

# Lecture 4: Constituent-Context Model

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# EM FOR GAUSSIAN MIXTURES

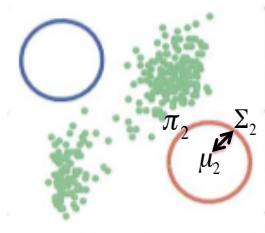
1. Initialize  $\mu_k$ ,  $\Sigma_k$   
 $\pi_k$ , one for each  
Gaussian  $k$

- Tip! Use K-means  
result to initialize:

$$\mu_k \leftarrow \mu_k$$

$$\Sigma_k \leftarrow \text{cov}(\text{cluster}(K))$$

$$\pi_k \leftarrow \frac{\text{Number of points in } k}{\text{Total number of points}}$$

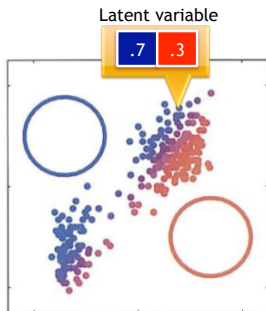


## EM FOR GAUSSIAN MIXTURES

2. **E Step:** For each point  $X_n$ , determine its assignment score to each Gaussian  $k$ :

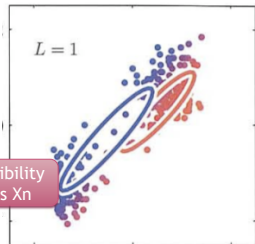
$$\gamma(z_{nk}) = \frac{\pi_k \mathcal{N}(x_n | \mu_k, \Sigma_k)}{\sum_{j=1}^K \pi_j \mathcal{N}(x_n | \mu_j, \Sigma_j)}$$

$\gamma(z_{nk})$  is called a “responsibility”: how much is this Gaussian  $k$  responsible for this point  $X_n$ ?



## EM FOR GAUSSIAN MIXTURES

3. **M Step:** For each Gaussian  $k$ , update parameters using new  $\gamma(z_{nk})$



Mean of Gaussian  $k$

$$\mu_k^{\text{new}} = \frac{1}{N_k} \sum_{n=1}^N \gamma(z_{nk}) \mathbf{x}_n$$

$$N_k = \sum_{n=1}^N \gamma(z_{nk})$$

Find the mean that “fits” the assignment scores best

# GENERAL EM ALGORITHM

1. Initialize parameters  $\theta^{old}$
2. **E Step:** Evaluate  $p(Z|X, \theta^{old})$
3. **M Step:** Evaluate

Hidden variables

Observed variables

$$\theta^{new} = \arg \max_{\theta} Q(\theta, \theta^{old})$$

where

$$Q(\theta, \theta^{old}) = \sum_{\mathbf{Z}} p(\mathbf{Z}|\mathbf{X}, \theta^{old}) \ln p(\mathbf{X}, \mathbf{Z}|\theta).$$

Likelihood

4. Evaluate log likelihood. If likelihood or parameters converge, stop. Else  $\theta^{old} \leftarrow \theta^{new}$  and go to E Step.

## Efficient EM for PCFGs: Inside-outside algorithm

- Hidden variables: the derivations that generated the observed sentences

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## Efficient EM for PCFGs: Inside-outside algorithm

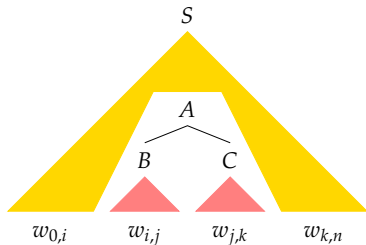
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- For the M-step, we only need to know the *expected relative frequency* of each rule in the derivation of the sentences;
- *expected relative frequency* of rules can be computed without computing the entire probability distribution over derivations!

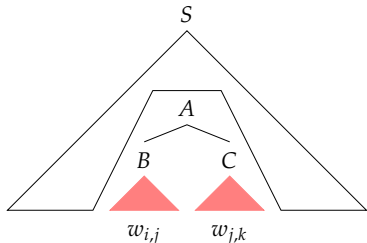
## Dynamic programming for $E_p(f_{A \rightarrow BC} | w)$

$$E_p(f_{A \rightarrow BC} | w) = \frac{\sum_{0 \leq i < j < k \leq n} P(S \Rightarrow^* w_{1,i} A w_{k,n}) p(A \rightarrow BC) P(B \Rightarrow^* w_{i,j}) P(C \Rightarrow^* w_{j,k})}{P_G(w)}$$



## Dynamic programming recursion

$$\begin{aligned}
 & P_G(A \Rightarrow^* w_{i,k}) \\
 &= \sum_{j=i+1}^{k-1} \sum_{A \rightarrow BC \in R(A)} p(A \rightarrow BC) P_G(B \Rightarrow^* w_{i,j}) P_G(C \Rightarrow^* w_{j,k})
 \end{aligned}$$



$P_G(A \Rightarrow^* w_{i,k})$  is called the *inside probability* of  $A$  spanning  $w_{i,k}$ .

## Language modeling using dynamic programming

- **Goal:** To compute  $P_G(w) = \sum_{\psi \in \Psi_G(w)} P_G(\psi) = P_G(S \Rightarrow^* w)$
- **Data structure:** A table called a *chart* recording  $P_G(A \Rightarrow^* w_{i,k})$  for all  $A \in N$  and  $0 \leq i < k \leq |w|$
- **Base case:** For all  $i = 1, \dots, n$  and  $A \rightarrow w_i$ , compute:

$$P_G(A \Rightarrow^* w_{i-1,i}) = p(A \rightarrow w_i)$$

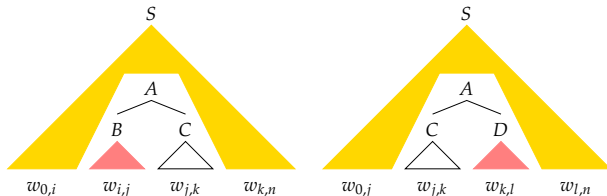
- **Recursion:** For all  $k - i = 2, \dots, n$  and  $A \in N$ , compute:

$$\begin{aligned} & P_G(A \Rightarrow^* w_{i,k}) \\ &= \sum_{j=i+1}^{k-1} \sum_{A \rightarrow BC \in R(A)} p(A \rightarrow BC) P_G(B \Rightarrow^* w_{i,j}) P_G(C \Rightarrow^* w_{j,k}) \end{aligned}$$



## Recursion in $P_G(S \Rightarrow^* w_{0,i} A w_{k,n})$

$$\begin{aligned}
 P(S \Rightarrow^* w_{0,j} C w_{k,n}) = & \\
 & \sum_{i=0}^{j-1} \sum_{\substack{A, B \in N \\ A \rightarrow BC \in R}} P(S \Rightarrow^* w_{0,i} A w_{k,n}) p(A \rightarrow BC) P(B \Rightarrow^* w_{i,j}) \\
 + & \sum_{l=k+1}^n \sum_{\substack{A, D \in N \\ A \rightarrow CD \in R}} P(S \Rightarrow^* w_{0,j} A w_{l,n}) p(A \rightarrow CD) P(D \Rightarrow^* w_{k,l})
 \end{aligned}$$



## Calculating “outside probabilities”

Construct a table of “outside probabilities”

$P_G(S \Rightarrow^* w_{0,i} A w_{k,n})$  for all  $0 \leq i < k \leq n$  and  $A \in N$

Recursion from *larger to smaller* substrings in  $w$ .

*Base case:*  $P(S \Rightarrow^* w_{0,0} S w_{n,n}) = 1$

*Recursion:*  $P(S \Rightarrow^* w_{0,j} C w_{k,n}) =$

$$\sum_{i=0}^{j-1} \sum_{\substack{A, B \in N \\ A \rightarrow B C \in R}} P(S \Rightarrow^* w_{0,i} A w_{k,n}) p(A \rightarrow B C) P(B \Rightarrow^* w_{i,j})$$

$$+ \sum_{l=k+1}^n \sum_{\substack{A, D \in N \\ A \rightarrow C D \in R}} P(S \Rightarrow^* w_{0,j} A w_{l,n}) p(A \rightarrow C D) P(D \Rightarrow^* w_{k,l})$$

## The EM algorithm for PCFGs

Input: a corpus of strings  $w = w_1, \dots, w_n$

Guess initial production probabilities  $p^{(0)}$

For  $t = 1, 2, \dots$  do:

1. Calculate *expected frequency*  $\sum_{i=1}^n \mathbb{E}_{p^{(t-1)}}(f_{A \rightarrow \alpha} | w_i)$  of each production:

$$\mathbb{E}_p(f_{A \rightarrow \alpha} | w) = \sum_{\psi \in \Psi_G(w)} f_{A \rightarrow \alpha}(\psi) P_p(\psi)$$

2. Set  $p^{(t)}$  to the *relative expected frequency* of each production

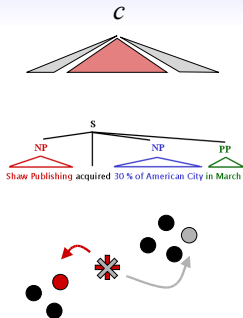
$$p^{(t)}(A \rightarrow \alpha) = \frac{\sum_{i=1}^n \mathbb{E}_{p^{(t-1)}}(f_{A \rightarrow \alpha} | w_i)}{\sum_{A \rightarrow \alpha'} \sum_{i=1}^n \mathbb{E}_{p^{(t-1)}}(f_{A \rightarrow \alpha'} | w_i)}$$

It is as if  $p^{(t)}$  were estimated from a visible corpus  $\Psi_G$  in which each tree  $\psi$  occurs  $\sum_{i=1}^n P_{p^{(t-1)}}(\psi | w_i)$  times.



# A Nested Distributional Model

- Klein and Manning (2002) propose a model that:
  - Ties spans to linear contexts (like distributional clustering)
  - Considers only proper tree structures
  - Has no symmetries to break (like a dependency model)



# Generative model

- $S$  is a sentence
- $B$  is a bracketing
- $P_{bin}(B)$  : uniform prob. over binary bracketings
- $\alpha_{ij}$  - parts-of-speech from  $i$  to  $j$
- $x_{ij}$  – context of  $\alpha_{ij}$
- $P(S, B) = P_{bin}(B) P(S|B)$
- $P(S|B) = \prod P(\alpha_{ij}|B_{ij})P(x_{ij}|B_{ij})$

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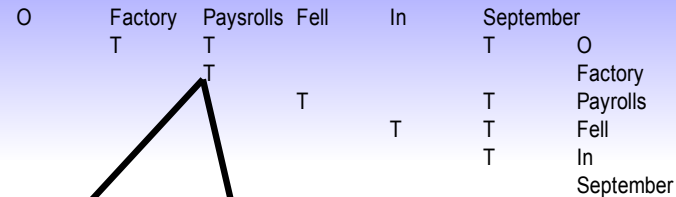
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O	Factory	Paysrolls	Fell	In	September	
	T	T			T	O
		T				Factory
			T		T	Payrolls
				T	T	Fell
					T	In
						September

(((Factory) (Payrolls)) (((Fell) ((In) (September))))))

NN NN VBP IN NN

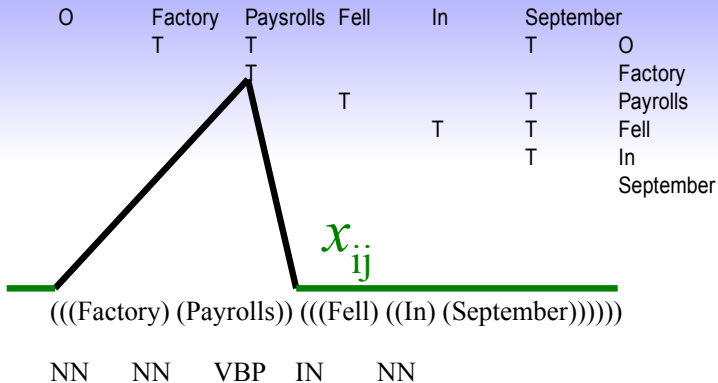
# Constituent...



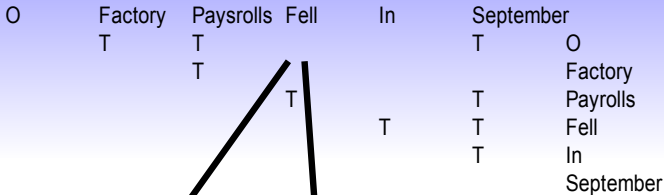
(((Factory) (Payrolls)) (((Fell) ((In) (September))))))

NN    NN    VBP    IN    NN

# ... Context



# Distituent...

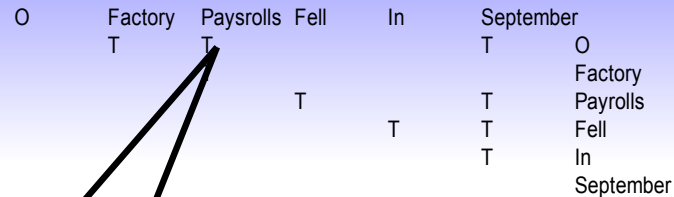


(((Factory) (Payrolls)) (((Fell) ((In) (September))))))

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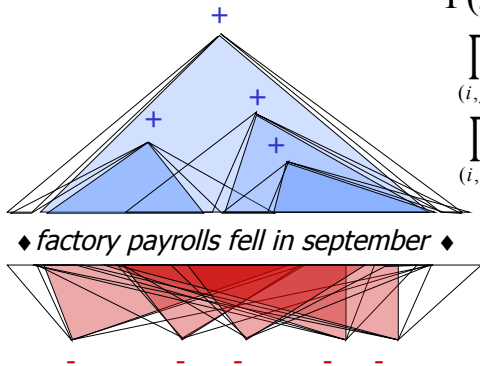
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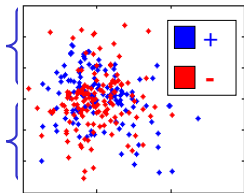
# Constituent-Context Model (CCM)



$$P(S|T) =$$

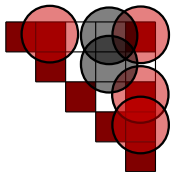
$$\prod_{(i,j) \in T} \left\{ \begin{array}{l} P(\phi_{ij} | +) P(\chi_{ij} | +) \\ P(\diamond\_ \diamond | +) \end{array} \right.$$

$$\prod_{(i,j) \in T} \left\{ \begin{array}{l} P(\phi_{ij} | -) P(\chi_{ij} | -) \\ P(\diamond\_ fell | +) \end{array} \right.$$

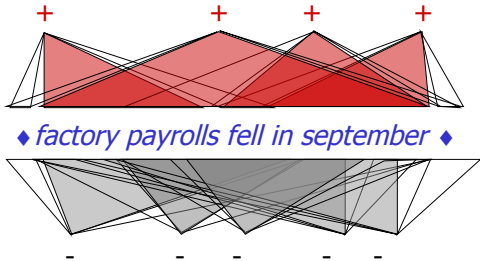


# Constituent-Context Model (CCM)

$$P(S, B) = P(B) P(S|B)$$



$P(B)$



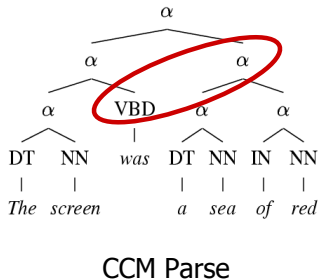
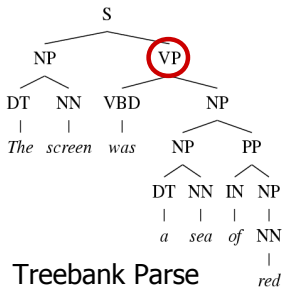
$P(S|B)$

## Constituent-Context Model (CCM; Klein and Manning 2002)

- Generative model where *every* possible constituent/distituent generates its yield as well as its context;
- Parameters of the model are the probabilities with which yields/contexts are generated;
- Parameters are initialized using a clever scheme (Klein, 2005);
- Parameters are optimized using EM & early stopping

# Results: Constituency

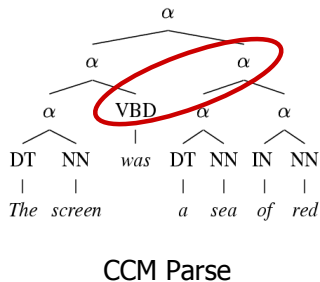
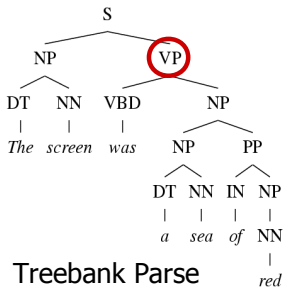
CCM: 71.9%



# Results: Constituency

Right-Branch

70.0



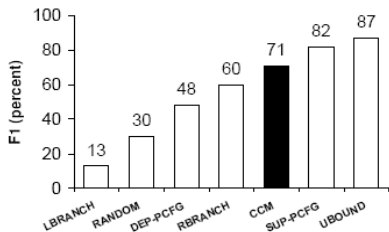
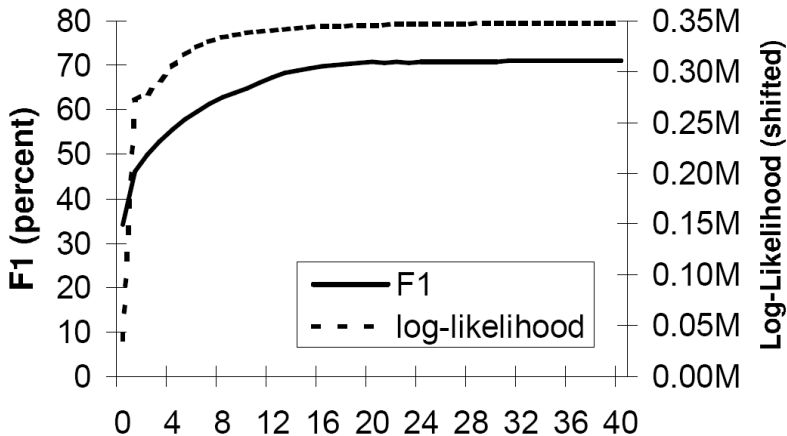


Figure 4: F<sub>1</sub> for various models on WSJ-10.

System	UP	UR	F <sub>1</sub>	CB
EMILE	51.6	16.8	25.4	<b>0.84</b>
ABL	43.6	35.6	39.2	2.12
CDC-40	53.4	34.6	42.0	1.46
RBRANCH	39.9	46.4	42.9	2.18
COND-CCM	54.4	46.8	50.3	1.61
CCM	<b>55.4</b>	<b>47.6</b>	<b>51.2</b>	1.45

Figure 6: Comparative ATIS parsing results.





# Spectrum of Systematic Errors

CCM  
analysis  
better



Treebank  
analysis  
better

Analysis	Inside NPs	Possesives	Verb groups
CCM	<i>the [lazy cat]</i>	<i>John ['s cat]</i>	<i>[will be] there</i>
Treebank	<i>the lazy cat</i>	<i>[John 's] cat</i>	<i>will [be there]</i>
CCM Right?	Yes	Maybe	No

*But the worst errors are the non-systematic ones (~25%)*

# How good is CCM?

- How good is CCM's f-score of 71.9% (63.2% with induced POS tags)
- It can be improved to 77.6% if enriched with dependency structure (Klein and Manning 2004)
- Yet, there shortcomings of CCM:
  - Initialization & stopping heuristics play a big role;
  - The generative mode is linguistically not plausible;
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# Maximum likelihood

