Logic4P: Spatial Logic for Polyhedra[1]

Mieke Massink (CNR-ISTI, Pisa, IT)

joint work with:

Nick Bezhanishvili (ILC, Univ. of Amsterdam, NL)
Vincenzo Ciancia (CNR-ISTI, Pisa, IT)
David Gabelaia (TSU Razmadze Math. Inst., Tbilisi, Georgia)
Gianluca Grilletti (Munich Center for Math. Phil., LMU München, DE)
Diego Latella (CNR-ISTI, Pisa, IT)

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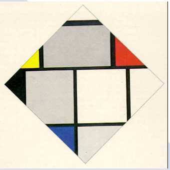
Reasoning about Space (and Time)

Reasoning about space is a very active topic of research in many areas of science

Physical Sciences

Ordinary/
Partial
Differential
Equations

Pure Mathematics

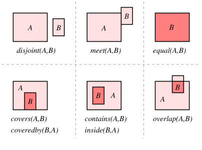


Topology
Modal Logics

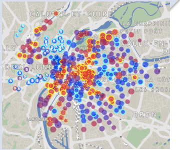
Decidability
Satisfiability

SPACE

Region
Connection
Calculus



Artificial Intelligence



Model checking

Collective Adaptive Systems

Origins in Logic: Spatial Interpretation of Modal Logic [McKinsey & Tarski,1944]

 Alfred Tarski John C. C. McKinsey 

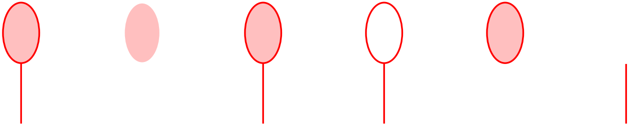
$$\Phi ::= p \mid \top \mid \bot \mid \neg\Phi \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \Box\Phi \mid \Diamond\Phi$$

A topological space (X, O)

- X a set of points
- O the set of open sets of X
- $\emptyset, X \in O$; O closed under arb. unions and fin. intersections

A *model* $\mathcal{M} = ((X, O), \mathcal{V})$

- (X, O) a topological space
- $\mathcal{V} : AP \to \mathcal{P}(X)$ a valuation function



p $\Box p$ $\Diamond p$ $\neg\Box p \wedge \Diamond p$ $\Diamond\Box p$ $p \wedge \neg\Diamond\Box p$

[Handbook of Spatial Logics, Aiello, Pratt-Hartmann and van Benthem (Eds.), Springer, 2007]

Spatial Logic for Closure Spaces (SLCS)[2]

A *closure space* is a pair $(X, \mathcal{C})$ with $\mathcal{C} : 2^X \to 2^X$ such that for each $A, B \subseteq X$:

- $\mathcal{C}(\emptyset) = \emptyset$
- $\mathcal{C}(A \cup B) = \mathcal{C}(A) \cup \mathcal{C}(B)$
- $A \subseteq \mathcal{C}(A)$
- ~~$\mathcal{C}(\mathcal{C}(A)) = \mathcal{C}(A)$~~

- $\mathcal{I}(A) = \overline{\mathcal{C}(\overline{A})}$
- A is *open* iff $A = \mathcal{I}(A)$
- A is *closed* iff $A = \mathcal{C}(A)$
- A is a *neighbourhood* of $x \in X$ iff $x \in \mathcal{I}(A)$

Spatial Logics for Closure Spaces (SLCS):

$$\phi \ ::= \ \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \rho\ \psi\ [\phi]$$

where $p \in AP$ is an atomic proposition and spatial reachability operator:

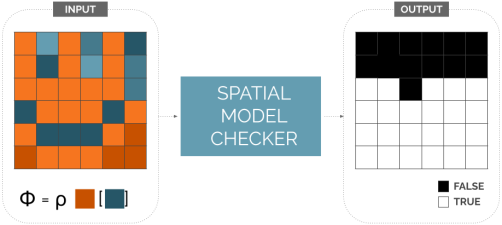
$$\mathcal{X}, x \models \rho\ \psi\ [\phi] \text{ iff there is path } \pi \text{, index } \ell \text{ s.t.}$$
$$\pi(0) = x \text{ and } \mathcal{X}, \pi(\ell) \models \psi \text{ and for all } j \text{ s.t. } 0 < j < \ell : \mathcal{X}, \pi(j) \models \phi$$

$\rho\ \psi\ [\phi]$ ("reach ψ through ϕ") is satisfied by a point if there is a path rooted in that point, leading to a point satisfying ψ and whose intermediate points all satisfy ϕ.

Graphs are (quasi discrete) closure spaces, *images* can be seen as *regular graphs*

[2] *[Ciancia,Latella,Loreti,Massink,LMCS16]*

Spatial Model Checking



Pixels are nodes of a regular graph $\mathcal{X} = \langle (X, \mathcal{C}), \mathcal{V} \rangle$ closure model

The 8-adjacency relation between pixels forms the edges of the graph

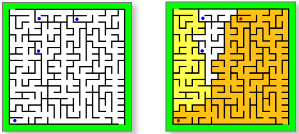
Formula Φ is an SLCS formula

White pixels satisfy Φ

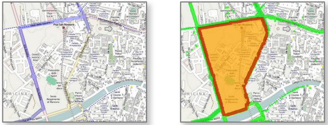
Black pixels do *not* satisfy Φ

The global model checker VoxLogicA provides an efficient implementation.

Applications with the spatial model checker VoxLogicA[3]

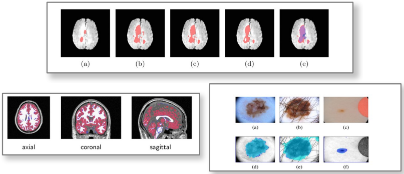


Reachability



Map annotation

GPS traces

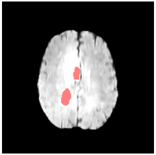


Medical Imaging

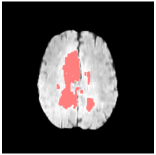
VoxLogicA is available at: `https://github.com/vincenzoml/VoxLogicA` or contact authors.

3 *[Belmonte,Ciancia,Latella,Massink,TACAS19] [Ciancia, Gilmore, Grilletti, Latella, Loreti, Massink, STTT 2018] [Banci Buonamici,Belmonte,Ciancia,Latella,Massink, STTT 2019 and ESMRBM19]*

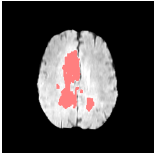
3D Magnetic Resonance Tumour Segmentation[4,5]



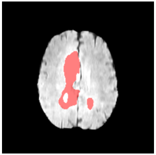
hyper intense
(hI)



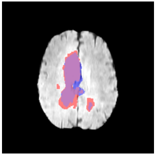
very intense
(vI)



grow(hI,vI)
(c)



similar texture
(d)



gtv=grow(c,d)
manual (blue)

```
let background = touch(intensity <.  0.1,border)
let brain = !background

let pflair = percentiles(intensity,brain)
let hI = pflair >.  0.95
let vI = pflair >.  0.86
let hyperIntense = flt(5.0,hI)
let veryIntense = flt(2.0,vI)

let growTum = grow(hyperIntense,veryIntense)
let tumSim = similarTo(growTum)
let tumStatCC = flt(2.0,tumSim >.  0.6)
let gtv = grow(growTum,tumStatCC)
```

background removal

threshholding

region growing and
texture similarity

Brain Tumor Segmentation Benchmark
(BraTS 2017) 213 cases

18 techniques on at least 100 cases
Similarity score Dice: avg. 0.88 (0.64-0.96)

Our score on 193 cases: avg. 0.85 (std. 0.10)

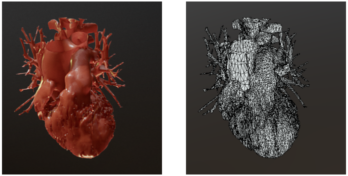
About 10 seconds on Intel Core I7 7700 (8 cores)
$\sim$ 9 million voxels

In line with state-of-the-art!

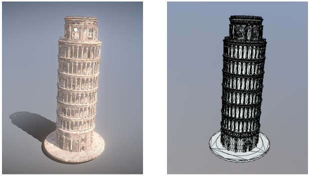
4 *[Belmonte,Ciancia,Latella,Massink,TACAS19]*

5 *[Banci Buonamici,Belmonte,Ciancia,Latella,Massink, STTT 2019 and ESMRBM19]*
Image: Brats17_2013_2_1 from BraTS 2017 database

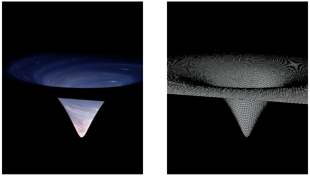
SLCS: Interpretation on Polyhedra



Human heart



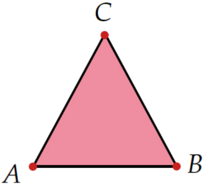
Tower of Pisa



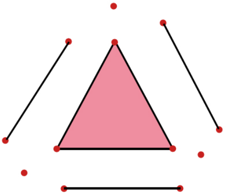
Black hole

Many images rendered as 3D triangular meshes
(screenshots from SketchFab)

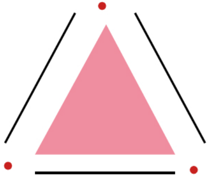
A simplicial complex and its (simplex) faces and cells:



Simplicial complex



Simplex (σ) faces



Cells ($\widetilde{\sigma}$)

Relation on cells of simplicial complex K:

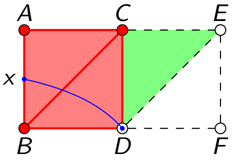
$$\widetilde{\preceq} \subseteq \widetilde{K} \times \widetilde{K} \text{ with } \widetilde{\sigma}_1 \,\widetilde{\preceq}\, \widetilde{\sigma}_2 \text{ iff } \sigma_1 \preceq \sigma_2 \text{ where } \preceq \text{ is the face relation}$$

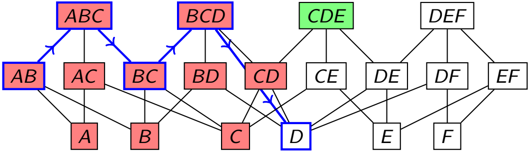
[6] *[Bezhanishvili,Ciancia,Gabelaia,Grilletti,Latella,Massink,CoRR abs/2105.06194, 2021]*

SLCS: Interpretation on Polyhedra[7]

Polyhedral model $\mathcal{X} = \langle P, \mathrm{K}, V\rangle$ with:
P a polyhedron and K a simplicial complex with $P = |\mathrm{K}|$;
$V : AP \to \mathcal{P}(P)$ a valuation s.t. $V(p)$ is a union of cells $\widetilde{\sigma}$ of $\widetilde{K}$ ....

... and its related Kripke model $\mathcal{M}(\mathcal{X})$ with face relation:





$\mathcal{X}, x \models \Box\phi$ $\quad$ $\mathcal{M}(\mathcal{X}), \widetilde{\sigma} \models \Box\phi$

$\mathcal{X}, x \models \gamma(\phi, \psi)$ $\quad$ $\mathcal{M}(\mathcal{X}), \widetilde{\sigma} \models \gamma(\phi, \psi)$

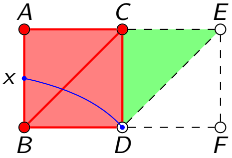
Let $\sigma \in \mathrm{K}$ be the unique simplex such that $x \in \widetilde{\sigma}$. For every formula ϕ of SLCS we have

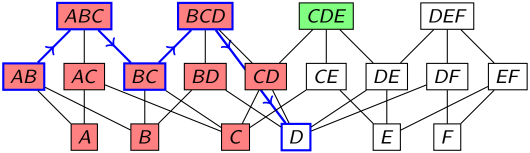
$$\mathcal{X}, x \models \phi \iff \mathcal{M}(\mathcal{X}), \widetilde{\sigma} \models \phi.$$

[7] [Bezhanishvili,Ciancia,Gabelaia,Grilletti,Latella,Massink,CoRR abs/2105.06194, 2021]

SLCS: Interpretation on Polyhedra[8]

Polyhedral model $\mathcal{X} = \langle P, \mathrm{K}, V \rangle$ with $P = |K|$ and its related Kripke model $\mathcal{M}(\mathcal{X})$:





$$\mathcal{X}, x \models \Box\phi \iff x \in \mathcal{I}_P(\llbracket \phi \rrbracket^{\mathcal{X}})$$

$$\mathcal{X}, x \models \gamma(\phi, \psi) \iff \text{exists a path } \pi \text{ such that } \pi(0) = x,\ \pi(1) \in \llbracket \psi \rrbracket^{\mathcal{X}} \text{ and } \pi((0,1)) \subseteq \llbracket \phi \rrbracket^{\mathcal{X}}$$

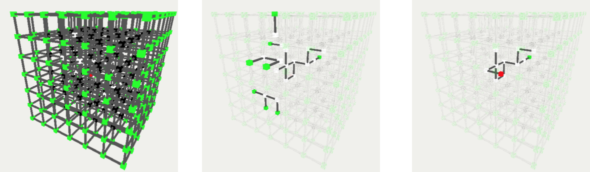
$$\mathcal{M}(\mathcal{X}), \widetilde{\sigma} \models \Box\phi \iff \forall \widetilde{\tau} \in \widetilde{\mathrm{K}}.\ \text{if } \widetilde{\sigma} \,\widetilde{\preceq}\, \widetilde{\tau} \text{ then } \mathcal{M}(\mathcal{X}), \widetilde{\tau} \models \phi$$

$$\mathcal{M}(\mathcal{X}), \widetilde{\sigma} \models \gamma(\phi, \psi) \iff \text{exists a } \pm\text{-path } \pi : \{0, \ldots, k\} \xrightarrow{\pm} \widetilde{\mathrm{K}} \text{ such that } \pi(0) = \widetilde{\sigma},\ \pi(k) \in \llbracket \psi \rrbracket^{\mathcal{M}} \text{ and } \pi(\{1, \ldots, k-1\}) \subseteq \llbracket \phi \rrbracket^{\mathcal{M}}$$

[8] *[Bezhanishvili, Ciancia, Gabelaia, Grilletti, Latella, Massink, CoRR abs/2105.06194, 2021]*

Tools and First Examples: PolyLogicA and PolyVisualizer

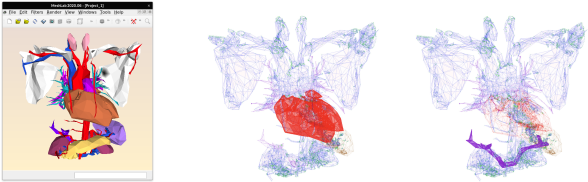
A 3D maze:



Size: 150K cells
Model checking time: 300ms
Parsing time: 4s
Building Kripke: 1s

```
whiteToGreen = through((white |
corridorWW | corridorWG), green)
```

Digital anatomy:



Size: 1.5M cells
Model checking time: 5s
Parsing time: 44s
Building Kripke: 32s

```
selectedVein = vein &
(!through(vein,heart))
```

Tools available at: `https://github.com/vincenzoml/VoxLogicA` or contact authors.

"There is nothing more practical than a good theory"
Kurt Lewin (1952)

Thanks! Questions?

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