Practice Exam FNCM13

Dynamical systems

1. Draw the phase portrait, label the relevant points/curves/regions, and describe the long term behavior of variable x for various initial conditions for the system described by the following differential equation:

dx/dt = (x-2)^2 - 4

2*. Consider the phase space diagram below (showing a parabola and a line, and two vectors on the right, of a two-dimensional systems of ODE’s). Indicate the equilibrium/equilibria and determine its/their stability. Explain.

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       / \         _|
      /   \
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    /       \      _
   /         \      |
  /           \
```

Single neuron models

3. The Fitzhugh-Nagumo model describes the dynamics of an idealized neuron with one equation (dV/dt, a 3d order polynomial) describing the change in electrical potential, and one equation (dR/dt, a linear equation) describing the change in the state of the ion channels. Depending on where the nullclines cross, the model defines a stable and a unstable equilibrium. For these two cases, draw qualitative phase spaces, including (labeled) nullclines, vector fields and two example trajectory per case, one after a small and one after a large perturbation.

Perceptron

4. Given the network presented below, compute the activation of the output layer for an input vector (0 1). Assume a threshold function that gives 0 when net input <0 and 1 otherwise.

```
       w=1
     O---O b=-0.5
 w=-1 \ / \w=1
       /   O b=-1.5
  w=1 / \ /w=1
     O---O b=1.5
       w=-1
```

5*. Consider the feature space below, with +'s and -'s indicating two categories. A single-layer perceptron cannot represent these two categories. (a) explain why. (b) Design a multilayer perceptron that can distinguish +'s and -'s. Draw the network architecture and the decision boundaries the hidden units need to have (no need to compute the actual weights). Hint: every neuron in the hidden layer is itself a perceptron, and its weights define a decision boundary in the feature space.

```
+++++++++++++
+++++++++++++
+++       +++
+++  ---  +++
+++       +++
+++++++++++++
+++++++++++++
```

Hopfield

6. Give the Hopfield learning rule.

Symbolic models

7. Given the initial and target configuration of a 3-ring Towers of Hanoi problem as below, give the steps the 'goal recursion' strategy has to go through, including the adding and removing of subgoals from the stack.

Initial state

```
   |    |    |
   |    |    |
 -----  |    |
------- |   ---

__________________
   A    B    C
```

Goal state

```
   |    |    |
   |   ---   |
   |  -----  |
   | ------- |

__________________
   A    B    C
```

Bayesian modelling

8. Given the two PCFGs G1 and G2 below, the data D, and a uniform prior probability over the two grammars, what is the posterior probability of each of them?

G1	G2	D
S -> S a (0.25)	S -> aa (0.25)	aaa
S -> a (0.75)	S -> aaa (0.25)	aa
	S -> aaaa (0.25)	aaa
	S -> aaaaa (0.25)	aaaa

Methodology

9. Give three criteria for good models; give 3 models of a neuron; explain briefly how each of these models scores on each of your criteria. Write at most half a page.

10*. (a) What is the binding problem? (b) Recall that autoencoder networks (discussed in the student presentations) are multilayer perceptrons where the desired output layer activation vector equals the input layer. When trained on images, we have seen that the hidden units start to represent particular features of the input, such as edges with a particular orientation. Discuss whether autoencoder networks suffer from the binding problem. Write at most half a page.