A/ \cup /{\simeq}A/ = /\neg A/$, as shown in fig. 4.

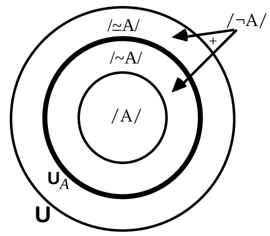

Figure 4

We now define [<Fx>], the *presuppositional extension* of the predicate *Fx*, as the set of entities that satisfy the preconditions of *Fx*, i.e. for which either *Fx* or *~Fx* yields truth. Clearly, if *Fx* has no preconditions, [<Fx>] = **I** and {Fx} = [[Fx]]. We define the universal quantifier ∀ as taking the precondition that {Fx} ∩ [<Gx>] ≠ Ø, and the update condition that {Fx} ∩ [<Gx>] ⊆ [[Gx]]. In other words, “All F is G” is true iff for all *e* ∈ {Fx} ∩ [<Gx>] (i.e. *e* ∈ {Fx} qualifies for the predicate Gx), *e* ∈ [[Gx]]. For example, “All Englishmen are rich” is true iff all members of {Englishman(x)} that qualify for the predicate “rich” are indeed rich. Since “rich” has a precondition of existence for its subject term, the class of imaginary Englishmen is automatically excluded from consideration. On the other hand, “All unicorns are imaginary” is true in this world, since the predicate “imaginary” has no existential precondition, so that [<imaginary(x)>] = **I**. Since {unicorn(x)} ∩ **I** = {unicorn(x)}, it is sufficient that {unicorn(x)} ⊆ [[imaginary(x)]], which is the case. Existential import is thus taken away from the existential quantifier and placed in the G-predicate. “Some Englishmen are rich” now entails the existence of Englishmen, but “Some gods are worshipped” does not entail the existence of gods, since “rich” is extensional but “be worshipped” intensional with regard to the subject term.

Whether the precondition {Fx} ∩ [<Gx>] ≠ Ø specified for ∀ should also be specified for (the negation of) ∃, depends on natural intuitions. If a sentence like “No unicorn likes hay” is deemed true in this world, which is far from unlikely, then the answer is No, otherwise Yes. Pending the availability of reliable data, we leave both options open. In fact, French speakers report that “Aucun unicorne n’aime le foin” is more likely to be taken to be false, whereas “Il n’y a pas d’unicorne qui aime le foin” is clearly true. There may thus be different varieties of the (negated) existential quantifier.

This account restores the equivalence of “not-all F is G” and “some F is not G”, which was lost in RAPC. In RAPC, ¬∀(Fx,Gx) ≢ ∃(Fx,¬Gx), since when [[Fx]] = Ø, ¬∀(Fx,Gx) is true while ∃(Fx,¬Gx) is false. In the intensional calculus, however, when the matrix predicate Gx is extensional, ~∀(Fx,Gx) is false in cases where [[Fx]] = Ø, owing to existential presupposition failure, while ∃(Fx,~Gx) is likewise false. When, on the other hand, Gx is intensional, it is immaterial whether or not [[Fx]] = Ø, since with intensional matrix predicates the conditions for both ∀ and ∃ apply to {Fx}, which is automatically nonempty (see above). Fig. 5 shows this more clearly: the shaded area represents /~∀(Fx,Gx)/, which coincides with /∃(Fx,~Gx)/. If the G-predicate has no preconditions, $\mathbf{U}_{\forall}$ = **U**, which leaves the equivalence intact. This again improves the empirical status of the calculus.

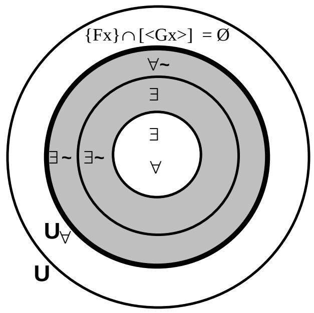

Figure 5

On the other hand, however, there is a problem (signalled by a number of authors, e.g. Zalta 1988, Castañeda in Haller 1985/6:58, Lejewski in Haller 1985/6:232), with the truth conditions of simple sentences of the form “Holmes is an Englishman”. We want to say that “Some Englishmen are imaginary” is true, since {Englishman(x)} ∩ [[imaginary(x)]] ≠ Ø. If we are then asked to mention an instance of an imaginary Englishman, we want to be able to produce, for example, Sherlock Holmes as imagined by Conan Doyle. Yet under the terms specified so far “Holmes is an Englishman” is (radically) false owing to presupposition failure, since to be an Englishman one first has to exist, which Holmes does not do. We need, therefore, a second interpretation under which “Holmes is an Englishman” is true, which makes this sentence ambiguous. Our solution consists, in principle, in adding the hedge “who/which qualifies for the (main) predicate Gx” not only to all quantified terms but also to instantiations adduced in a chain of argument. Not only would “Some Englishmen are imaginary” then be read as “Some Englishmen who qualify for the predicate “imaginary” are imaginary”, but “Holmes is an Englishman” would then likewise be read as “Holmes is an Englishman who qualifies for the predicate “imaginary”, or “Holmes is a case in point”. However, the mechanism for the distribution of such hedges has not been developed yet (see Castañeda in Haller 1985/6:58 for a similar view, but without formal elaboration).

One possible implication for the study of human cognition should be mentioned here. It appears that human cognition does not naturally develop the concept of null set until a very high degree of mathematical abstraction is achieved. The question is whether a satisfactory logic and semantics of quantification can be developed without the help of Ø to account for the lower levels of abstraction where natural language operates. As has been shown above, intensional predicate calculus eliminates Ø as an option for the R-set (F-predicate). It remains to eliminate Ø for the matrix set (G-predicate): a sentence like “Some logicians are 450 years old”, where the predicate “450 years old” is uninstantiated, must be processable and result in the value False. This may be achieved by treating the quantifiers as binary predicates over pairs of R-sets and predicate intensions (satisfaction conditions of predicates). “All F is G” now means: “all members of {Fx} that qualify for Gx satisfy the conditions of Gx”. One notes that this would be a return to the Aristotelian notion of a proposition as the mental assignment of a property to an entity or set of entities. It would also place the medieval theory of distributive supposition in a new formal light.

There is, furthermore, the peculiar fact that, at a non-reflective default level of cognitive operation, *some* is naturally interpreted as “partial”, and thus equivalent with “partial not”. The corresponding extensional calculus is shown in fig. 6 (with “**P**” for ‘partial’ ‘and “=“ for ‘equivalent’):

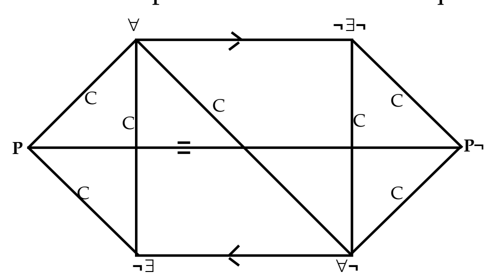

Figure 6

It may be assumed that at this default level of abstraction the notion ‘subset’ (denoted by “some F”) is defaultwise interpreted as ‘proper subset’. The combination of the ‘no-null-set’ hypothesis with the ‘proper subset’ hypothesis opens an interesting perspective on further research into the logical properties of cognition.

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Completeness and Decidability of the Logic of Relativized End-Extension Set Algebras

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Abstract

The most popular concrete algebras for predicate logic are (generated subalgebras) of cylindric algebras. This research note is part of a larger project, that looks at an alternative class of concrete algebras, which use end-extensions instead of cylindrifications. An earlier manuscript gave reason to think that the generated subalgebras of the usual, full case would be essentially isomorphic. This resarch note shows some similarities between the two classes of algebras in the case of relativized algebras: there is a class of relativized end-extension set algebras, which has some of the key properties of relativized cylindric set algebras: models of the logic contain essentially arbirary sets of assignments and the key diference between the axiomatization of the full and relativized case is the restriction of vacuous generalization to atomic formulas, and membership in the logic is decidable.

Introduction

The most popular concrete algebras for predicate logic are (generated subalgebras) of cylindric set algebras. Nevertheless, there have been repeated attempts in the literature to explore alternative classes of algebras for predicte logic such as the algebras of sets of finite sequences of Craig (1974), or the many-sorted cylindric algebras of Bernays (1959). The present note is part of this tradition, exploring the case of a class of algebras closely related to but different from cylindric set algebras, which are called end-extension set algebras. End-extension algebras are different from cylindric set algebras in that sets of ω sequences are related to one another by the predecessor operation instead of cylindrification. The corresponding propositional modal logic was axiomatized in Ben Shalom (1995) and a one-to-one translation was suggested between its formulas and predicate logic formulas (up to alphabetical variants).

A full end-extension set algebra is defined by:

$$\langle \mathcal{B}({}^{\omega}U), \mathrm{c}, \mathrm{Id}_i \rangle_{i-1<\omega}$$

with

$$\mathrm{c} =_{def} \mathrm{pred}$$

i.e., for every $x \in {}^{\omega}U$, $\mathrm{pred}x =_{def} \{\mathrm{pred}\alpha \mid \alpha \in x\}$, where $(\mathrm{pred}\alpha)_i =_{def} \alpha_{i+1}$, for all $\alpha \in {}^{\omega}U, i < \omega$.

$$\mathrm{Id}_i =_{def} \{\alpha \in {}^{\omega}U \mid \alpha_0 = \alpha_i\}$$

Following a convention in the cylindric algebra literature, we will use the following abbreviation:

$$\mathrm{s}_i x =_{\mathrm{def}} \mathrm{c}(\mathrm{Id}_i \wedge x)$$

Generated subalgebras:

Generators of algebras for a first-order language with a set $\mathcal{R}$ of relation symbols and $\rho{:}\mathcal{R} \to \omega$ a rank-function on $\mathcal{R}$ are of the form:

$$\{\mathrm{R} \times {}^{\omega}U \mid R \in \mathcal{R}, \mathrm{R} \in {}^{\rho(R)}U\}$$

Axiomatization

Axioms beyond the minimal modal logic K

$$[+1]: \quad t = \mathrm{c}[+1]t$$

$$U: \quad \mathrm{c}(\mathrm{Id}_i \wedge x) = -\mathrm{c}(\mathrm{Id}_i \wedge -x)$$

$$\mathrm{Id}: \quad \mathrm{Id}_i = \mathrm{c}(\mathrm{Id}_1 \wedge \mathrm{Id}_{i+1})$$

$$\rho: \quad x_j = \underbrace{\mathrm{c}(\mathrm{Id}_j \wedge \ldots \mathrm{c}(\mathrm{Id}_j \wedge}_{j \text{ times}} x_j \underbrace{)\ldots)}_{j \text{ times}}$$

Relativized set algebras:

A relativized end-extension set algebra is defined by:

$$\langle \mathcal{B}(V), \mathrm{c}, \mathrm{Id}_i \rangle_{i-1<\omega}$$

$$V \subseteq \text{ unraveling of } \langle {}^{\omega}U, \mathrm{c} \rangle$$

$$V \text{ closed under } \mathrm{s}_i, i-1<\omega$$

$$\mathrm{c}, \{\mathrm{Id}_i\}_{i-1<\omega} \text{ restricted to } V$$

with unraveling defined by the following:

Definition 1 *Let s be a point in the domain of a model $\mathcal{M} = \langle S, R \rangle$. $\mathcal{M}_s = \langle S_s, R_s \rangle$, the* unraveling of $\mathcal{M}$ around s *is defined inductively by:*

$$\begin{array}{lcl} S_0 & = & \{(s,0)\} \\ S_{n+1} & = & S_n \cup \{(as', n+1) \mid (a,n) \in S_n,\ a_0 R s'\} \\ S_s & = & \bigcup_{i\in\omega} S_i \\ (a,l) R_s (a',l') & \text{iff} & a' = as' \text{ for some } s' \in S \end{array}$$

Generated subalgebras:

Generators of a relativized algebra for a first-order language with a set $\mathcal{R}$ of relation symbols and $\rho{:}\mathcal{R} \to \omega$ a rank-function on $\mathcal{R}$ are of the form:

$$\{((\mathrm{R} \times {}^{\omega}U) \times \omega) \ \cap\ V \mid R \in \mathcal{R}, \mathrm{R} \in {}^{\rho(R)}U\}$$

Axiomatization

$[+1]' : \quad t = \mathrm{c}[+1]t \qquad\qquad t$ atomic or negated atomic

$U : \quad \mathrm{c}(\mathrm{Id}_i \wedge x) = -\mathrm{c}(\mathrm{Id}_i \wedge -x)$

$\mathrm{Id} : \quad \mathrm{Id}_i = \mathrm{c}(\mathrm{Id}_1 \wedge \mathrm{Id}_{i+1})$

$$\rho : \quad x_j = \underbrace{\mathrm{c}(\mathrm{Id}_j \wedge \ldots \mathrm{c}(\mathrm{Id}_j \wedge}_{j \text{ times}} x_j \underbrace{) \ldots)}_{j \text{ times}}$$

Theorem 1 $\Pi' =_{def} K[+1]'U\mathrm{Id}\rho$ *is an axiomatization of the logic of relativized end-extension set algebras.*

proof:

Appendix A.

Theorem 2 *The logic of relativized end-extension set algebras is decidable.*

proof:

Appendix B.

Thus, relativized end-extension set algebras have some of the key properties of relativized cylindric set algebras, as summarized in van Benthem (1996): models of the logic contain essentially arbitrary sets of assignments, the key difference between the axiomatization of the full and relativized case is the restriction of vacuous generalization to atomic formulas, and membership in the logic is decidable.

A Completeness

Let γ^0 be a set of $\mathcal{L}$ formulas that is Π'-consistent.

We construct a tree model $\mathcal{M}^t = (S^t, R^t, V^t)$ and a point $s^t \in S$ such that

$$\mathcal{M}^t \models_{s^t} \gamma \text{ for all } \phi \in \gamma^0$$

Step 1: Since Q is countable, γ^0 can be extended to a maximally Π'-consistent set γ' in the usual way. Enumerate $\mathcal{L}$. Define

$$\begin{array}{rcl} \gamma_0 & = & \gamma^0 \\ \gamma_{n+1} & = & \begin{cases} \gamma_n \cup \{\phi_n\} & \text{if } \gamma_n \vdash_{\Pi'} \phi_n \\ \gamma_n \cup \{\neg\phi_n\} & \text{otherwise} \end{cases} \end{array}$$

$$\gamma' = \bigcup_{n \geq 0} \gamma_n$$

Step 2: Let $\mathcal{M}^{\Pi'}_{\Gamma'} = \langle S^{\Pi'}_{\gamma'}, R^{\Pi'}_{\gamma'}, V^{\Pi'}_{\gamma'} \rangle$ be the unraveling of the canonical model of Π' around γ', and for every (Σ, i) in $\mathcal{M}^{\Pi'}_{\gamma'}$ let $a_{\Sigma,i}$ be a new object.

Lemma 1 *For every $\delta, \delta' \in Id$,*

$$\vdash_{\Pi'} \Box(\delta \rightarrow \delta') \leftrightarrow \Box(\delta \leftrightarrow \delta') \quad [1]$$

$$\vdash_{\Pi'} \Diamond(\delta \wedge \delta') \leftrightarrow \Box(\delta \leftrightarrow \delta')$$

[1] All proofs in the appendices are in the corrseponding propositional modal logic, and $\Box$ is the universal modal operator.

Proof:

$$\begin{array}{lll} \Box(\delta \to \delta') & & \vdash_{\Pi'} \\ \Diamond(\delta \wedge \delta') & U & \vdash_{\Pi'} \\ \Diamond(\delta' \wedge \delta) & & \vdash_{\Pi'} \\ \Box(\delta' \to \delta) & U & \end{array}$$

□

Define the following binary relation on Id

$$\delta \sim \delta' \text{ iff } \Box(\delta \leftrightarrow \delta') \in \gamma'$$

$\sim$ is an equivalence relation on Id.

Let

$$A_{\gamma'} =_{def} \ Id/{\sim} \cup \{a_{\Sigma,i} \mid (\Sigma, i) \in \mathcal{M}_{\gamma'}^{\Pi'}\}$$

Step 3: Define a function f from $\mathcal{M}_{\gamma'}^{\Pi'}$ into pairs of the form (s, l), where s is a sequences of length ω over $A_{\gamma'}$, $l \in \omega$, by induction on i:

We write $f(\Sigma, l)$ for $f((\Sigma, l))$.

$$\begin{array}{rcl} f_0(\gamma', 0) & = & (\ldots[\delta_k]\ldots[\delta_2][\delta_1], 0) \\ f_{n+1}(\Sigma\gamma, n+1) & = & \begin{cases} (ss_m, n+1) & \text{if } \ f_n(\Sigma, n) = (s, n), \ \delta_{m+1} \text{ is the minimal } \delta \in \gamma \\ (sa_{\Sigma\gamma, n+1}, n+1) & \text{if } \ f_n(\Sigma, n) = (s, n), \ \text{no such } \delta \text{ exists} \end{cases} \\ f & = & \bigcup_{i \in \omega} f_i \end{array}$$

Definition 2 *$\mathcal{M}^t = \langle S^t, R^t, V^t \rangle$, is defined by:*

$$\begin{array}{lll} S^t & = & \{f(\Sigma, l) \mid (\Sigma, l) \in \mathcal{M}_{\gamma'}^{\Pi'}\} \\ (s,l)R^t(s',l') & \text{iff} & s' = sa, a \in A_{\gamma'}, l' = l+1 \\ f(\Sigma,l) \in V^t(q) & \text{iff} & (\Sigma,l) \in V_{\gamma'}^{\Pi'}(q) \end{array}$$

Step 4: f is a bisimulation between the domains of $\mathcal{M}_{\Gamma'}^{\Pi'}$ and $\mathcal{M}^t$.

Lemma 2 *Let $(s,l) \in \mathcal{M}^t, (\Sigma, l) \in \mathcal{M}_{\gamma'}^{\Pi'}$ such that $(s,l) = f(\Sigma, l)$. For all $i, k \in \omega$,*

$$s_i = s_k \text{ iff } (\Sigma, l) \vdash_{\Pi'} \Box(\delta_{i+1} \leftrightarrow \delta_{k+1})$$

Proof:

By induction on l.

$l = 0$: by the construction.

$l = m+1$: by the construction, there is an $(s',m) \in \mathcal{M}^t, (\Sigma', m) \in \mathcal{M}_{\gamma'}^{\Pi'}$ such that $(s',m)R^t(s,m+1)$, $(\Sigma',m)R_{\gamma'}^{\Pi'}(\Sigma, m+1)$, and $(s',m) = f(\Sigma',m)$.

Subcase 1: $i, k \geq 1$

$$\begin{array}{lll} s_i = s_k & & \Leftrightarrow \\ s'_{i-1} = s'_{k-1} & & \Leftrightarrow \\ (\Sigma', m) \vdash_{\Pi'} \Box(\delta_i \leftrightarrow \delta_k) & \text{IH} & \Leftrightarrow \\ (\Sigma', m) \vdash_{\Pi'} \Box\Box(\delta_{i+1} \leftrightarrow \delta_{k+1}) & [+1]' & \Leftrightarrow \\ (\Sigma, m+1) \vdash_{\Pi'} \Box(\delta_{i+1} \leftrightarrow \delta_{k+1}) & & \end{array}$$

Subcase 2: If $i = k = 0$, we are done. Otherwise, wlg $k > i = 0$

By the construction of $\mathcal{M}_{\gamma'}^{\Pi'}$, let δ_n be the minimal δ in Σ_0'.

$$\begin{array}{lll} s_0 = s_k & & \Leftrightarrow \\ s_n = s_k & & \Leftrightarrow \\ s_{n-1}' = s_{k-1}' & & \Leftrightarrow \\ (\Sigma', m) \vdash_{\Pi'} \Box(\delta_n \leftrightarrow \delta_k) & \text{IH} & \Leftrightarrow \\ (\Sigma, m+1) \vdash_{\Pi'} \delta_k & & \Leftrightarrow \\ (\Sigma, m+1) \vdash_{\Pi'} \Box(\delta_{i+1} \leftrightarrow \delta_{k+1}) & \text{Id} & \end{array}$$

□

Lemma 3 *For every* $(\Sigma, l) \in S_{\gamma'}^{\Pi'}$, $\delta_i \in Id$ *there is exactly one* $(\Sigma\gamma, l+1) \in S_{\gamma'}^{\Pi'}$ *such that* $(\Sigma\gamma, l+1) \vdash_{\Pi'} \delta_i$.

Proof:

Such a $(\Sigma\gamma, l+1)$ exists because $(\Sigma, l) \vdash_{\Pi'} \Box(\delta_i \to \top) \leftrightarrow \Diamond(\delta_i \wedge \top)$. Suppose there are two: $(\Sigma\gamma, l+1)$ and $(\Sigma\gamma'', l+1)$, then for every ϕ, $(\Sigma, l) \vdash_{\Pi'} \Box(\delta_i \to \phi) \leftrightarrow \Diamond(\delta_i \wedge \phi)$, so $(\Sigma\gamma, l+1)$ and $(\Sigma\gamma'', l+1)$ verify the same modal formulas, so γ and γ'' are the same Π'-maximal set. □

Lemma 4 f *is* $1-1$.

Proof:

For every $i \in \omega$, f_i maps objects of the form (Σ, i) to objects of the form (s, i), so it is enough to prove that every f_i is $1-1$.

By induction on i:

$i = 0$: obvious.

$i = m + 1$: By the IH and the construction, the only relevant case is of the form $(sa, m+1) = f(\Sigma\gamma, m+1) = f(\Sigma\gamma'', m+1)$, with δ_{k+1} being the minimal $\delta \in \gamma$ and $\delta_{k'+1}$ being the minimal $\delta \in \gamma''$. So by the construction, $s_k = s_{k'}$, so by lemma 2 $(\Sigma, m) \vdash_{\Pi'} \Box(\delta_{k+1} \leftrightarrow \delta_{k'+1})$, so by lemma 3, $(\Sigma\gamma, m+1) = (\Sigma\gamma'', m+1)$. □

By definition 1 and lemma 4, f is a bisimulation between the domains of $\mathcal{M}_{\Gamma'}^{\Pi'}$ and $\mathcal{M}^t$.

Step 5: $\mathcal{M}^t$ is a relativized end-extension model.

Lemma 5 *For every $(s, l) \in S^t$ and δ_i*

$$(s, l) \models \delta_i \text{ iff } s_0 = s_i$$

Proof:

By lemma 2 and Id. □

Let $a_1, a_2, \ldots, a_h \in A_{\gamma'}$ be a positive number of element such that $a_1, a_2, \ldots, a_h$ appear together in the string part s of some $(s, l) \in S^t$. Let $t_{a_1, a_2, \ldots, a_h}$ be the (s, l) with the minimum l, such that every $a_g, 1 \leq g \leq h$ appears in s. By the construction such a $t_{a_1, a_2, \ldots, a_h}$ exists and is unique.

For every g, $1 \leq g \leq h$, let k_g be the difference between the level of $t_{a_1, a_2, \ldots, a_h}$ and t_{a_g}.

Lemma 6 *For every* $(sa_1a2\ldots a_j, l) \in S^t$ *and* p

$$(sa_1a2\ldots a_j, l) \models p \quad \text{iff}$$

$$f^{-1}(t_{a_1,a_2,\ldots,a_j}) \models \Box(\delta_{i_1+1} \rightarrow \Box(\delta_{i_2+2} \rightarrow \ldots \Box(\delta_{i_j+j} \rightarrow p \underbrace{)\ldots)}_{j \text{ times}}$$

Proof:

Let k the difference between the levels of $t_{a_1,a_2,\ldots,a_j}$ and $(sa_1a2\ldots a_j, l)$.

Subcase 1: $sa_1a2\ldots a_j = t_{a_1,a_2,\ldots,a_j}b_1b2\ldots b_m, m \geq j$.

$$\left.\begin{array}{l} t_{a_g} \\ \vdots \\ t_{a_1,a_2,\ldots,a_j} \end{array}\right\} k_g$$

$$\left.\begin{array}{l} b_1 \\ b_2 \\ \vdots \\ b_m \end{array}\right\} k$$

For all g, $1 \leq g \leq j$

$$(sa_1a_2\ldots a_j)_{j-g} = (sa_1a_2\ldots a_j)_{k+k_g} \qquad \Rightarrow$$

$$sa_1a_2\ldots a_j \models \Box(\delta_{j-g+1} \leftrightarrow \delta_{k+k_g+1}) \quad \text{lemma 2} \quad \Rightarrow$$

$$sa_1a_2\ldots a_j \models \Box^g(\delta_j \leftrightarrow \delta_{k+k_g+g}) \quad [+1']$$

So by lemma 1, we can replace every δ_{k+k_g+g} in the second line below by δ_j.

$$f^{-1}(t_{a_1,a_2,\ldots,a_j}) \models \Box(\delta_{k_1+1} \to \Box(\delta_{k_2+2} \to \ldots \Box(\delta_{k_j+j} \to p \underbrace{)\ldots)}_{j \text{ times}} \qquad \Leftrightarrow$$

$$f^{-1}(sa_1a_2\ldots a_j) \models \Box(\delta_{k_1+1+k} \to \Box(\delta_{k_2+2+k} \to \ldots \Box(\delta_{k_j+j+k} \to p \underbrace{)\ldots)}_{j \text{ times}} \quad [+1]' \quad \Leftrightarrow$$

$$f^{-1}(sa_1a_2\ldots a_j) \models \Box(\delta_j \to \Box(\delta_j \to \ldots \Box(\delta_j \to p \underbrace{)\ldots)}_{j \text{ times}} \qquad \Leftrightarrow$$

$$f^{-1}(sa_1a_2\ldots a_j) \models p$$

Subcase 2: $sa_1a2\ldots a_j = t_{a_1,a_2,\ldots,a_j}b_1b2\ldots b_m, m < j$.

For every $g < j - m$, $j - g = k + k_g$ by assumption. The rest is as in case 1. □

By step 4 f is a bisimulation between the domains of $\mathcal{M}^{\Pi'}_{\Gamma'}$ and $\mathcal{M}^t$, and by step 5 $\mathcal{M}^t$ is a relativized end-extension model. So $\mathcal{M}^t$ is the desired relativized end-extension model and Π' is complete.

B Decidability

Definition 3 *A clipped relativized end-extension model for $\mathcal{L}$ is a triple $\mathcal{M} = (T, R, V)$, if is are a relativized end-extension model $\mathcal{M}' = (T', R', V')$ with root (s, l), $n, k \in \omega$ such that*

$$\begin{aligned} T &= \{(s', l') \in T' \mid l' \leq n + l\} \\ R &= R' \cap (T \times T) \\ V(p) &= V'(p) \cap T \end{aligned}$$

and for every s_i in the root (s,0) of T, $s_i = s_k$ for all $i \geq k$.

We call n the *depth* of the model $\mathcal{M}$.

Theorem 3 *For $\mathcal{L}$ with a finite P, every clipped relativized end-extension model can be bisimulated onto a finite clipped relativized end-extension model with the same root.*

Proof:

Let $\mathcal{M} = (T, R, V)$ be a clipped relativized end-extension model with root (s, l) and depth n.

The theorem is proved by induction on n:

$n = 0$: obvious

$n = k + 1$:

Step 1: Let U be the set of sons of (s, l). Every $(s', l + 1) \in U$ is the root of a clipped relativized end-extension model with depth n, so by the IH, it can be bisimulated onto a finite clipped relativized end-extension model with the same root.

Step 2: Let U' be the set of finite clipped relativized end-extension models obtained from relativized end-extension models in U in Step 1.

For every two relativized end-extension models $\mathcal{M}$ and $\mathcal{M}'$ in U' with roots t and t', define:

$$\mathcal{M} \sim \mathcal{M}' \textit{ iff } t, t' \textit{ verify the same modal formulas up to depth } k$$

$\sim$ is an equivalence relation. Since P is finite, and there is an h beyond which $\delta_i \leftrightarrow \delta_h$ for every $i \geq h$, it has only a finite number of classes. For each such class, let $(s'', l+1)$ be the root of a representative of the class. By lemma 5, for every member of a class there is a bisimulation that connects its root and the root of the representative.

Step 3: By composing the bisimuations in step 1 and 2, we get a bisimulation that connects every $(s', l+1)$ in the original relativized end-extension model to the root of one of a finite number of clipped relativized end-extension models with roots in U. Moreover, this bisimulation is onto, since for very representative in step 2, the bisimulation between it and itself can be assumed to be identity. The desired finite clipped relativized end-extension model is the one determined by the original root and the finite set of finite clipped relativized end-extension models with roots in U as its sons. $\square$

By theorem 1, every formula ϕ is false in a relativized end-extension model $\mathcal{M}$ iff it is false in some finite clipped relativized end-extension model M'. Since Π' is complete for the class of relativized end-extension models models, non-membership in Π' is semi-decidable. As Π' is axiomatized,membership in Π' is semi-decidable too. So membership in Π' is decidable. $\square$

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A Reinterpretation of Syntactic Alignment

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1 Introduction: Harmonic Alignment in Syntax

Harmonic Alignment was proposed by Prince and Smolensky (1993) as a mechanism to establish a correspondence between different harmony scales within the overall framework of Optimality Theory ("OT" henceforth). They specifically address the combination of the phonological sonority hierarchy with the hierarchy of syllable positions. In recent work, Judith Aissen has taken up this idea as a mean to formulate insights from the functionally oriented markedness theory in morphology and syntax within OT syntax (cf. Aissen 1999, 2000). Though based on earlier work in typology like Silverstein (1981) , Aissen manages a formalization of a mechanism that promises an account of much that seems quaint and bizarre about natural languages when considered from the perspective of e.g. a designer of computer languages or logical formalisms.

Suppose a linguistic item can be classified according to two features, A and B. Suppose furthermore that A has two possible values, $A1$ and $A2$, while B has n possible values, $B1 \dots Bn$, for some $n \geq 2$. Finally, the values of each both features are ranked according to their prominence. Lets say that $A1$ is more prominent than $A2$, and Bi is more prominent than Bj iff $i < j$. Formally, we thus have the prominence scales

(1) $A1 > A2$

and

(2) $B1 > B2 > \cdots > Bn$

Harmonic alignment means that these scales induce a partial ordering on combinations of these features. A combination of a prominent A with a prominent B is harmonic, and so is a combination of a non-prominent A with a non-prominent B. Combinations of a prominent A with a non-prominent B or vice versa are non-harmonic. More precise, the two prominence hierarchies induce the following harmony sub-hierarchies:

(3) a. $A1/B1 \succ A1/B2 \succ \cdots \succ A1/Bn$
b. $A1/Bn \succ A1/Bn-1 \succ \cdots \succ A1/B1$

Aissen uses this mechanism to align formal markedness hierarchies (esp. the hierarchy of grammatical roles) with substantive markedness hierarchies like the definiteness hierarchy or the person hierarchy. The fundamental observation pertaining to harmonic alignment in syntax is that a considerable variety of regularities across languages can be expressed by making reference just to some upper part of the harmony partial order. We give a few examples for the purpose of illustration; the interested reader is referred to Aissen's papers for more comprehensive discussion.

Differential Object Marking Many languages with overt case marking mark some objects, but not others. Bossong (1985) calls this phenomenon "Differential Object Marking" (DOM). According to Aissen (2000), DOM always applies to the top section of a markedness hierarchy that is obtained by multiplying the scale of grammatical functions with some substantive scale like definiteness. Object marking may be optional for this top section and obligatory for the bottom section, it may be prohibited at the top and optional at the bottom, or it is obligatory at the bottom and excluded at the top. Language particular forms of DOM furthermore differ insofar as different substantive scales may be used, and the split may occur at different positions. Let us consider some examples. The scale of grammatical functions and the definiteness hierarchy are given in (4a,b); harmonic alignment leads to the harmony scales in (c) and (d).

(4) a. Subj > Obj
    b. pronoun > names > definite > specific indefinite > non-specific indefinite
    c. Subj/pronoun > Subj/name > Subj/def > Subj/spec > Subj/non-spec
    d. Obj/non-spec > Obj/spec > Obj/def > Obj/name > Obj/pronoun

Any split of the hierarchy in (4d) is attested in instances of DOM in certain languages.[1] Catalan, for instance, obligatorily marks object pronouns with *a*, while full NP objects are unmarked. In Pitjantjatjara (an Australian language), pronouns and proper nouns are case marked when they are objects while other NPs aren't. Hebrew marks only definite objects, and Turkish only specific ones. As borderline cases, one might add languages without any case marking like Kalkatungu (Pama-Nyungan) and languages with obligatory case marking like written Japanese, which select improper segments of the harmony hierarchy.

Similar observations can be made with regard to the animacy hierarchy and with regard to the Cartesian product of these two hierarchies.

Split ergativity The person specification of NPs induces another hierarchy. Simplifying somewhat, it says that the local persons (1st and 2nd) outrank 3rd person. Harmonic alignment thus yields the sub-hierarchies in (5).

(5) a. Subj/local $\succ$ Subj/3rd
    b. Obj/3rd $\succ$ Obj/local

These patterns underly split ergative case marking in languages like Dyirbal where the choice between the nominative/accusative system and the ergative/absolutive system is based on person. The table in figure 1 (which is taken from Aissen 1999) shows the basic case marking pattern for Dyirbal.

	Unmarked	Marked
Local persons	Subject	Object
3rd person	Object	Subject (of transitive)
Case	Nominative/Absolutive	Accusative/Ergative

Figure 1: Case marking system of Dyirbal

Briefly put, Dyirbal only marks non-harmonic arguments, i.e. local objects and 3rd person subjects. It thus represents a combination of DOM with Differential Subject Marking.

[1]See Aissen (2000) for examples and references.

2 OT Formalization

Prince and Smolensky (1993) develop a simple trick to translate harmony scales into OT constraints: for each element x of a scale we have a constraint $*x$ ("Avoid x!"), and the ranking of these constraints is just the reversal of the harmony scale. For the person/grammatical function interaction discussed above, this looks schematically as follows (adapted from Bresnan *et al.* 2001):

(6)

Prominence scales	Harmonically aligned scales	OT constraint sub-hierarchies
Subj > Obj	Subj/local $\succ$ Subj/3rd	*Subj/3rd $\gg$ *Subj/local
local > 3rd	Obj/3rd $\succ$ Obj/local	*Obj/local $\gg$ *Obj/3rd

The idea is that the constraint rankings in the third column represent universal sub-hierarchies which are to be respected by any language particular total constraint ranking.

Bresnan *et al.* (2001) present an interesting application of these constraint sub-hierarchies pertaining to person/voice interaction in Lummi, a Salish language spoken in British Columbia. There passivization is obligatory iff the agent of a two-place relation is expressed by third person and the patient by a local person. To express the proposition *The man knows me*, only the Lummi counterpart of (7b) is possible, (7a) is excluded:

(7) a. *The man knows me
b. I am known by the man

The alignment sub-hierarchy **Subj/Pat* $\gg$ **Subj/Ag*—which arises from harmonically aligning *Subj* > *Obj* with *Agent* > *Patient*—universally favors the active over the passive. On the other hand, the sub-hierarchy **Subj/3rd* $\gg$ **Subj/local* disfavors third person subjects. Languages differ as to how they resolve possible conflicts between these preferences. Lummi is characterized by the ranking **Subj/3rd* $\gg$ **Subj/Pat* $\gg$ **Subj/Ag*. This favors (7b) over (7a) and thus accounts for this grammaticality pattern. English, in comparison, ranks **Subj/3rd* lower than **Subj/Pat* and thus displays no categorical person/voice interaction of this kind. (Instead constraints referring to discourse features like topicality play a role that enforce passive under certain conditions.)

The applications of harmonic alignment that were discussed in the previous section are not covered yet by this OT treatment. Dyirbal, for instance, does not prohibit third person subjects, but it makes marking of those subjects obligatory. Generally, the common pattern of the examples is that non-harmonic combinations must be morphologically marked and harmonic combinations are unmarked. To formalize this idea in OT, Aissen employs a formal operation called "constraint conjunction" which she attributes to Paul Smolensky. If C_1 and C_2 are constraints, $C_1\&C_2$ is another constraint which is violated iff both C_1 and C_2 are violated. Crucially, $C_1\&C_2$ may outrank other constraints C_i that in turn outrank both C_1 and C_2. So the following constraint ranking is possible:

$$C_1\&C_2 \gg C_3 \gg C_4 \gg C_1 \gg C_5 \gg C_2$$

Furthermore, two general constraints play a role:

- "*∅" is violated if a morphological feature is not marked
- "*STRUC" is violated by any morphological marking

Each constraint resulting from harmonic alignment is conjoined with *∅, and the ranking of the conjoined constraints is isomorphic to the ranking induced by alignment. (Also the conjoined constraints outrank each of their conjuncts.) The alignment of the person hierarchy with the scale of grammatical functions thus for instance leads to the following universal constraint sub-hierarchies:

(8) *∅ & *Subj/3rd ≫ *∅ & *Subj/local
*∅ & *Obj/local ≫ *∅ & *Obj/3rd

Interpolating the constraint *STRUC at any point in any linearization of these sub-hierarchies leads to a pattern where morphological marking indicates non-harmony. The choice of the threshold for morphological marking depends on the relative position of *STRUC. The Dyirbal pattern, for instance, corresponds to the following constraint ranking.

(9) *∅ & *Subj/3rd ≫ *∅ & *Obj/local ≫ *STRUC ≫ *∅ & *Subj/local ≫ *∅ & *Obj/3rd

3 Some problems

The basic idea of harmonic alignment is conceptually attractive, and it explains a variety of typological generalizations in an elegant way. It is also quite natural to employ OT to formalize the cross-linguistic parameterization of the relevant harmony hierarchies. Nevertheless we find some aspects of the particular OT implementation that Aissen uses conceptually not fully satisfactory. In this section we will point out some issues that strike us problematic. The remainder of the paper will suggest a solution to some of them, while others have to be left open for further research.

To start with, Harmonic Alignment as such is only defined if one of the two scales to be aligned is binary. However, there are natural configurations where both inputs have more elements. In the previous sections, we tacitly confined the hierarchy of grammatical functions to subject and object, but the full scale is much more articulated; it comprises at least the following elements:

(10) subject > direct object > indirect object

Suppose we want to align this hierarchy with the animacy hierarchy

(11) human > anim > non-anim

For the subject and the indirect object, we presumably get a copy and a mirror image of the animacy hierarchy:

(12) a. subject/human ≻ subject/anim ≻ subject/non-anim
b. i-object/non-anim ≻ i-object/anim ≻ i-object/human

It is unclear though what the harmony hierarchy for the direct object should be. Both (13a) and (b) can be justified

(13) a. d-object/human ≻ d-object/anim ≻ d-object/non-anim
b. d-object/non-anim ≻ d-object/anim ≻ d-object/human

At the present time, we have to leave this issue open.

The next points concern the nature of the OT constraints that implement Harmonic Alignment. It seems to be highly unnatural to assume constraints like "Avoid pronominal subjects" or "Avoid indefinite objects!" Technically this is harmless because they are always dominated by constraints that are effectively their negation. Nevertheless one rather does without constraints that exclude the least marked configurations one can imagine.

Likewise, the concept of constraint conjunction is technically compatible with the overall OT architecture, but it nonetheless does not fit in very naturally. It is one of the basic assumption of OT that one violation of a given constraint cannot be countered by arbitrarily many violations of lower constraints. Constraint conjunction undermines this. Consider the following constraint ranking:

(14) $C_1 \& C_2 \gg C_3 \gg C_1 \gg C_2$

Effectively, this amounts to saying that violations of C_1 and C_2 each separately count less than a violation of C_3, but violations of C_1 and C_2 together sum up and are more severe than a single violation of C_3.

While this might be a marginal technical point, it appears to be *ad hoc* which constraints are conjoined with each other. The intuitive correlation of Harmonic Alignment and morphological marking is quite simple: Mark non-harmonic combinations! The OT formalization of this insight rests on the assumption that the constraints that are obtained from aligning markedness scales are conjoined with *∅. It would be equally possible though to conjoin them with *STRUC instead. To take an example, if we exchange *∅ and *STRUC in (9), we obtain the constraint hierarchy

(15) *STRUC & *Subj/3rd ≫ *STRUC & *Obj/local ≫ *∅ ≫ *STRUC & *Subj/local ≫ *STRUC & *Obj/3rd

This constraint hierarchy describes the mirror image of Dyirbal, i.e. a language where only 3rd person objects and local person subjects are case marked. Briefly put, this hypothetical Anti-Dyirbal has case marking only on non-harmonic NPs. To our knowledge, no such language exists. Even stronger, the markedness regularities that Harmonic Alignment attempts to formalize in fact exclude such a language.

What is really at stake here is the status of constraints in OT. We are sympathetic with the hypothesis of Haspelmath (1999) that

> "the grammatical constraints are not innate, and are not part of Universal Grammar. They arise from general constraints on language use, which for the most part are in no way specific to language." (Haspelmath 1999:204)

As we will argue in the remainder of the paper, the markedness facts addressed by Aissen lend themselves in fact fairly naturally to the kind of functional explanation envisaged by Haspelmath.

4 Two Experiments

A way of explaining why morphology will appear on disharmonic elements (like human pronoun objects or non-specific subjects) is functional. The morphology marks the element as a subject or object and this helps the recognition of the elements as subjects and objects. Without the morphology, there would be a bias to interpret the elements as harmonic, i.e. recognize the human pronoun as a subject or recognize the non-specific NP as an object.

The bias would derive from the distribution in normal use of language. If human pronouns are normally interpreted as subject, interpreting the human pronoun as a subject is better than interpreting it as an object. We can see this as a conflict between two defeasible constraints,[2] one, **Generation** enforcing faithful interpretation of the morphology (adding a marker for a semantic property not in the input is bad), the other, **Bias** preferring the normal reading, where normal is defined as the reading that is available in most of the cases. There are two options for the interpretation of the second constraint. We could think of it as a question of yes and no: an interpretation is either normal or not, or it could be a question of preferences: the normal interpretation is preferred to the degree to which it is normal.

In either case, we would get a preference for normal interpretations. This means that when a semantic input is realized by means of disharmonic elements its preferred interpretation will be different from the input and by the weakest interpretation of bidirectionality that the realization is not available unless another and stronger constraint overrides **Bias**. Bidirectionality minimally requires that a good realization for an input is one that will (preferentially) be interpreted as that input and that would be the problem of disharmonic elements: they are syntactically allowed but their surface characteristic prefer an interpretation as an harmonic element. The combination of **Generation** and the extra morphology overrides this preference and allows the interpretation as a disharmonic element. Besides we take it that morphological marking should only be used if required by these constraints. Let us tentatively stipulate a constraint **Economy** (roughly corresponding to Aissen's *STRUC), that is violated by morphological marking. For the purposes of this paper, we will assume just these three constraints, ordered in the way indicated in (16).

(16) **Generation** $\gg$ **Bias** $\gg$ **Economy**

This explanation only works if in fact there is a bias towards harmonic elements in the natural distributions in language use. In this section, we present two corpus investigations which confirm that hypothesis and a third rather speculative argument to show that that sort of distribution is to be expected on the basis of three universal tendencies.

The first corpus we looked at is a large annotated corpus, the Wall Street Journal corpus, consisting of text taken from the newspaper. Here we have about 250,000 NPs, divided by the annotators into subjects and non-subjects. There is a majority of non-subjects here since non-direct objects cannot be distinguished from direct objects. By looking at the head nouns of NPs, these can be divided into human and inanimate NPs (the Wall Street Journal does not discuss animals very frequently). We can also make an approximate division into pronouns, definite NPs, specific NPs and non-specific NPs by classifying formal characteristics like determiners, name or non-name. But this remains a bit of a black art of dubious reliability: bare NPs can be non-specific and specific indefinites as well as definite NPs (names of kinds or persons), and a proper classification would be very costly. There are only a couple of thousand pronouns. Another question that can well be raised about this corpus is its representativity for natural language use: it is monologue and the topic seems to be almost exclusively the state of the economy.

What we expect to find is that disharmonic combinations have a lower frequency than would be expected, i.e. than the frequency of the either element in the combination. For example, we expect

$$p(OBJ|HUM) < p(OBJ|NP)$$

and

$$p(HUM|OBJ) < p(HUM|NP)$$

[2]The idea of doing this in this particular way is due to Jason Mattausch

And this is borne out. $p(OBJ|HUM) = 42\%$ and $p(OBJ|NP) = 75\%$, while $p(HUM|OBJ) = 10\%$ and $p(HUM|NP) = 13\%$.

But since we are in the business of interpretation, we want to know whether we can predict the abstract category (the syntactic function) from the surface property (a feature like HUM is given with the recognition of the NP). And we can derive from the above that assuming that a human NP is a subject pays off: the probability that the human NP is is a subject is 58%.

82% of the pronouns are subjects while there is only 25% probability of a being a subject in the corpus.

Definites (without the pronouns) slightly increase the probability of being an object (88% vs. 75%), while the other NP objects have exactly the same frequency as the objects (75%).

Indefinites slightly raise the probability of the NP being an object: it increases from 75% to 90%.

We find here strong evidence for two rules: assume that pronouns are are subjects and assume that humans are subjects, especially if we make the assumption that the probabilities for being a subject and a non-subject should be corrected to 50-50 (a high frequency of non-direct objects comes from long sentences which are not expected in the natural spoken language environment). On the object side we find a tendency that indefinites are objects and the reflexes of the two rules that bias towards a assuming a subject. But pronouns are low-frequent in the corpus (5%) as are human NP (13%) which makes it hard to see effects from lexicality (non-pronouns) or inanimacy.

The following table gives the relevant results.

$p(subj\|np)$	=	25%
$p(obj\|np)$	=	75%
$\mathbf{p(subj\|X)}$	$\gg$	$\mathbf{p(subj\|np)}$
$p(subj\|pro)$	=	88%
$p(subj\|hum)$	=	58%
$\mathbf{p(obj\|X)}$	$\gg$	$\mathbf{p(obj\|np)}$
$p(obj\| - def)$	=	87%
$p(obj\|inan)$	=	90%

No effects for $p(obj|indef) = 75\%$.

In summary, we find strong effects in the subjects but less clear effects in the object. This may well be due to the relative scarcity of pronouns and human NPs in the corpus. Failure to find an effect for indefinites may be due to the difficulties of finding a good heuristics for that class. But we get confirmation of our expectation: that harmony in NPs is connected with frequency: harmony boosts frequency.

Our second experiment used a much more suitable corpus, *Samtal i Goeteborg* (conversations in Gothenburg) which is a collection of taped and transcribed conversations obtained by asking Gothenburgians to record some everyday conversation they were engaged in. Oesten Dahl used the corpus to obtain the data for his Dahl (2000) and in the course of that entered about 10% of the utterances into a database with annotations that were perfectly suited for our task.

The main difference with the WSJ corpus is that pronouns are highly frequent (72%) and that human NPs abound (54%). Another difference is the much smaller number of NPs (13692) and having only direct objects, so that now the subjects are in the majority.

We get the following data. I use *ego* for the egocentric pronouns *I, you, we* and their alternants, *3pro* for the other pronouns, *-def* for the non-definite NPs, *lexdef* for the non-pronominal NPs that are definite.

$p(subj\|NP)$	$=$	77%
$p(obj\|NP)$	$=$	23%
$\mathbf{p(A\|B)}$	$\gg$	$\mathbf{p(A\|NP)}$
$p(subj\|hum)$	$=$	97%
$p(subj\|ego)$	$=$	97%
$p(obj\|-def)$	$=$	87%
$p(obj\|lexdef)$	$=$	32%
$p(obj\|inan)$	$=$	46%
$\mathbf{p(A\|B)}$	$\approx$	$\mathbf{p(A\|NP)}$
$p(obj\|def)$	$=$	15%
$p(obj\|3pro)$	$=$	17%

The Aissen lattice is completely reconstructed by probabilities with which subjecthood is predicted from the category. We obtain the following linear order from those probabilities (we order by the value of p(subj—X)).

(17) *human pronoun > inanimate pronoun > human lexical definite > inanimate lexical definite > human non-definite > inanimate non-definite.*

This is a linearization of Aissen's partial order and fully consistent with *human > inanimate and pronoun > definite > non-definite.*

It works out less neatly in the object ($p(obj|X)$). We get the ordering

(18) *inanimate pronoun > inanimate non-definite > inanimate lexical definite > human non-definite > human lexical definite > human pronoun*

This is consistent with *human > inanimate*, but does not respect *pronoun > non-definite* or *pronoun > lexical definite* on the inanimates. (It does on the human NPs). Whether we should be worried about this is debatable.

The data this time give robust effects both in the subject and the object though still stronger effects in the subject. The harmonic NPs are much more frequent than clearly disharmonic ones. And the corpus is clearly a natural one: it is the sort of language use that we engage in on a daily basis and that forms the basis for language learning. Is this enough to conclude that the frequencies are the same all over the world? We can adduce rather similar results (in another language, in another genre) of our previous experiment. Preliminary investigation of the SUSANNE corpus (a syntactically annotated collection of written English from different genres, cf. Sampson 1995), and the CHRISTINE corpus (transcriptions of spontaneous English dialogues which are annotated according to the SUSANNE scheme) show similar patterns, with CHRISTINE rather close to Samtal (with the exception of the indefinites which in Samtal have a much stronger preference for being objects) and SUSANNE like the WSJ corpus in having only minor effects in the object.

But as a case for universality it does not really add up to very much. There is however a way we can explain the data which does not seem to appeal to the peculiarities of Swedish or English.

It is generally accepted that subjects are the most agentive syntactic function. And the proto-agent properties of Dowty (1991) all have the tendency to make the referent more and more human. There is therefore a universal explanation of the fact that subjects tend to be human and humans tend to be subjects.

Similarly, objects tend to be foci or comments. This in turn makes it likely that new (both in the sense of material new to the context or material that is not in the current discourse topic). And these things are realized by lexical NPs and if they are fully new, by indefinite lexical NPs. How strong are these effects? Well, it seems we can assume the frequencies we found can be taken as representative. And can we then predict the other frequencies? Well, if we make some particular but not unreasonable assumptions.

The following picture is made by BayesBuilder a free software package that allows the modeling of dependencies.[3] It pictures statistical dependencies between our parameters and causal assumptions about how they influence each other. The bottom node (subject) is the part where the predictions are made: the NP is a subject or not with a certain probability. The picture shows the situation when an arbitrary NP is made: it has 28% probability of being egocentric, 16 percent of being indefinite, 18% of being definite, 38% of being a third person pronoun. Then there are dependencies between being lexical and human again determined by the corpus. The factors that push up the probability of being an object (indefiniteness and lexicality) are summated in the object box. The subject probabilities are computed from the object box and the human box: if they conflict they are put at 50-50, otherwise (for humans) at 97% for being subject, (for objects) at 100% for not being a subject and for the case when no factor applies at the overall probability for subjects (74%). BayesBuilder allows one to make more specific assumptions (e.g. lexical human) and then computes the resulting probability for the item being a subject. If one does that, the network gives the values we measured in the corpus within a couple of percentage points.

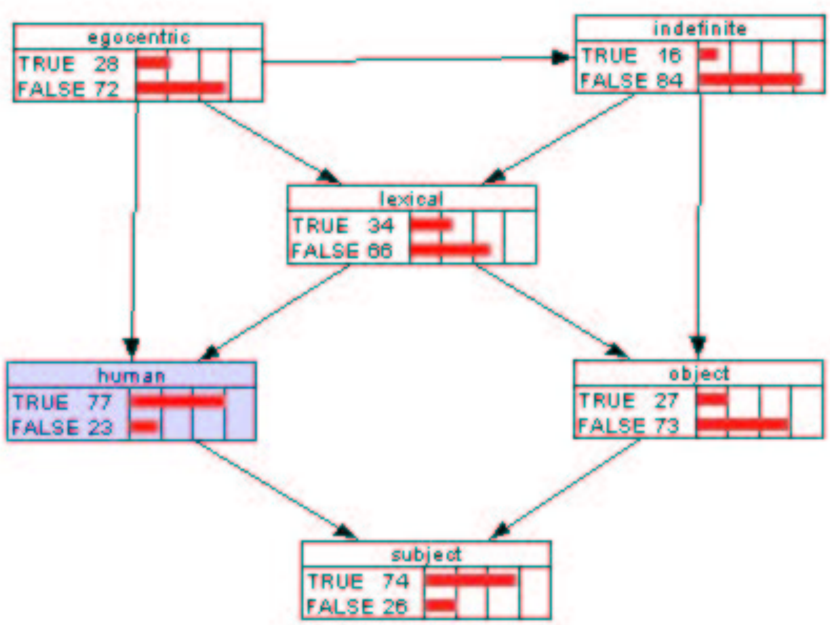

Figure 2: Bayesian Network

The assumptions and dependencies that we assume in the model are not unreason-

[3]Developed at Nijmegen University: http://www.mbfys.kun.nl/snn/Research/bayesbuilder/

able, but remain to some extent arbitrary. We show that a causal model is possible, based on the three factors indicated, not that we have found the true explanation.

If our basic line of thinking is right though, we show that the frequencies depend on other and deeper alignments: the alignment between subject and agentivity and the alignment of object with focus and comment. The frequencies are a surface effect of those alignments.

5 Theoretical repercussions

The results and considerations from the last section suggest that Aissen's syntactic alignment patterns can completely be explained in a functional way. Syntactic harmony basically means that a linguistic item conforms to the expectations that can be derived from the statistical patterns of language use. To take an example, it is a good heuristic to assume that a pronoun is a subject. Subject pronouns are in line with the corresponding expectation. Unmarked object pronouns would constantly risk to be misinterpreted as subjects. It is thus a good idea for a speaker to mark a pronoun with object case if it is supposed to be an object. As indicated above, we can formalize this intuition by assuming two constraints, **Bias** and **Generation**. **Bias** is fulfilled if an NP has the grammatical function that its semantic characteristics indicate—it is fulfilled by pronominal, definite, human etc. subjects and by indefinite, lexical, non-animated etc. objects, and violated by indefinite subjects, first person objects and the like. This is only one of its functions: it basically makes sure everything means what it normally means. **Generation** favors an interpretation that is faithful to the morphological case of the NP—accusative NPs are object, absolutive NPs are subjects etc. In general, it makes sure that all the syntactic rules are followed in generating the sentence. Finally, by a simple economy consideration, morphological marking should only be used when necessary—this is Aissen's constraint ***STRUC**, which we rename **Economy**.

The universal ranking of these constraints is

$$\textbf{Generation} \gg \textbf{Bias} \gg \textbf{Economy}$$

This has to be paired with a particular version of bidirectional evaluation. Let us say that a input-output pair is *optimal* iff 1. the output is optimal for the constraints encapsulated in **Generation** 2. there is no alternative input that is preferred by **Bias** and 3. there is no output that is equally good for the same input but which is shorter. (Note that this way of optimization is non-standard; OT usually takes solely the speaker perspective, but we assume that the speaker has to take in the hearer's point of view in order to make sure she will be understood. This makes our notion a special case of bidirectionality in the sense of Blutner 2001, though a different one from the two notions considered there.)

Now suppose we are in a language that has an accusative morpheme at its disposal. Suppose you are the hearer and you have to interpret a pronoun. There are two scenarios: the pronoun may be unmarked or carry accusative case. The unmarked pronoun is preferably interpreted as subject and the marked one as object.

(19)

			Generation	**Bias**	**Economy**
Pron+∅	☞	Subj			
		Obj		*!	
Pron+ACC		Subj	*!		*
	☞	Obj		*	*

In both cases, from a pure speaker's perspective, there is a preference for the unmarked case. But bringing in the hearer in the **Bias** constraint brings a preference for the marked accusative pronoun.

While this account is tempting and, we feel, in a sense correct, it cannot be adopted in an unqualified way. It fails in two respects. First, optimization is not restricted to single NPs. We always have to consider at least an entire clause, and there other devices (like word order) may suffice to disambiguate, if they are not occupied for other expressive tasks. Even if this may seem to undermine our case for a language like English, it does not really undermine the functional account that we are advocating because the robust parsing required for speech recognition is impossible without redundancies and has to cope with sentence fragments. But it is an argument against the OT formalization just sketched.

Second, and more severely, the above account by itself has nothing to say about the typological pattern of DOM and DSM ("Differential Subject Marking") that Aissen discusses. It predicts that DOM applies to all non-harmonic combinations. There is no space for cross-linguistic parameterization like the fact that Hebrew marks all definite objects and Turkish all specifics. Assuming separate constraints for each alignment cell like Aissen seems to be in fact inevitable.

Nonetheless the discussion above gives a clue for a Haspelmath style functional motivation of the constraints involved. Starting from the proposal above, we suggest to split up **Bias** into separate interpretative constraints like "Pronouns are subjects", "Definites are subjects", "Indefinites are objects" etc. These constraints still express parsing heuristics that are founded in the statistical patterns of language use. The problem is however that we have nothing to say about those languages where the marking has to occur on perfectly harmonic elements, such as non-specific object NPs in accusative languages or subject pronouns as in pure ergative languages.

The next modification concerns the optimization algorithm as such. Above we used a version of bidirectional OT in which we derive why in certain circumstances subject and object markers appear on subjects and objects. This in essence gives us an explanation of optional subject and object marking. The constraint **Bias** is not just taking in regularities about the kinds of NPs that are subject and object, but also other kind of preferences. E.g. an inanimate and a human NP as arguments of the verb "to please" will almost certainly have the inanimate NP as the subject and the human NP as the object and would not have to be marked since the two effects of **Bias** obliterate each other.

Now it is rather clear that while optional case marking for NPs with a fixed set of features exists these are far less frequent than obligatory systems: an object must be case marked if it has certain features. Most case marking is obligatory. This can only be explained as a grammaticalization process: an optional marking possibility becomes required by the grammar.

Bias itself offers a way in which this process can be explained. Let us assume that the language marks 50% of its human object NPs counterbalancing frequency statistics which without the marking would make the human NP a subject with a probability of 60%. But the marking has an effect on these statistics: unmarked NPs are now subjects with not 60% probability but with 75% and marking becomes more necessary as a result, thus further reducing the probability that an unmarked NP is an object. Once an optional marking strategy becomes non-exceptional and if it is functional, the marking makes itself more necessary and will normally become obligatory. It is then for the language learner at some point not distinguishable from a generation rule that requires marking certain combinations of features. As its original functional motivation and the process of self-reinforcement are not transparent to new language learners, learning it as a generation rule becomes the only option for new language learners.

This explains why we normally find —next to bias-driven optional systems— obligatory rules of case marking. We predict —but have not investigated this empirically— that optional case marking only occurs where the frequency of the case-marked NPs is low and there is no functional pressure.

Another prediction of our functional explanation is that one finds marking in the disharmonic cases only. A language that marks low prominent subjects and high prominent objects seems to be the best way out of the predicament caused by **Bias**.

But this is only so if the language has both subject markers -ERG and object markers -ACC. Assume an input meaning "the apple hits John". In the following tableau we rank the relevant possibilities.

(20)

		GENERATION	**BIAS**	**ECONOMY**
☞	apple John hit: hit(j,a)			
	apple John hit: hit(a,j)		*	
	apple-ERG John hit: hit(j,a)	*		
☞	apple-ERG John hit: hit(a,j)			
	apple John-ACC hit: hit(j,a)	*		
☞	apple John-ACC hit: hit(a,j)			
	apple-ERG John-ACC hit: hit(j,a)	*		*
	apple-ERG John-ACC hit: hit(a,j)			*

This is the optional system: accusative marking on John, or ergative marking on apple are the preferred options. (This changes when the marking is grammaticalized. In that case only the option from the last line of the tableau is open. The others do not pass **Generation**.)

In the optional system with both subject and object markers, we can see that all possibilities can be dealt with. High prominent subject and high prominent object is disambiguated by case on the object. Low prominent subject and low prominent object receives case on the subject. And high prominent subject and low prominent object does not need case-marking. Split ergative languages have grammaticalized the situation described here.

But if there is no subject marker, this changes. If we want to distinguish two low prominent arguments, we can only do this by case marking the object, even though low prominent objects are harmonic. Accusative languages are grammaticalizations of this situation.

If there is no object marker, the language has to distinguish two high-prominent arguments and grammaticalization of this results in a pure ergative language.

Now there is no need to put the events leading to this at the beginning of time. Case morphemes phonetically erode. We predict that if a subject marker disappears object marking will be extended and vice versa. The Spanish object marking of human NPs offers an example of how a preposition is reemployed as an object marker. (The lack of functional pressure that keeps it restricted to that class is due to subject marking by agreement and additional object marking by clitic doubling.)

6 Change

Very schematically, we can distinguish languages according to their behavior with subject and object marking. There are languages which mark all subjects ("Coherent Subject Marking": CSM), some subjects (DSM), all objects (COM) and some objects (DOM). And these can be combined. In addition a language can mark nothing. Assuming no other marking strategies, the language then is more or less able to distinguish subjects and objects, when these have high or low prominence. The following table shows the possibilities for the eight types to make clear which are the subjects and the objects given **Bias**.

(21)

subj-obj	high-high	high-low	low-high	low-low
∅	−	+	−	−
DOM	+	+	+	−
DSM	−	+	+	+
DOM+DSM	+	+	+	+
COM	+	+	+	+
CSM	+	+	+	+
COM+CSM	+	+	+	+

The empty system and the only DOM and only DSM systems are defective: they not mark certain situations. Other dimensions can be used (passivization or word order) but that also means that these are no longer free for other purposes like topic marking. Leaving aside those possibilities, such systems will be likely to develop marking.

Systems like CSM+COM overdo their marking on the other hand and can be said to be uneconomical: they force more marking effort on their users than is required by the functionality of the marking. The only three systems that have a good balance between the functional needs and economy are DOM+DSM, CSM and COM. But even such stable systems may change under the influence of phonetic erosion.

Five types of linguistic change are important in this perspective. The first is the process of *annexation*. If a lexical device can sometimes make a distinction which is useful for the interpretation of the utterance, a usage of that item may arise in which its purpose is to mark that distinction and not to convey its lexical meaning. Examples of such annexation are the use of *already* in Singapore English as a marker of perfectivity (in which its meaning "earlier than expected" is obliterated) or the use of the Latin preposition *ad* to mark objects in Spanish (obliterating its locative meaning).

There is something marked about this use of the marker: the lexical meaning of the marker is still available and this is rightly seen as analogous to epenthesis: the addition of extra phonetic material to the sentence. Semantic epenthesis can be formulated as follows. Do not put material in the sentence which has a meaning that is not in the input.

And epenthesis only occurs for a reason. In this case, we can identify that reason with our constraint **Bias**: non-use of the marker would mean that the default reading (subject or in the case of Singapore English Fong (2001) the non-perfective or futurate reading) will be generated.

Annexation is a process that turns a lexical item into an optional grammatical marker.

Optional marking reinforces **Bias**: the marked occurrences do not count and need to be subtracted. To use the example sketched above again, if 60% of the unmarked human NPs are subject, then 40 percent are objects. But if object human NPs are marked in 50% of the cases that means that unmarked human NPs are subjects in 75% of the cases and that the bias against object interpretation grows. This leads to further marking and even stronger bias against unmarked object human NPs. Unless the marking is exceptional or there is a choice between different markers, optional marking is not very stable.

Necessary marking can be seen as the extreme case where there is no possibility of non-biased interpretation. This is proper *grammaticalization*: the rule to use the marker when some condition applies becomes part of **Generation**. The complex semantics of these conditions is no doubt due to the categorization process that has to be built into the generation process itself.

Spread is the extension of necessary marking to a weaker condition. It can again be seen as a result of changing biases due to grammaticalization of markers or the annexation of a marker: unmarked NPs become more probable subjects the more often objects are marked, unless there are opposite biases or factors that guarantee that the proper readings are reached all the same. Spread needs optional marking and optional marking needs **Bias**-related misinterpretations.

Reinterpretation is a process by which optional marking becomes a method for expressing the trigger of marking rather than the case. If you have to be prominent to be marked for object, the speaker can make you prominent by object marking you. Object-marking is an economy transgression unless there is **Bias** against the intended interpretation. But violating economy is a good way of drawing attention to the referent of the expression that violates economy. That in turn can be used for different reasons, so that assigning prominence can be politeness, expression of respect, elevation of status for literary purposes, but can also become a conventional indication of semantic properties like specificity. (Again grammaticalization: **Bias** rigidly assigns the specificity to the object marked NP, the absence of the marker starts meaning non-specificity, the appearance of the object marker may become obligatory.)

Phonetic and semantic erosion is the last relevant changing process, but it is due to other factors. Grammatical markers can lose stress and unstressed elements can lose their phonetic profile to the point of obliteration. Grammaticalized markers can become more ambiguous and vague under spread. While we have discussed a number of stable situations, stability can be threatened by these phonetic and semantic obliteration processes. CSM may entirely disappear forcing new annexations or the spread of DSM, DOM may disappear forcing DSM to become CSM or new annexations to occur. The history of languages is cyclic.

While the Aissen system offers a good and concise way of describing the different grammatical systems with the necessary finegrained-ness, a proper explanation of DSM and DOM must take **Bias** and history into account.

7 Conclusion

The above is fairly speculative, but offers the beginning of an account of the typological observations of Aissen. We start out with a functional reinterpretation of subject and object marking and then explain how the grammars of particular languages come to contain certain marking rules, with application conditions as predicted by Aissen. The grammaticalization process has to live on an Aissen application condition (top or bottom of the prominence partial order), since such application conditions define areas where the frequency of subjects or objects is higher than in its complement.

We will follow up this work with a more thorough investigation of the historical processes that we assumed in the last section and try to evaluate them on real typological data. It is clear that much more needs to be done than we have been able to do here.

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