Outline of the project

Quantifiers, games, and complexity

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17th November 2006

Outline

1. Generalized Quantifiers
2. Branching quantifiers
3. Complexity and difficulty

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Instead of introduction

- Every poet has low self-esteem.
- Some dean danced nude on the table.
- At least 3 grad students prepared presentations.
- An even number of the students saw a ghost.
- Most of the students think they are smart.
- Less than half of the students received good marks.
- An equal number of logicians, philosophers, and linguists climbed Elbrus.

Lindström definition

Definition

A generalized quantifier is a class Q of structures of a finite relational signature which is closed under isomorphism. The type of Q can be identified with a finite sequence $(n_1, \ldots, n_k)$ of natural numbers.

Few examples to make it clear

- $K_{\exists} = \{(|M|, R) : R \subseteq |M| \wedge R \neq \emptyset\}$.
- $K_{\forall} = \{(|M|, R) : R = |M| \wedge R \neq \emptyset\}$.
- $K_{\exists^{=m}} = \{(|M|, R) : R \subseteq |M| \wedge card(R) = m\}$.
- $K_{D_n} = \{(|M|, R) : R \subseteq |M| \wedge card(R) = kn\}$.
- $K_{Most} = \{(|M|, R_1, R_2) : card(R_1 \cap R_2) > card(R_1 - R_2)\}$.
- $K_{Equal} = \{(|M|, R_1, \ldots, R_n) : card(R_1) = \ldots = card(R_n)\}$.

Games for elementary quantifiers

- If $\psi := \exists x \varphi(x)$, then Eloise chooses an element $d \in |M|$ and the game continues for the formula $\varphi(d)$.
- If $\psi := \forall x \varphi(x)$, then Abelard chooses an element $d \in |M|$ and the game continues for the formula $\varphi(d)$.
- If $\psi := \exists^{=m} x \varphi(x)$, then Eloise chooses subset $A \subseteq M$, such that $card(A) = m$, and Abelard chooses $d \in A$ and the game continues for the formula $\varphi(d)$.

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Hintikka's-like sentences

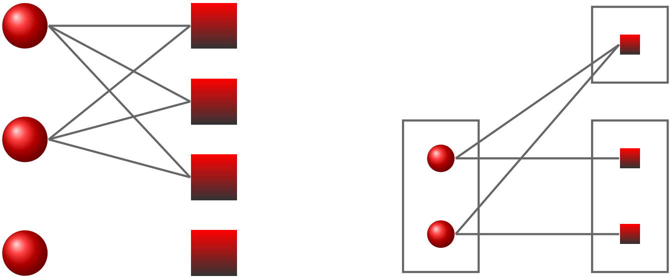
1. Some relative of each villagers and some relative of each townsmen hate each other.
2. Most villagers and most townsmen hate each other.
3. Exactly half of all villagers and exactly half of all townsmen hate each other.

HINTIKKAS'S THESIS

Hintikka claims that we need branching quantifiers to express their meaning.

1. $\begin{matrix} \forall x \exists y \\ \forall z \exists w \end{matrix} ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))).$
2. $\exists f \exists g \forall x \forall z ((V(x) \wedge T(z)) \Rightarrow R(x, f(x)) \wedge R(z, g(z)) \wedge H(f(x), g(z)))).$
3. $\begin{matrix} \text{MOST } x : V(x) \\ \text{MOST } y : T(y) \end{matrix} H(x,y).$
4. $\exists A \exists B[\text{MOST}x(V(x), A(x)) \wedge \text{MOST}y(T(y), B(y)) \wedge \forall x \forall y (A(x) \wedge B(y) \Rightarrow H(x,y))].$

Illustrations



GTS and Subgame semantics I

- If $\psi := \begin{matrix} \forall x \exists y \\ \forall z \exists w \end{matrix} \varphi(x)$ then Abelard chooses an element $a \in |M|$ and Eloise chooses an element $b \in |M|$, and then Abelard chooses $c \in |M|$ and Eloise chooses independently $d \in |M|$.
- GTS is counterintuitive, for instance $\varphi \vee \varphi$, $\varphi \wedge \varphi$, and φ are not equivalent.

Objective

Investigate subgame semantics as an alternative. Compare it with strategic interpretation of Henkin quantifiers.

GTS and Subgame semantics II

Objective

Formulate game-theoretical (subgame) semantics for all branching quantifiers.

Objective

Investigate linguistic plausibility of various interpretations for branching sentences in natural language.

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Monadic quantifiers and automata

definability	example	recognized by
FO	exactly 6	acyclic FA
$FO(D_n)$	even	FA
Pr	most	PDA

Table: Quantifiers and complexity of corresponding algorithms.

Important: FA do not have a memory, PDA have stack - which is considered a form of memory.

Neuroimaging study

- Comprehension of FO and non-FO quantifiers recruit right inferior parietal cortex – the region of brain associated with number knowledge.
- Non-FO quantifiers recruit right dorsolateral prefrontal cortex – the part of brain associated with executive resources and working memory.

Objective

Find psychologically plausible explanation of these results.

Complexity of branching quantifiers

Theorem

Henkin quantifier defines NP-complete class of finite models.

Theorem

Branching MOST *defines NP-complete class of finite models.*

Objective

What is the source of such complexity of those constructions?

Theorem

Ramsey quantifiers define NP-complete class of finite models.

Complexity, difficulty and games

Objective

Study evaluation games in connection with the way people understand quantifier sentences.

Objective

Try to use higher-order games, like signaling games, to investigate connection between difficulty and complexity.

For Further Reading I

T. Janssen
Independent choices and the interpretation of IF-logic.
JOLLI 11: 2002.

M. Mostowski, J. Szymanik
Semantical bounds for everyday language.
Semiotica, to appear.

N. Gierasimczuk, J. Szymanik
Hintikka's Thesis Revisited.
preliminary report, see: ILLC Preprint Series, 2006.

For Further Reading II

C. McMillan et al.
Neural Basis for Generalized Quantifiers.
Neuropsychologia, 43,2005.

M. Sevenster
Branches of imperfect information: logic, games, and computation.
PhD Thesis, ILLC 2006.

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Neuropsychologia, to appear.