Multi-valued and Modal Truth Diagrams

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Slogan!

There is more than one way

to represent truth!

Especially in non-classical logics!

Why?

A Novel Graphical Representation for Propositional Logic

Truth Diagrams were developed by Peter Cheng recently for a novel graphical representation for propositional logic.

They can be viewed as an alternative to truth tables, following the traditions of Frege, Wittgenstein, Venn, Pierce or Gardner.

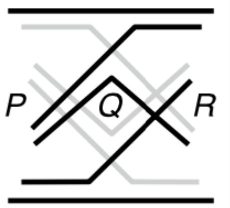
Reference

“Truth Diagrams Versus Extant Notations for Propositional Logic”, Peter C.-H. Cheng, *Journal of Logic, Language and Information*, vol.29, no.2, pp.121–161, 2020.

Truth Diagrams for Propositional Logic: An Example

Consider the formula $P \rightarrow (Q \wedge R)$.

The upper half of the diagram represents the part for TRUE whereas the lower half represents the part for FALSE.
The formula $P \rightarrow (Q \wedge R)$ fails when P is TRUE and $Q \wedge R$ is FALSE.
These three cases are represented by the light-gray lines.



The formula holds for all other five cases which are represented by the black lines. For example, the cases where all three propositions are FALSE, or TRUE.

Limitations

The propositional cases, as expected, are relatively straight-forward.

Two truth values are easy to represent and read.

Nevertheless, it is not easy to immediately “guess” the connectors and the formula, given the diagram.

Extensions

It can be anticipated that for multi-valued logics, the graphical complexity of the diagrams would explode.

Similarly, for some other non-classical operators, take Peirce's arrow (logical nor) for instance, we can have some fun diagrams.

Non-Classical Extensions

In non-classical logics, however, we often have an *asymmetry* between truth, falsity and some other truth values.

Validities are also defined more carefully. *Designated Truth Values* (DTVs) are those truth values that are preserved in valid inferences. Two logics may differ on their designated truth values despite the fact that they may have the same truth table.

Truth diagrams can graphically show the difference, unlike truth tables. This makes the truth diagrams essential for the graphical representation of truth in non-classical logic.

How?

Some Examples of Non-Classical Logics

Consider Kleene's and Priest's three-valued Systems. We represent the third truth value by B for Both.

	$\neg$
T	F
B	B
F	T

$\wedge$	T	B	F
T	T	B	F
B	B	B	F
F	F	F	F

$\vee$	T	B	F
T	T	T	T
B	T	B	B
F	T	B	F

$\rightarrow$	T	B	F
T	T	B	F
B	T	B	B
F	T	T	T

If the DTV is taken as T only, we have the (intuitionistic) Kleene's logic.

If the DTV is taken as T and B, we have the (paraconsistent) Logic of Paradox of Priest.

Designated Truth Values on Truth Diagrams

DTVs matter.

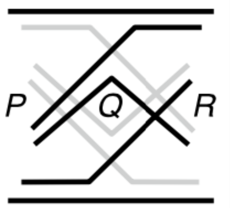
We can represent them on Truth Diagrams with different colour.

Similarly, for the three truth-values, we can designate three positions on the diagram. Top for TRUE, bottom for FALSE and middle for BOTH (or NEITHER, intuitionistically).

An Example for Logic of Paradox

Recall the very formula $P \to (Q \wedge R)$ in propositional logic.

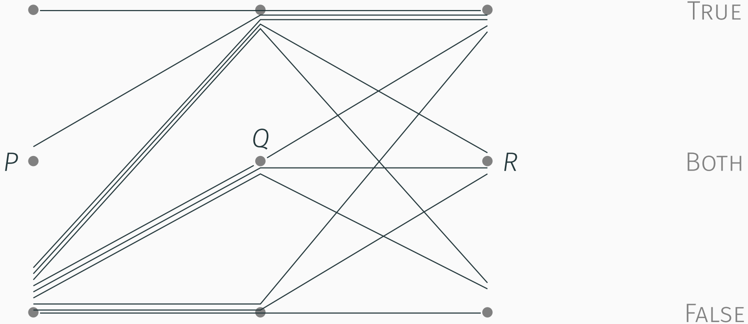
The upper half of the diagram represents the part for TRUE whereas the lower half represents the part for FALSE.



The formula holds for all other five cases which are represented by the black lines. For example, the cases where all three propositions are FALSE, or TRUE.

A Truth Diagram for Logic of Paradox: Step 1

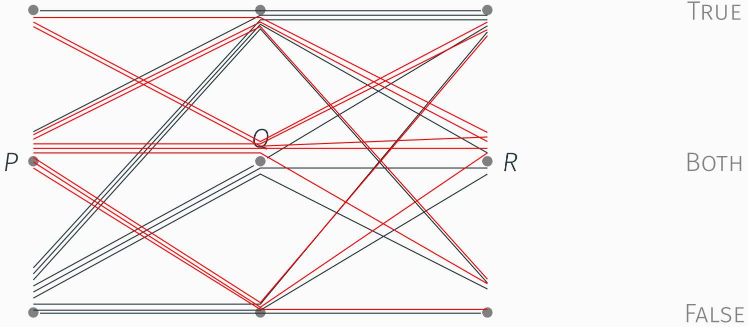
Here is the truth diagram for $P \to (Q \wedge R)$ in Logic of Paradox.



Black lines represent True.

A Truth Diagram for Logic of Paradox: Step 2

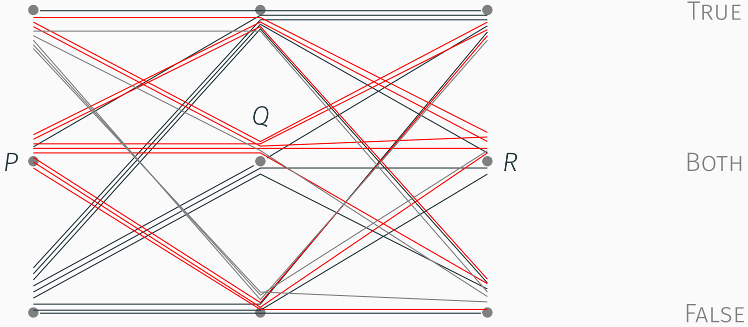
Here is the truth diagram for $P \to (Q \wedge R)$ in Logic of Paradox.



Black lines represent TRUE whereas the red lines represent the paradoxical truth value BOTH.

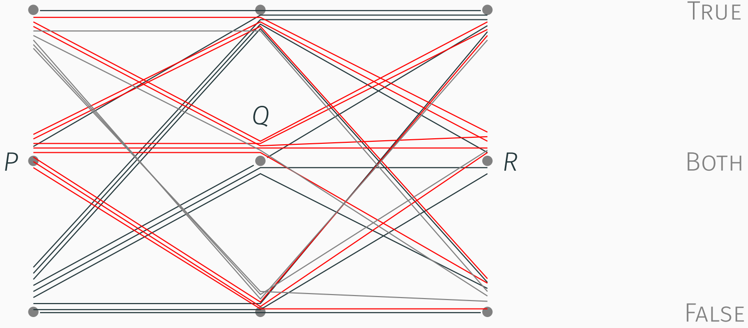
A Truth Diagram for Logic of Paradox: Step 3

Here is the truth diagram for $P \to (Q \wedge R)$ in Logic of Paradox.



Black lines represent True, the red lines the paradoxical truth value Both and the gray lines False.

A Truth Diagram for Logic of Paradox

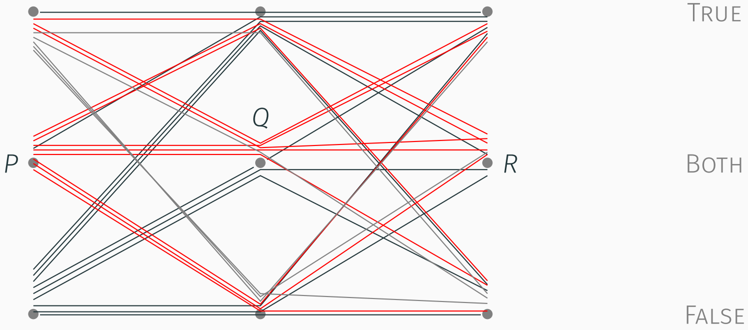


The truth diagram for $P \rightarrow (Q \wedge R)$ shows that the formula is true for 11 truth conditions, paradoxical for 11 truth conditions, and false for 5 truth conditions out of 27 possibilities in Logic of Paradox.

Red and black lines represent the DTVs Both and True, respectively.

Logic of Paradox vs Kleene's – Graphically

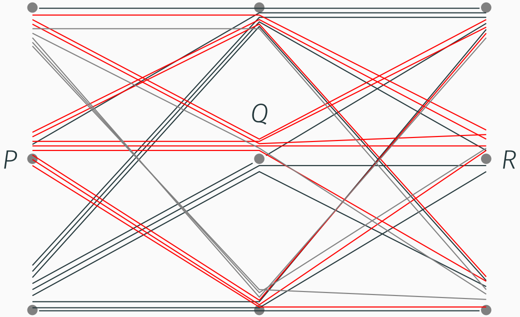
Truth diagrams allow us to distinguish Priest's Logic of Paradox and Kleene's system graphically, unlike truth tables.



DTV in this case is represented only by the black lines.

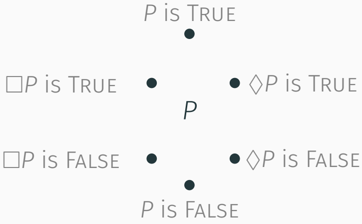
Needless to say, multi-valued logics make the truth diagrams graphically more complicated.

vs



Truth Diagrams in Modal Logic

For (classical) modal logic K, I propose to employ additional four positions around a propositional letter.

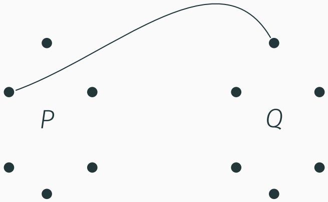


Top (that is 12 o'clock) for "true", bottom (that is 6 o'clock) for "false", left-upper-middle (that is 10 o'clock) for "necessarily true", right-upper-middle (that is 2 o'clock) for "possibly true", left-lower-middle for (that is 8 o'clock) "necessarily false" and right-lower-middle (that is 4 o'clock) for "possibly false".

An Example for Modal Logic K

Consider the formula $\Box P \wedge Q$ in modal logic K.

Here is its truth diagram.



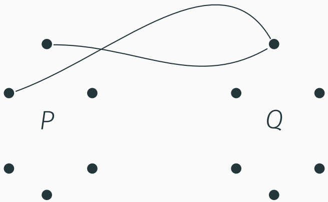
Gray lines for falsity are omitted for easy read.

An Example for Modal Logic S5

Consider the very formula $\Box P \wedge Q$ in modal logic S5 this time.

However, in S5, we have $\Box\varphi \to \varphi$ which suggests that in addition to $\Box P$ being satisfied, P itself is also satisfied.

Therefore, the truth diagram differs from that of in K.



Truth Diagrams for Plethora of Modal Logics

Frame properties of modal logics open up a new area for applications of truth diagrams. Today, we quickly looked at two of them.

Analysing the behaviour of truth diagrams for other major modal logics remains a future task.

What is Next?

Conclusion

This is still a work in progress.

There is a lot to be done.

The immediate task is to extend truth diagrams to few other major non-classical logics, including connexive and four-valued Belnap-Dunn logic as well as to other well-known logics. This will open up new avenues to represent modal attitudes, including epistemic and deontic.

Next Steps

Formally, the completeness of this system will be given.

Then, the focus will be the diagrams themselves. Composition and de-composition of truth diagrams, the rules that govern these operation and their Hintikkan semantical meaning will be the next steps of this project.

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Thank you!

Talk slides are available at my website

CanBaskent.net/Logic