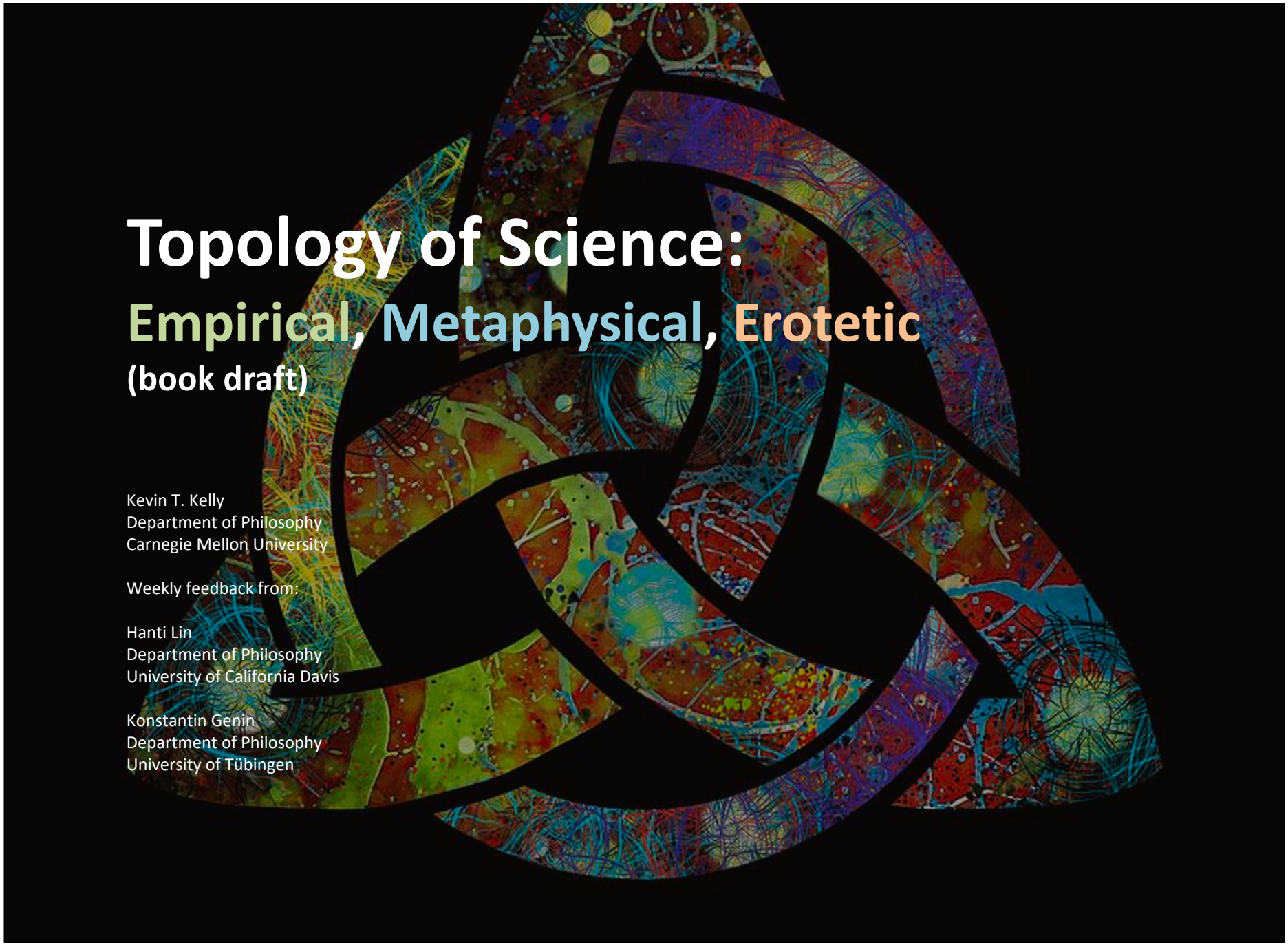

Topology of Science:
Empirical, Metaphysical, Erotetic
(book draft)

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Philosophy of Science

Relevance

- What matters
- Type of inference problem
- Lower bound on content

Philosophy of Science

Empirical information

- Verification
- Refutation
- Underdetermination
- Learnability
- Simplicity and Ockham's razor

Relevance

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Philosophy of Science

Reality

- Natural science vs. Data-science
- Miracles and Luck
- Negligibility

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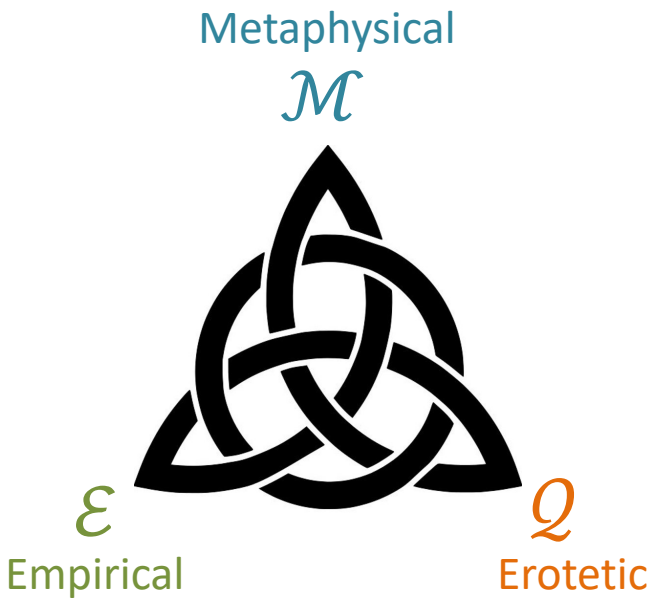
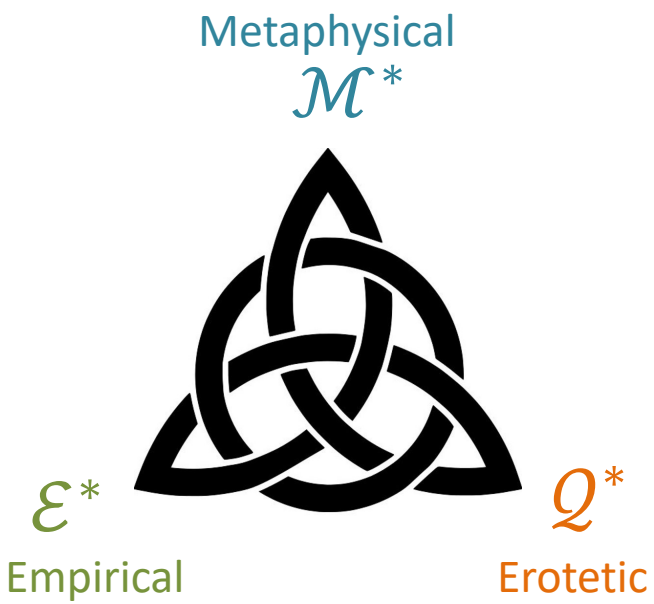
Three Topological Bases



Three Topologies



Three Topologies

Topology of
real similarity

$\mathcal{M}^*$

$\mathcal{E}^*$

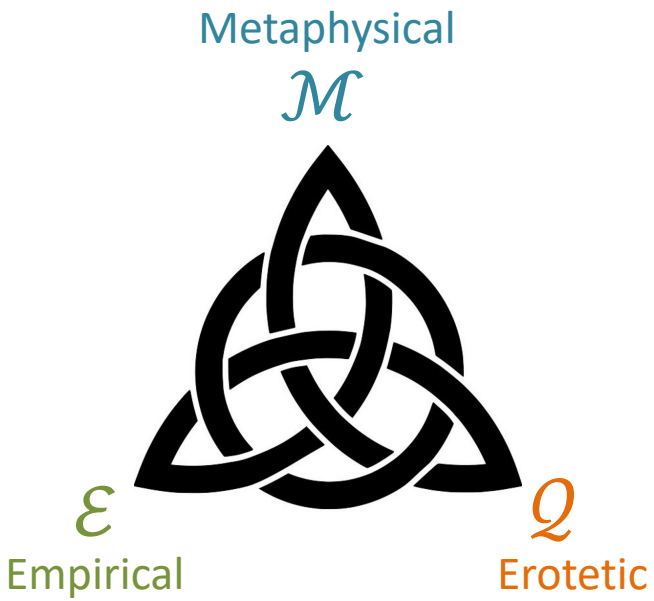
Topology of
empirical similarity

$\mathcal{Q}^*$

Topology of
relevant similarity

Context of Inquiry

$$\mathfrak{C} = (W, \mathcal{M}, \mathcal{E}, \mathcal{Q})$$



1. EROTETIC BASIS

Erotetic Basis

- Elements of $\mathcal{Q}$ are answers to a question.
- Answers should be concluded eventually, if true.

$\mathcal{Q}$ is a countable topological basis.

1. At worst, vacuous information W is requested.
2. Requests accumulate.
3. Requests are expressible.

Erotetic Basis

Allows for overlapping answers.

1. One-sided questions:

   verification $= \{A, W\}$;

   refutation $= \{\neg A, W\}$,

   decision $= \{A, \neg A\}$.

2. Replace hopeless catch-all hypothesis with W.
3. Quantitative (estimation) questions have open intervals as answers.
4. Science writ large: answers accumulate across disciplines.

Erotetic Operators

Inquiry	Erotetic Topology
"You should conclude A"	int A
"You should deny A"	ext A
"You needn't deny A"	cl A
"You needn't decide A"	bdry A
"You needn't conclude A, even though it's true"	frnt $\neg A$

Erotetic Properties

Inquiry	Erotetic Topology
"A is positively relevant"	A is open
"A is negatively relevant"	A is closed
"A is relevant"	A is clopen

2. EMPIRICAL BASIS

Empirical Basis

- Elements of $\mathcal{E}$ are empirical information states.

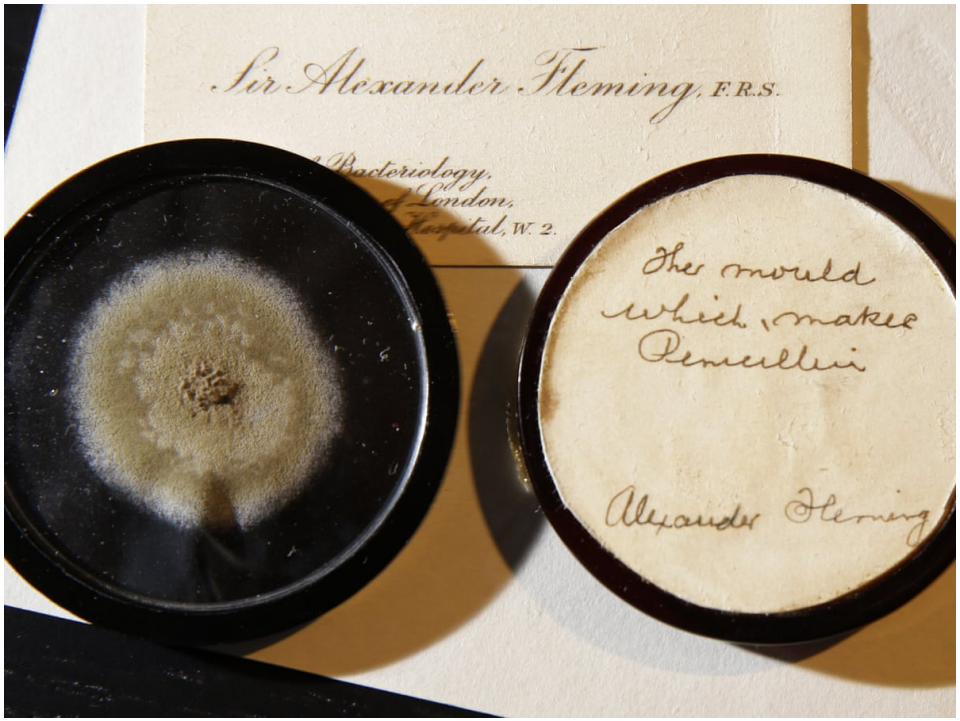
$\mathcal{E}$ is a countable topological basis.

1. At worst, vacuous information W is available.
2. Available information accumulates.
3. Scientific information is recordable.

$\mathcal{E}_w$ = the set of all empirical information states true/possible in w.

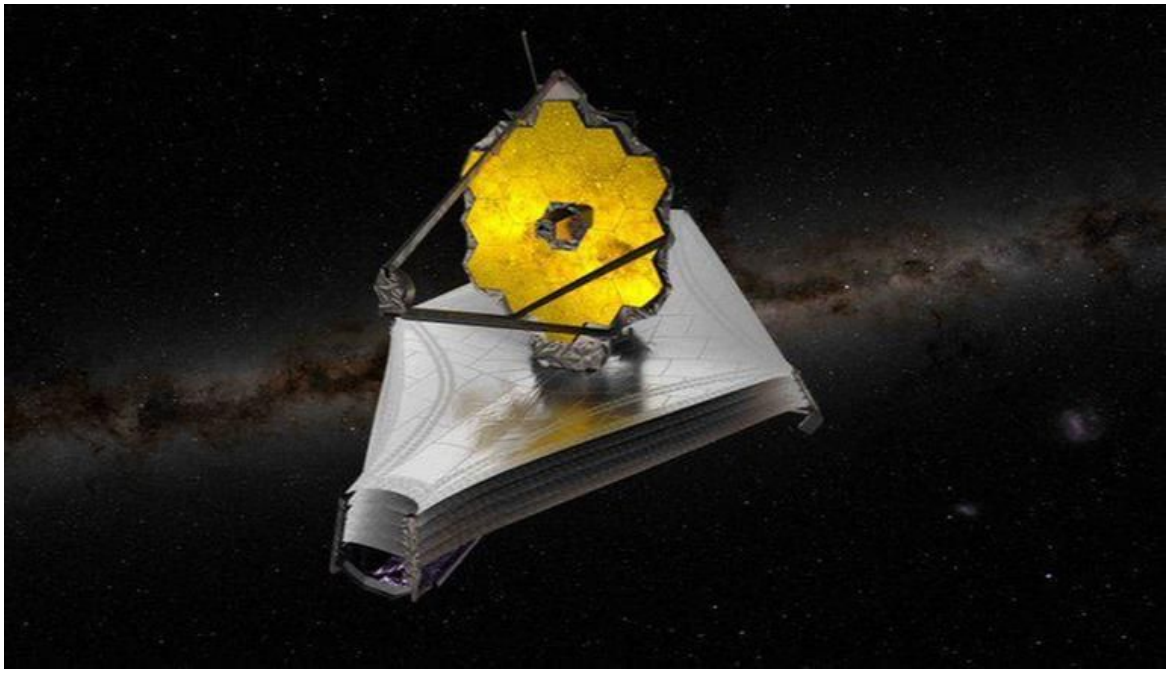
Serendipity

- $E \in \mathcal{E}_w$ says you might obtain E in w by luck.



Achievable Information

- $E \in \mathcal{E}_w$ says diligence will yield at least E in w.
- Familiar normative requirement on experimental results.



Empirical Modalities

Inquiry	Empirical Topology
"A will be verified"	int A
"A will be refuted"	ext A
"A will be decided"	$\neg$bdry A
"A will never be verified"	cl$\neg A$
"A will never be refuted"	cl A
"A will never be decided"	bdry A
"A is false but will never be refuted" = Popper's problem	frnt A $\quad (= \text{cl}\, A \setminus A)$
"A is true but will never be verified" = Hume's problem	frnt $\neg A$

Empirical Properties

Inquiry	Erotetic Topology
"A is verifiable"	A is open
"A is refutable"	A is closed
"A is decidable"	A is clopen
"A is verifutable"	A is locally closed

Ero-Empirical Modalities

Inquiry	Ero-Empirical Topology
"A will be irrelevantly verified"	int $A \backslash$ int A
"The problem of induction arises relevantly for A"	frnt$\neg A \cap$ int A
Etc.	

Learnability

Inquiry	Empirical Topology
"Q is answerable infallibly"	Each answer to Q is $\mathcal{E}$-open
"Q is answerable in the limit with elimination of false reasons"	Each answer to Q is $\mathcal{E}$-sigma-constructible (= countable union of differences of opens)
"Q is answerable in the limit with elimination of false answers"	Each answer to Q is $\mathcal{E}$-sigma-constructible and co-sigma-consructible.

Popper's Analysis of Simplicity

$A \leq B$

iff every information state that falsifies B falsifies A.

iff $A \subseteq \text{cl}\, B$. ← LOL, It's topological!

Two flaws.

1. W is strictly more complex than every other proposition, so mere suspension of judgment violates Ockham's razor!
2. Maybe A is simpler than B somewhere but not everywhere.

Empirical Simplicity

Improvement:

- $A \lhd B =$ "A is strictly simpler than B"
  $= A \cap \text{frnt } B$.
- A is Ockham given E iff
  no B is possibly simpler than A given E.

Prop. The following are equivalent.

1. A is Ockham given E.
2. A is closed (refutable) given E.

Ockham Necessity Theorem

Prop. Suppose that method M:

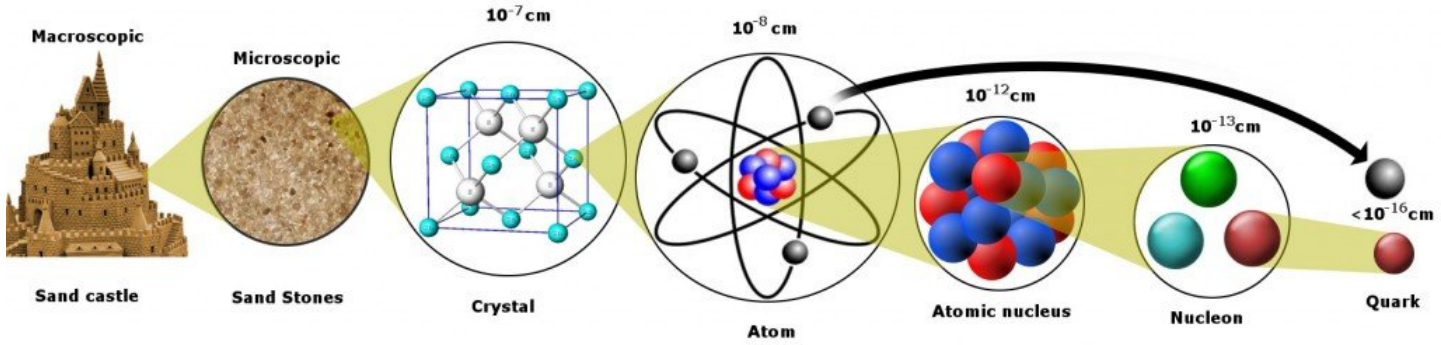
- answers Q,
- eliminates false reasons,
- never drops a true reason.

Then M concludes an Ockham reason for each answer.

3. METAPHYSICAL BASIS

Scientific Realism

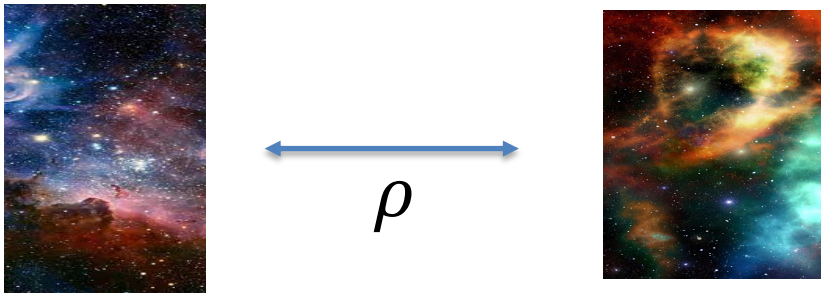
- Scientific realists think science can penetrate beneath the appearances.



- To address realism, one must **represent** **hidden reality.**

Metaphysical Basis

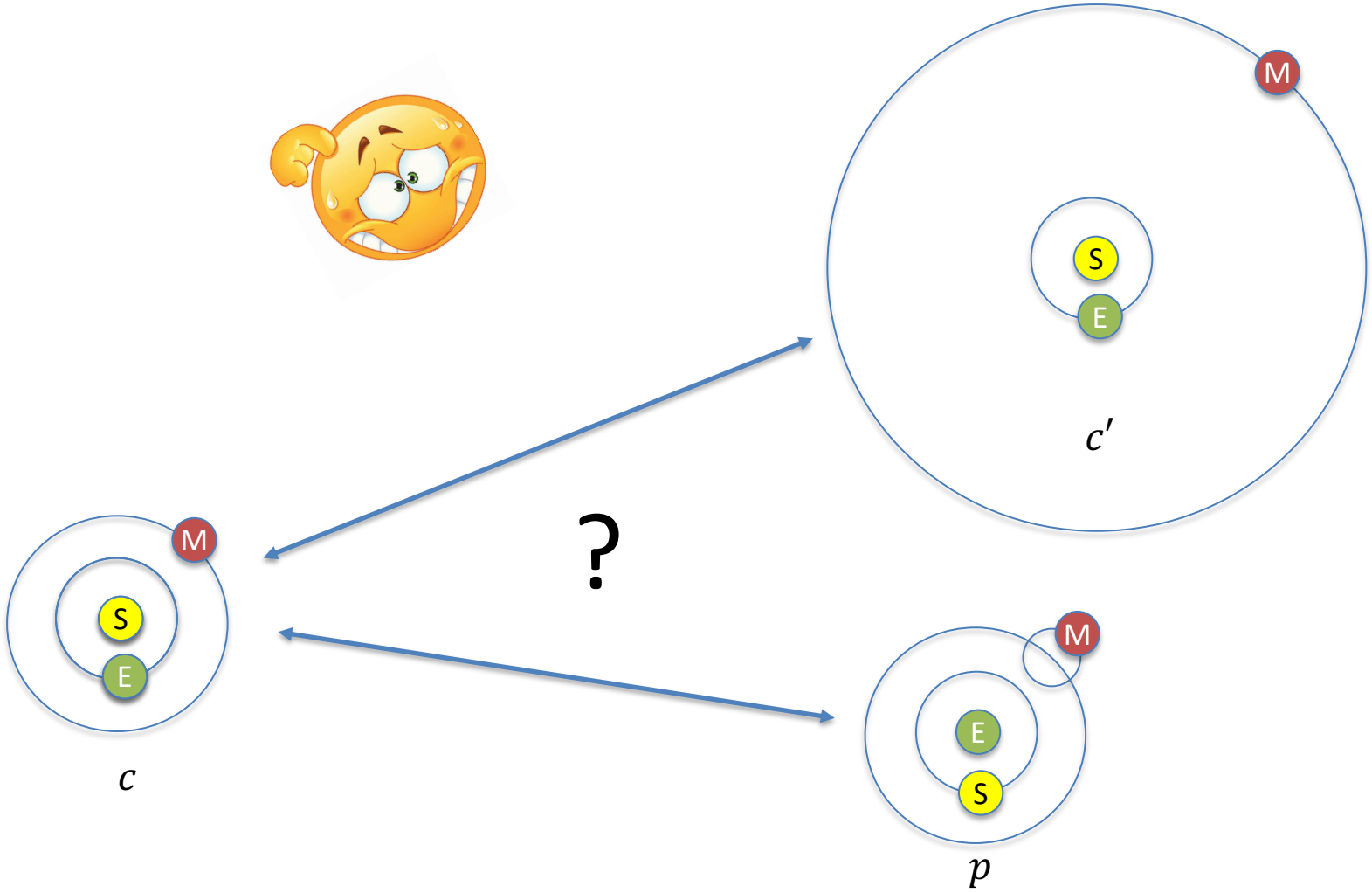
- Worlds are more or less similar.
- ρ is the (dis-)similarity metric.



- $\mathcal{M}$ is the set of all open metric balls.

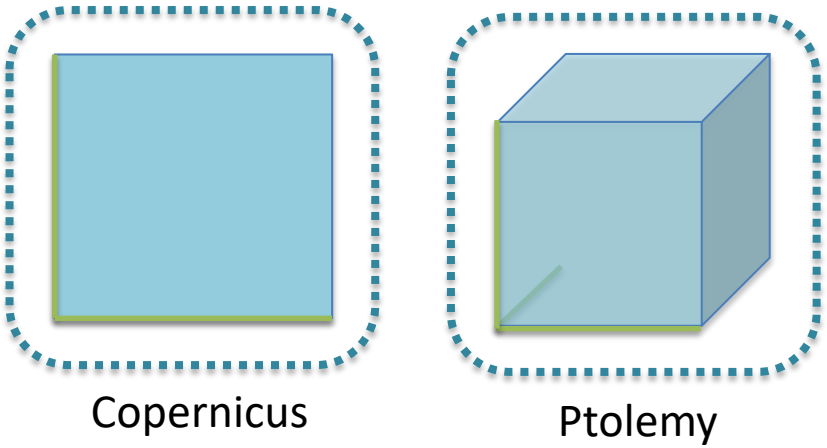
Nice, but Hopeless

Which world is more similar to c?



Metaphysical Topology

- Hopeless comparisons are sidestepped by the induced topology $\mathcal{M}^*$.
  - Across models, discrete difference.
  - Within models, standard metric topology.
  - That determines the metaphysical topology uniquely.



Metaphysical Modalities

Inquiry	Topology
"A is securely true."	int A
"A is securely false."	ext A
"A has a secure truth value."	$\neg$bdry A
"A is or is arbitrarily close to being false."	cl$\neg A$
"A is or is arbitrarily close to being true."	cl A
"A is brittle."	bdry A
"A is barely false."	frnt A
"A is barely true."	frnt $\neg A$

Metaphysical Properties

Inquiry	Metaphysical Topology
"A is natural" = "A cannot be barely true"	A is open

Natural:

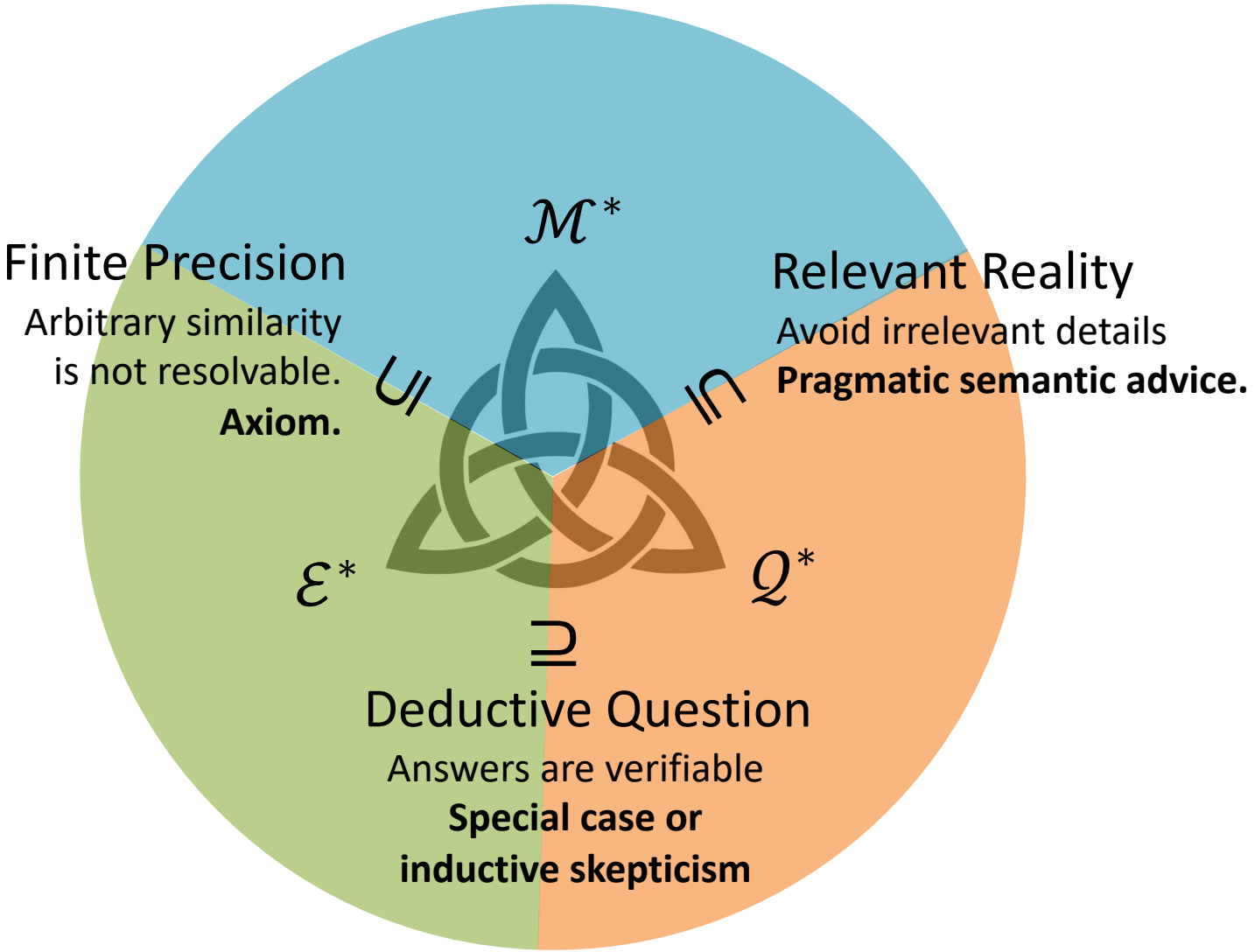
- Open interval estimates (properly open).
- Models (clopen).
- Paradigms = countable disjunctions of models (clopen).

Unnatural:

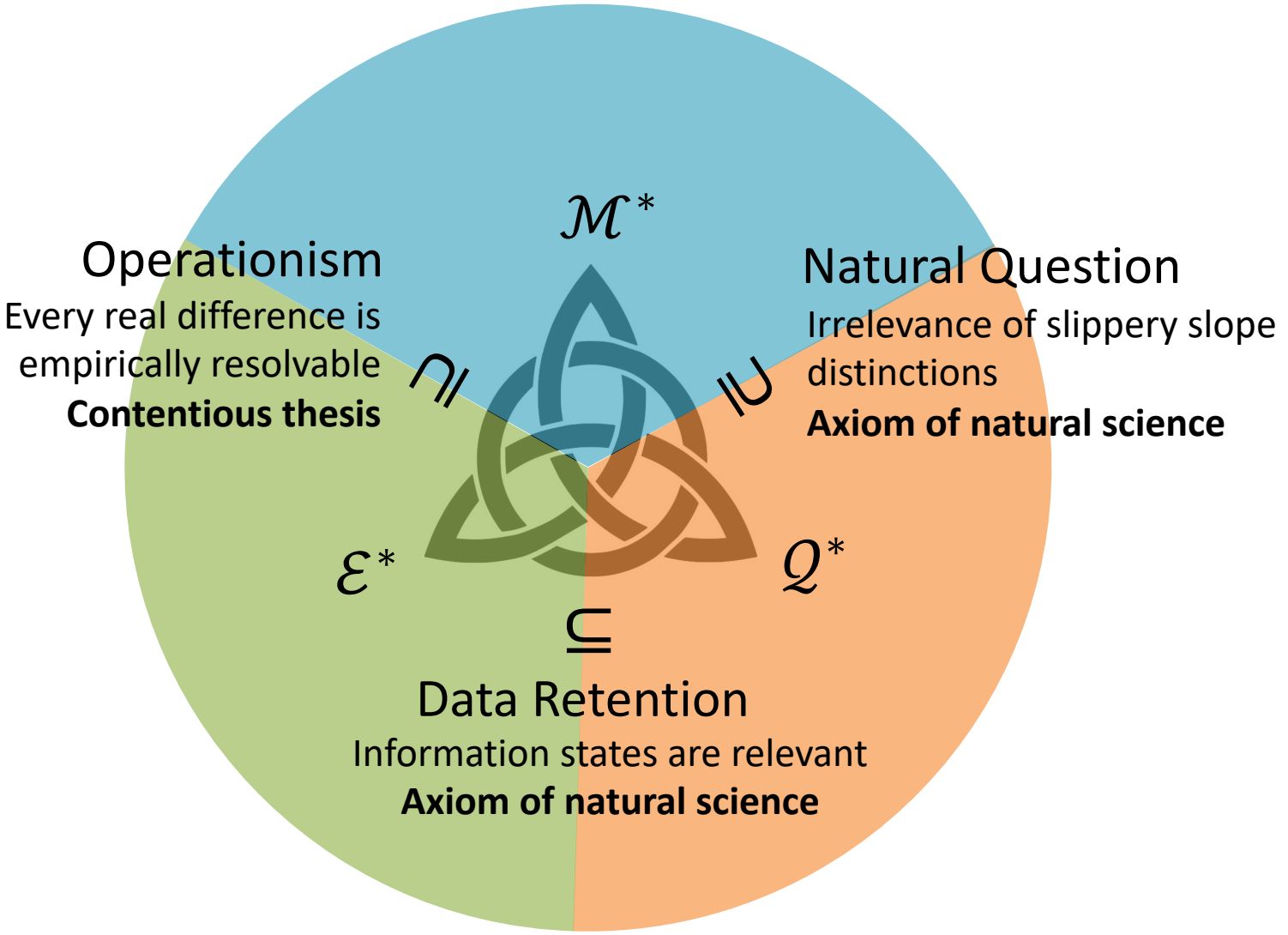
- Arbitrary thresholds.
- Arbitrary quantitative models with no interpretation.
- Arbitrary parameter settings.

LINKS

Clockwise


$\mathcal{M}^*$
Finite Precision
Arbitrary similarity
is not resolvable.
Axiom.
$\sqsubseteq$
Relevant Reality
Avoid irrelevant details
Pragmatic semantic advice.
$\sqsupseteq$
$\mathcal{E}^*$
$\mathcal{Q}^*$
$\sqsupseteq$
Deductive Question
Answers are verifiable
Special case or
inductive skepticism

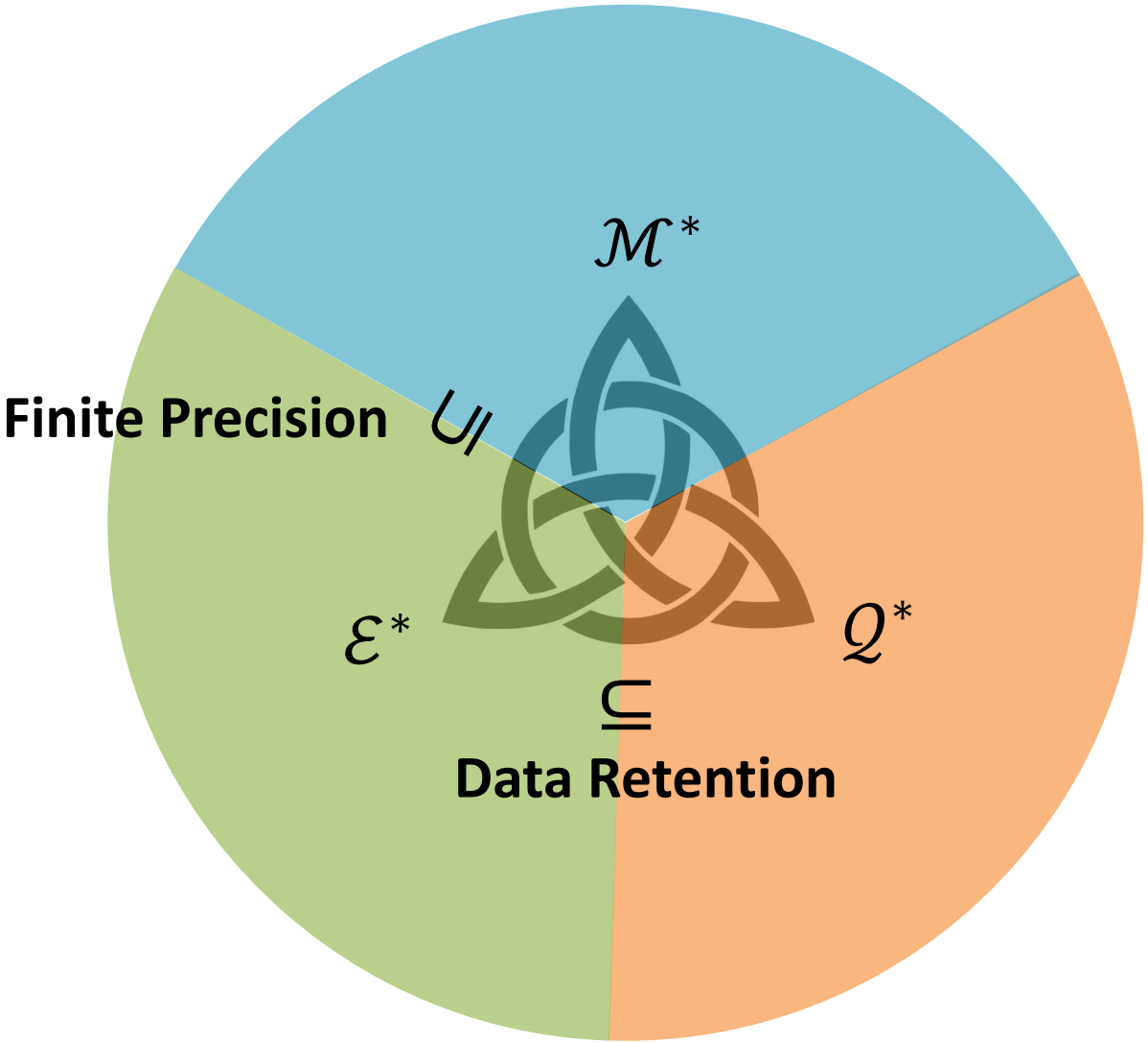
Counter-Clockwise



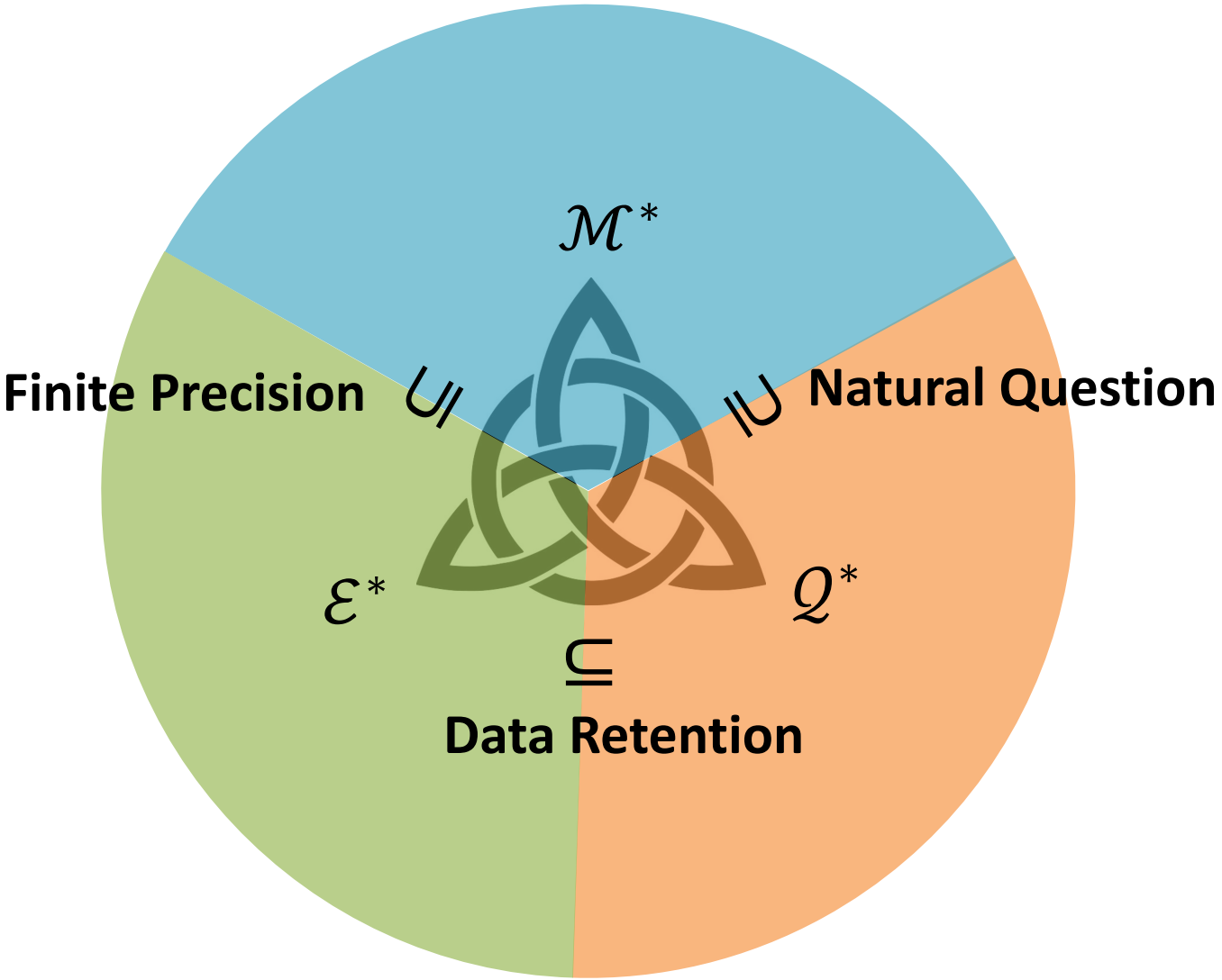
Transitive Implications

Finite Precision	$\wedge$	Relevant Reality	$\Longrightarrow$	Data Retention
Finite Precision	$\wedge$	Deductive Question	$\Longrightarrow$	Natural Question
Relevant Reality	$\wedge$	Deductive Question	$\Longrightarrow$	Operationism
Operationism	$\wedge$	Natural Question	$\Longrightarrow$	Deductive Question
Operationism	$\wedge$	Data Retention	$\Longrightarrow$	Relevant Reality
Data Retention	$\wedge$	Natural Question	$\Longrightarrow$	Finite Precision

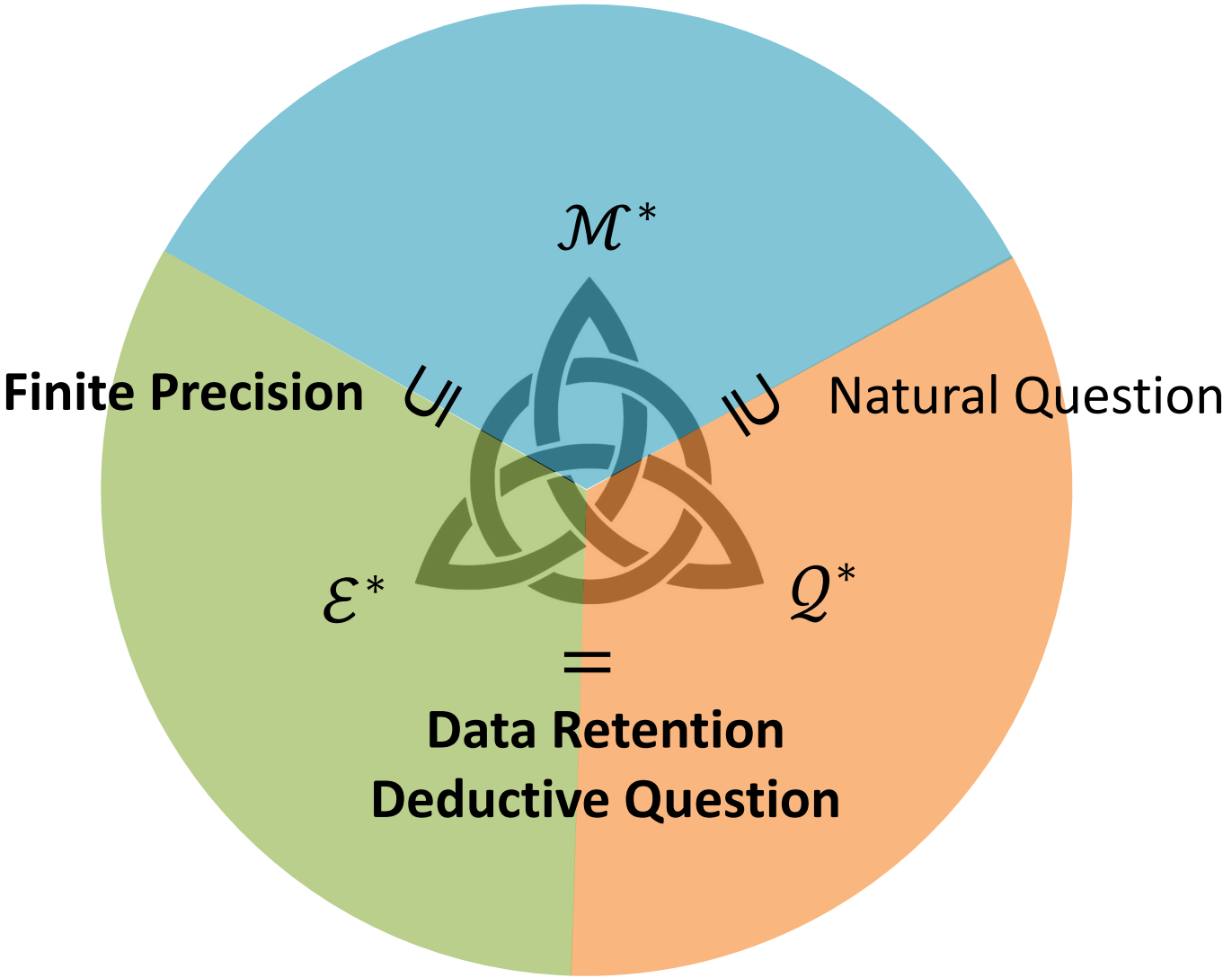
All Contexts



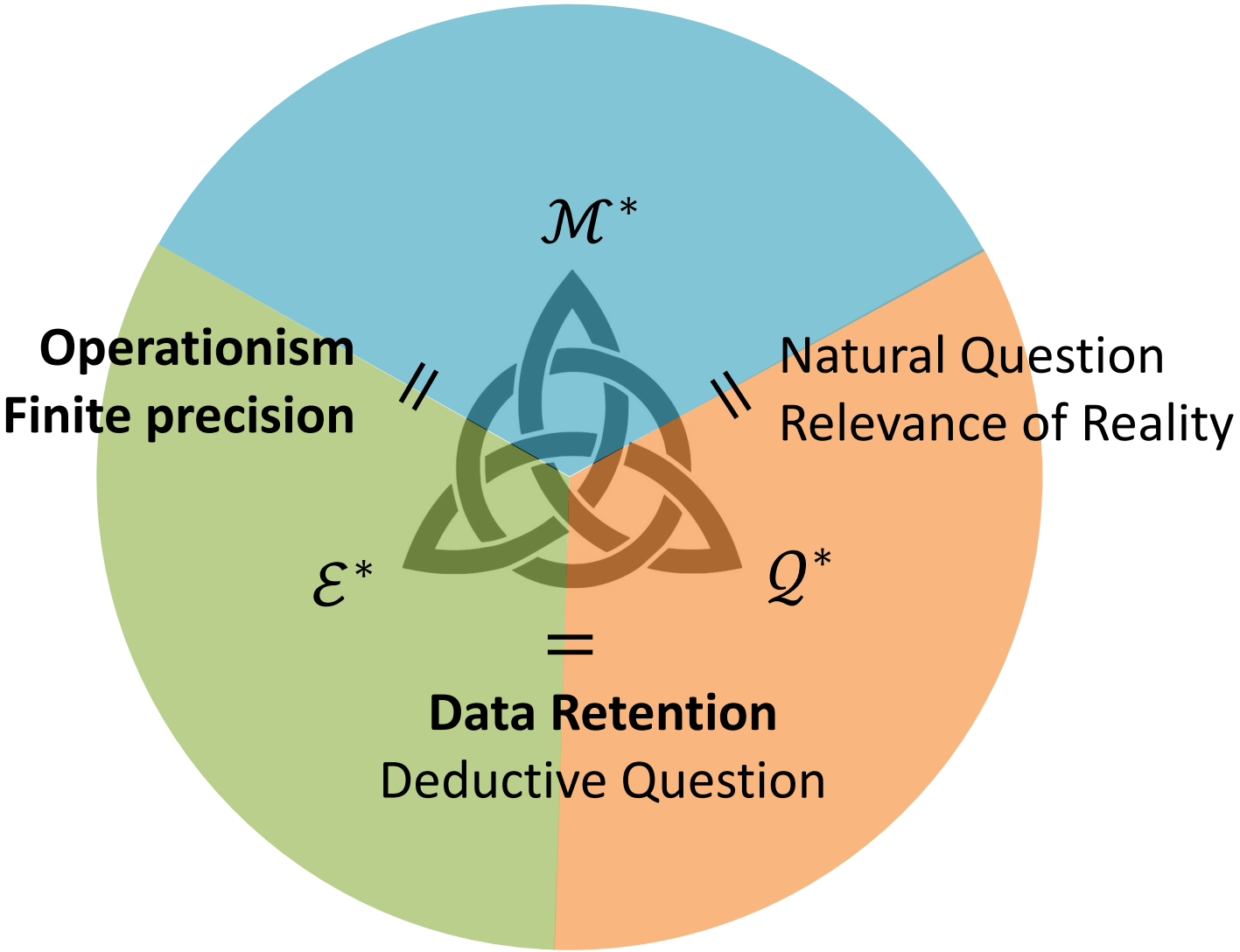
Realist Contexts



Deductive Contexts



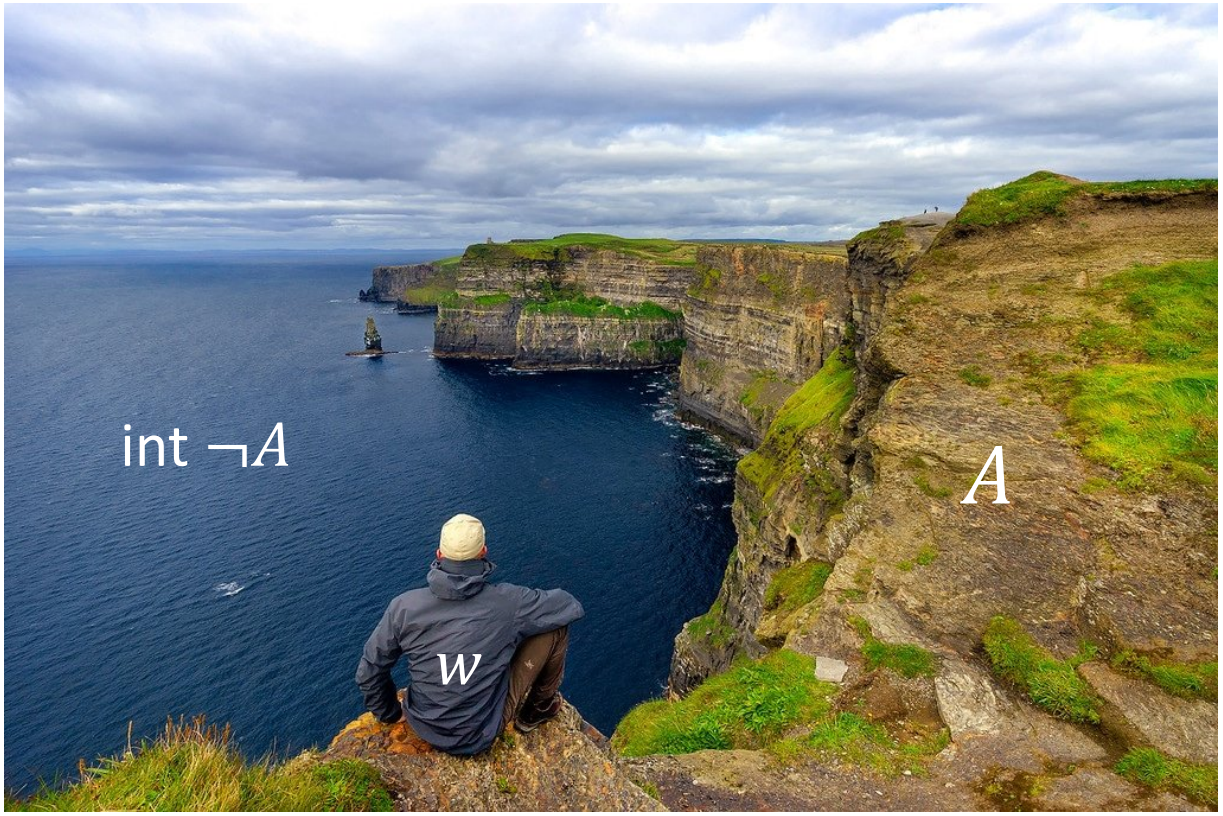
Operationist Contexts



MIRACLES AND FINE-TUNING

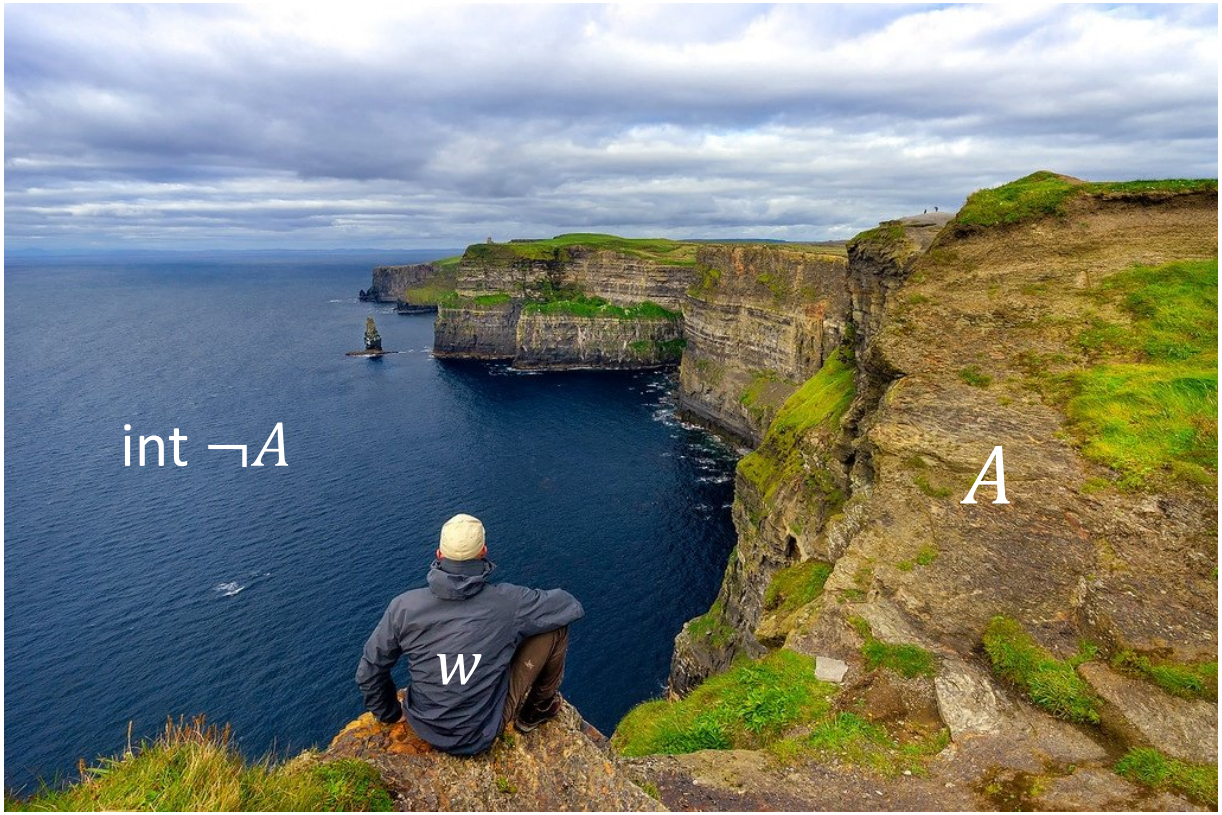
Fine-tuning

- Truth of A teeters at the edge of a sea of falsehood.
- mir A = A ∩ cl int $\neg A$.



Miracles

- A is miraculous iff $A \subseteq \text{mir } A$.



Famous Scientific Miracles

- The morning star is on the same orbit as the evening star.
- Mars' epicycle is perfectly synchronized with the sun's deferent.
- Reflecting telescopes produce exactly the same illusions as refracting telescopes.
- Light is distinct from EM radiation, even though they have exactly the same speed.
- It matters whether the coil or the magnet is moving, even though the current is exactly the same.

NEGLECT

Realists Neglect Miracles

- Hidden realities go beyond all possible empirical information.
- Realism neglects miraculous possibilities of error.
- Anti-realism refuses to.

Metaphysical Negligibility

- A is nowhere dense iff int cl A = ∅.
- The nowhere dense propositions are a non-trivial ideal.
  1. Closed under subset.
  2. Closed under finite union.
  3. Exclude W.
- So nowhere density is a concept of negligibility.
- Unlike prior probability, it is a semantic/metaphysical concept of negligibility.

Realism Theorem

Prop. A is nowhere dense iff A is miraculous.

So the miraculous propositions are exactly the negligible ones!

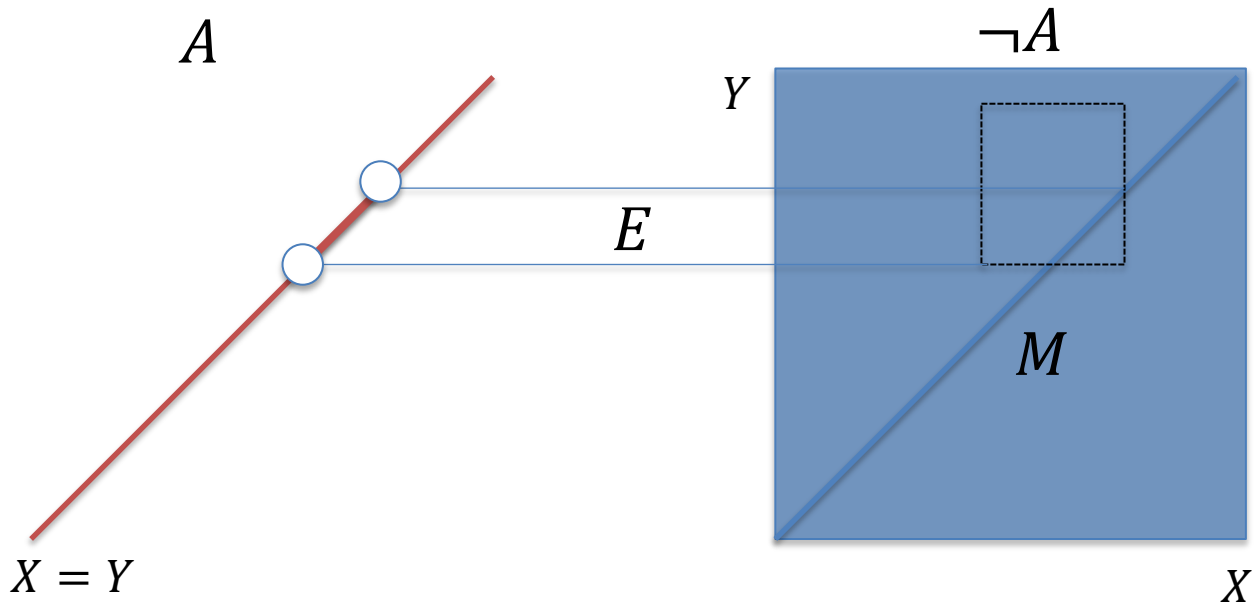
Example: Theoretical Identification

X = maximum elongation of Hesperus.

Y = maximum elongation of Phosphorus.

A = "the two planets are identical".

M = "they are different, but $X = Y$ anyway".

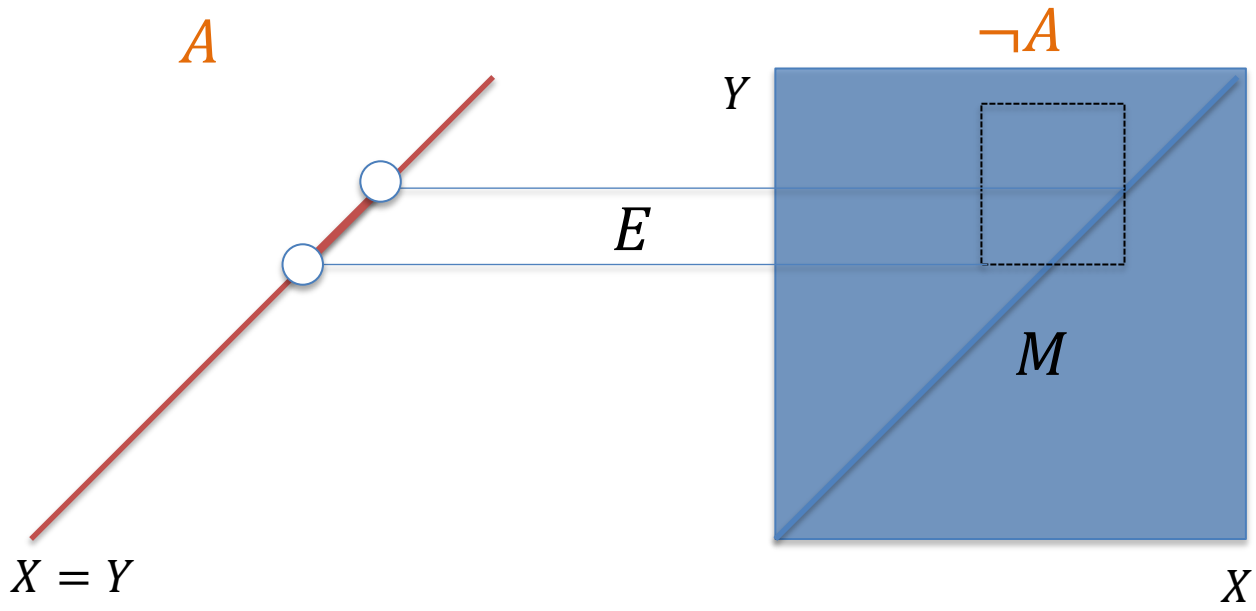


Example: Theoretical Identification

M is miraculous/negligible, so unnatural.

$A, \neg A$ are open/natural, so not miraculous/negligible.

Natural question: $\{A, \neg A\}$.



Realism Vindicated

- M is empirically identical to A.
- But M is negligible and A is not.
- Neglecting M, Ockham's razor mandates A.