The Problem of the Divided Majority

Preference Aggregation and Uncertainty

Đura-Georg Granić, University of Cologne, Germany

<georg.granic@uni-koeln.de>

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The Divided Majority

The Divided Majority

- Three Candidates: Red, Blue and Green
- Electorate (group, committee, state, etc.) is characterized by the following preference profile

Type of Voter	$\sharp$ Voters	Preferences
Grues	2	Green $\succ$ Blue $\succ$ Red
Reds	3	Red $\succ$ Blue $\sim$ Green
Bleens	2	Blue $\succ$ Green $\succ$ Red

- Reds voters constitute a weak majority
- Red is the worst outcome for an absolute majority of voters
- **Coordination Problem:** Grues and Bleens can avoid the '*bad*' outcome if they coordinate

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- Central to the analysis of electoral systems since at least Jean Charles de Borda (1781), Marie Jean Nicolas Caritat Marquis de Condorcet (1785)
- Condorcet-Winner (Loser) is defined as an alternative that can beat (that is beaten by) any other alternative in pairwise comparison:
  - ◊ 4 voters prefer Green over Red, 4 voters prefer Blue over Red, Red is a Condorcet-Loser
- Infamous real world examples exist...

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The Divided Majority

Type of Voter	♯ Votes received	Preferences
Gore	48.84 %	Gore $\succ$ Nader $\succ$ Bush
Bush	48.85 %	Bush $\succ$ Gore $\sim$ Nader
Nader	1.64 %	Nader $\succ$ Gore $\succ$ Bush

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  - ◊ An absolute majority of voters prefer Gore over Bush and Nader over Bush, Bush is a Condorcet-Loser
- Infamous real world examples exist... like the United States presidential election in Florida, 2000

Research questions

RQ1: Coordination Failures and Condorcet-Efficiency?

RQ2: Informational Structure?

RQ3: Individual level of sophistication?

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  Is coordination efficient, i.e., does coordination take place on the Condorcet-Winner?

RQ2: Informational Structure?

RQ3: Individual level of sophistication?

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RQ3: Individual level of sophistication?

- How strategic do voters act?
  What is the impact of the underlying information structure on these results?

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Why Lab experiments?

- Field Experiments:
  - Offer invaluable data and evidence for the actual feasibility, and show that changes in voting methods alter the results, and that the methods are well accepted by voters (see Alós-Ferrer and Granić (2012), Baujard and Igersheim (2009) and Laslier and Van der Straeten (2008))
  - Suffer from potential self-selection biases and lack of fully identifying participants' preferences
- Laboratory Experiments:
  - Controlled environment allows us to test certain properties that cannot be tested in the field
  - Design of the experiment is based on Forsythe et al. (1993) and Forsythe et al. (1996)
  - Experiments with single-peaked preferences and spatial representation: Dellis et al. (2010), Van der Straeten et al. (2010)

Design of the Experiment

Design

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- Voting methods:
  - Approval Voting (AV): Each voter can approve of as many alternatives as he/she likes. The alternative with the most approvals wins the election
  - Borda Count (BC): Each voter distributes 3, 2, 1, and 0 points among the alternatives. The alternative with the most points wins
  - Plurality Voting (PV): Each voter can cast one vote, a simple majority is enough to win the election

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Design

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- Voting methods:
  - Approval Voting (AV)
  - Borda Count (BC)
  - Plurality Voting (PV)
- Information structure:
  - Full information (FI): Participant know the payoffs (not the identities) of their group members
  - Incomplete information (II): Participant know their own payoff only (more on this later)

Design contd

- Each session: 28 participants, randomly divided into 4 groups (7 participants each)
- Each group participates in 8 elections with 4 available alternatives
- Participants are informed about the election results and their corresponding payoffs
- After 8 elections: randomly reassign the participants into 4 new groups and another series of 8 elections starts
- Each participant plays 3 series of 8 elections (96 elections per session in total)
- The experiment was conducted in the University of Konstanz' own computer laboratory (Lakelab) using the computer software z-Tree (Fischbacher, 2007)

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Induced Preference Profile

Number of Participants	Payoffs in ECU A	B	C	D	Induced Preferences
2	100	40	60	80	$A \succ D \succ C \succ B$
3	40	100	60	80	$B \succ D \succ C \succ A$
2	60	40	100	80	$C \succ D \succ A \succ B$

- Condorcet-Winner and Condorcet-Loser
  - D is the unique Condorcet-Winner, it beats every other alternative in a pairwise comparison
  - B is the unique Condorcet-Loser, it loses against every other alternative in a pairwise comparison

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- In light of RQ1:
  - Coordination failures arise if B wins an election, B should win less often under AV and BC than under PV
  - Coordination should take place on the Condorcet-Efficient alternative D

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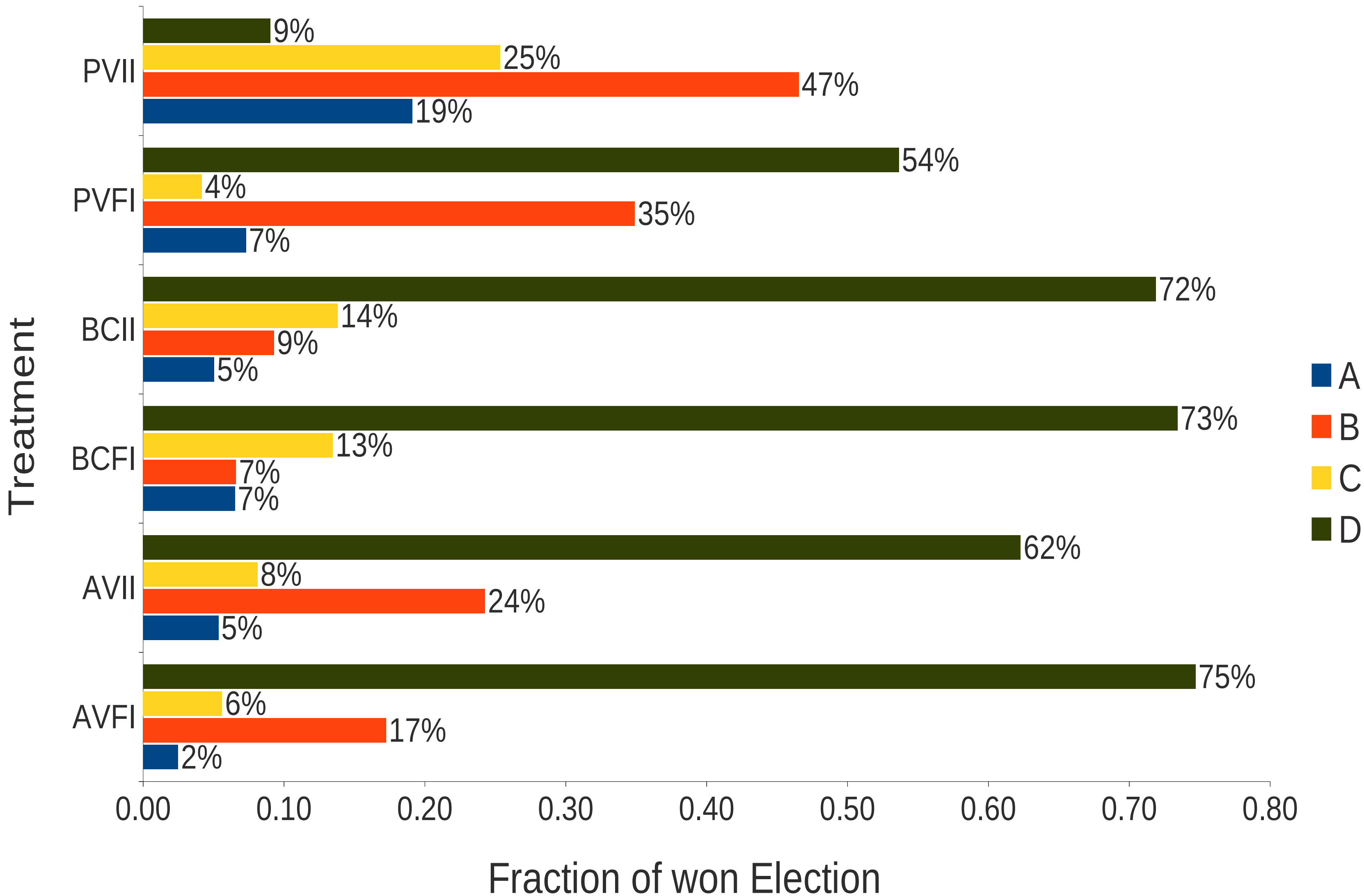
Results

Aggregate Data: Election Outcomes



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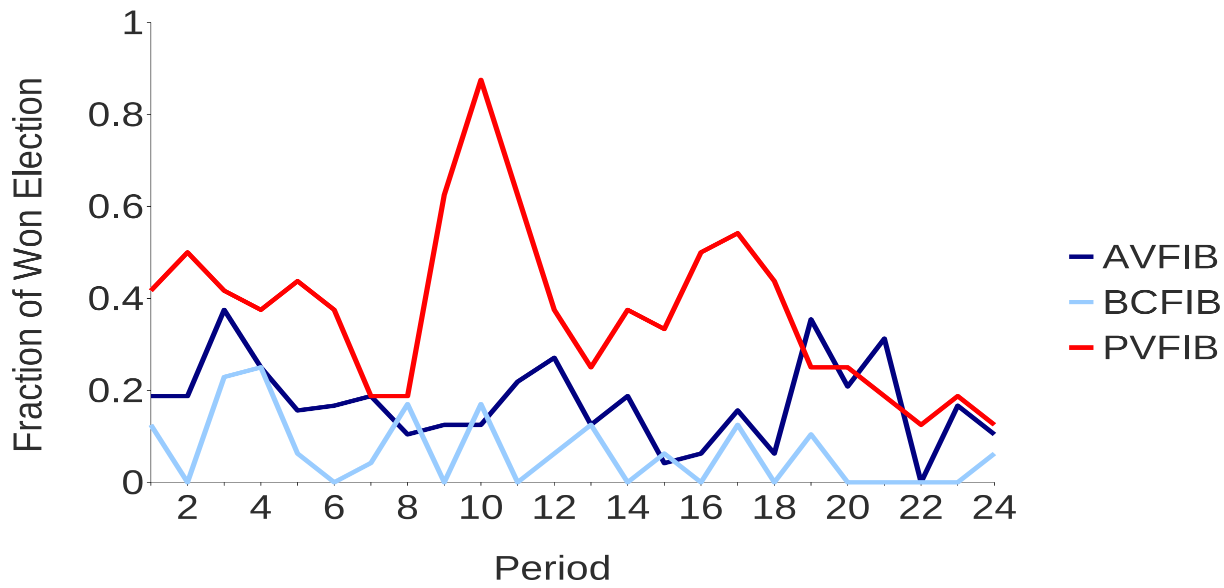
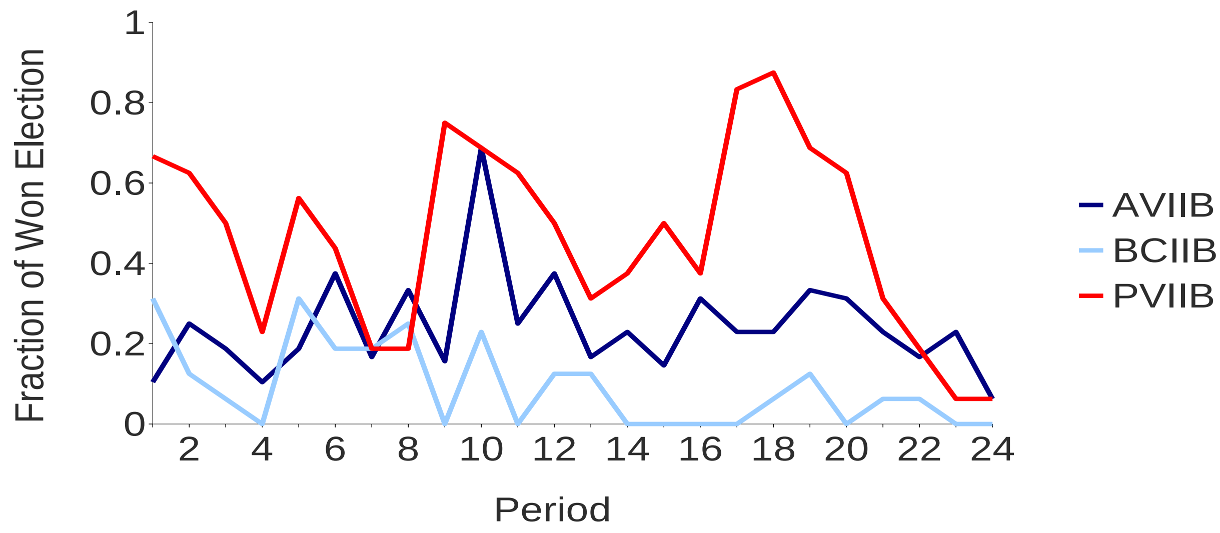
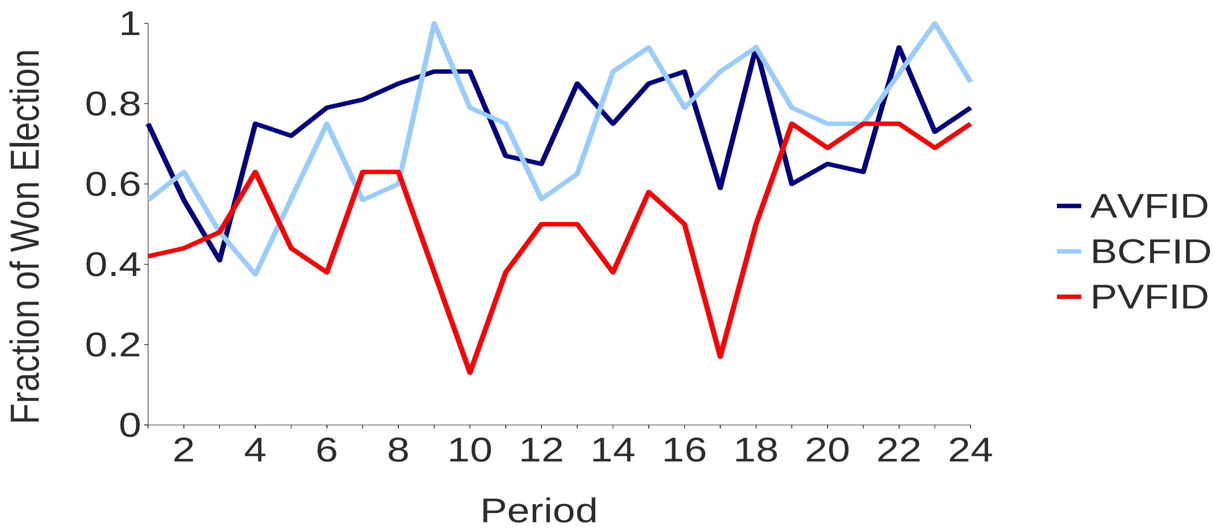
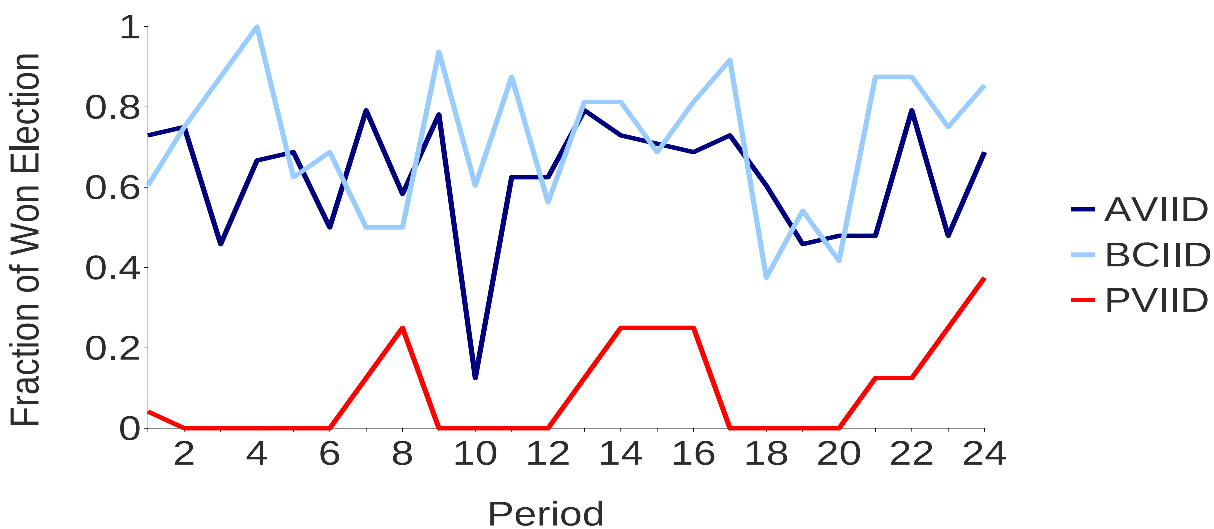
Aggregate Data: Coordination Failures



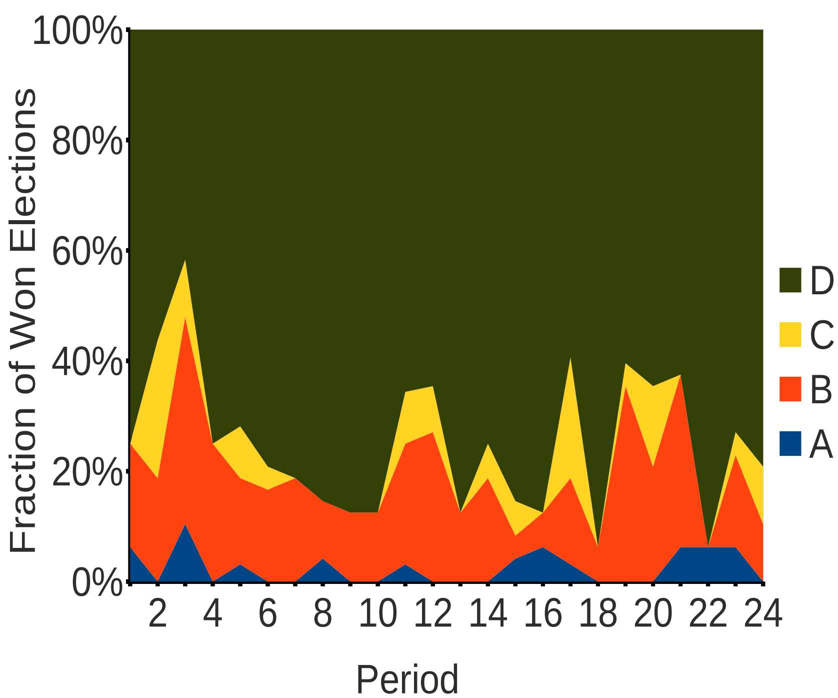


Aggregate Data: Condorcet Efficiency

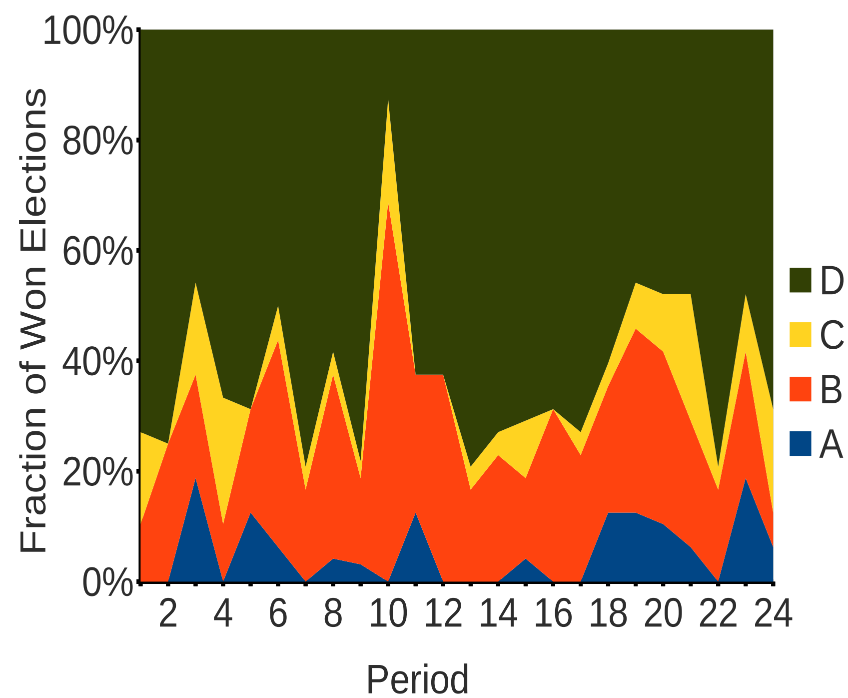




Aggregate Data: AV

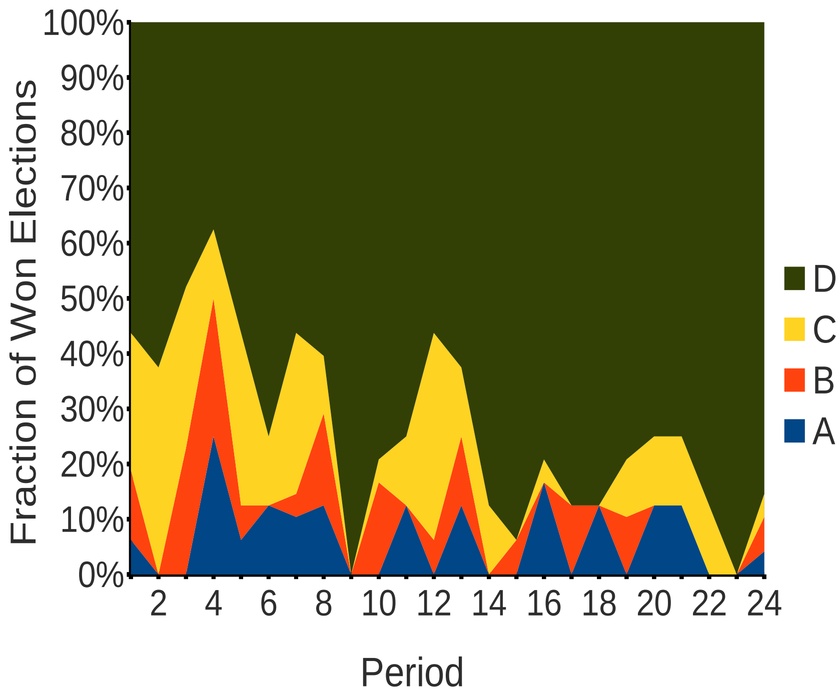


(a) AVFI

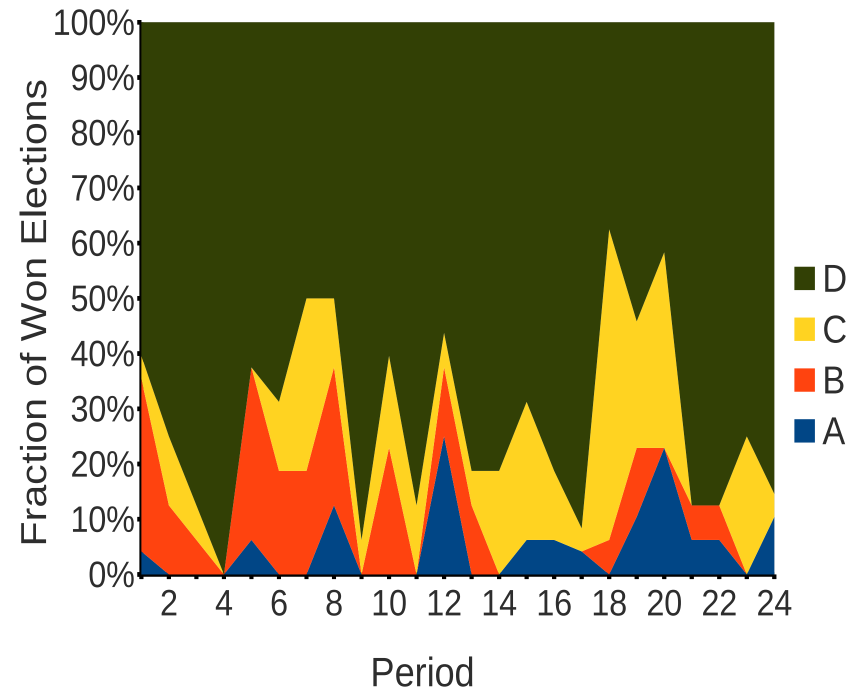


(b) AVII

Aggregate Data: BC

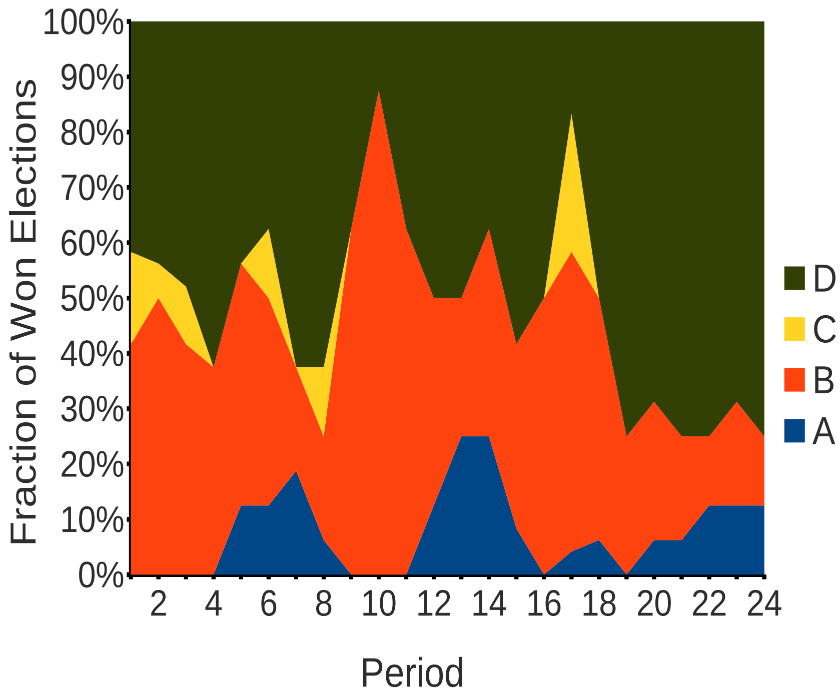


(c) BCFI

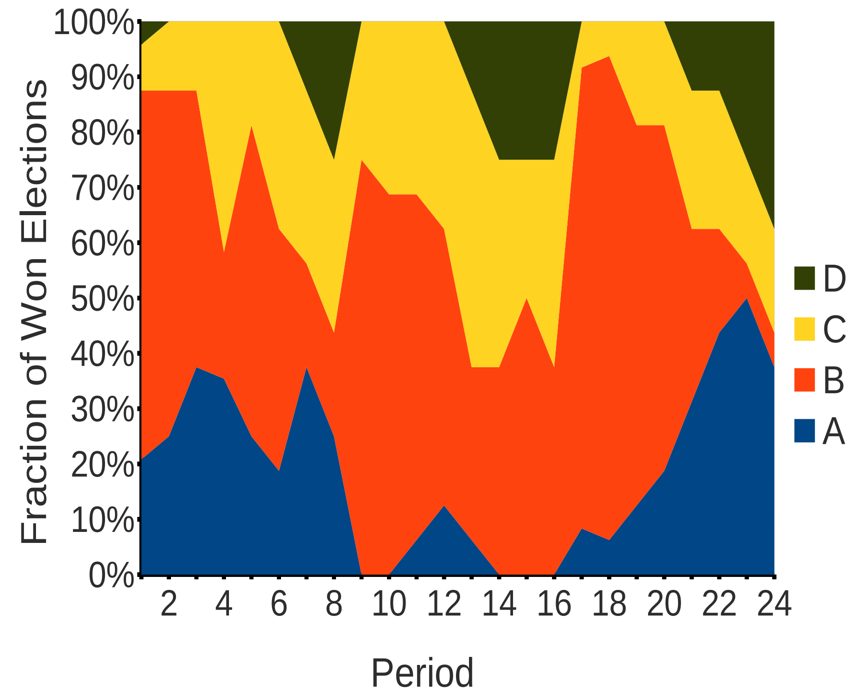


(d) BCII

Aggregate Data: PV



(e) PVFI



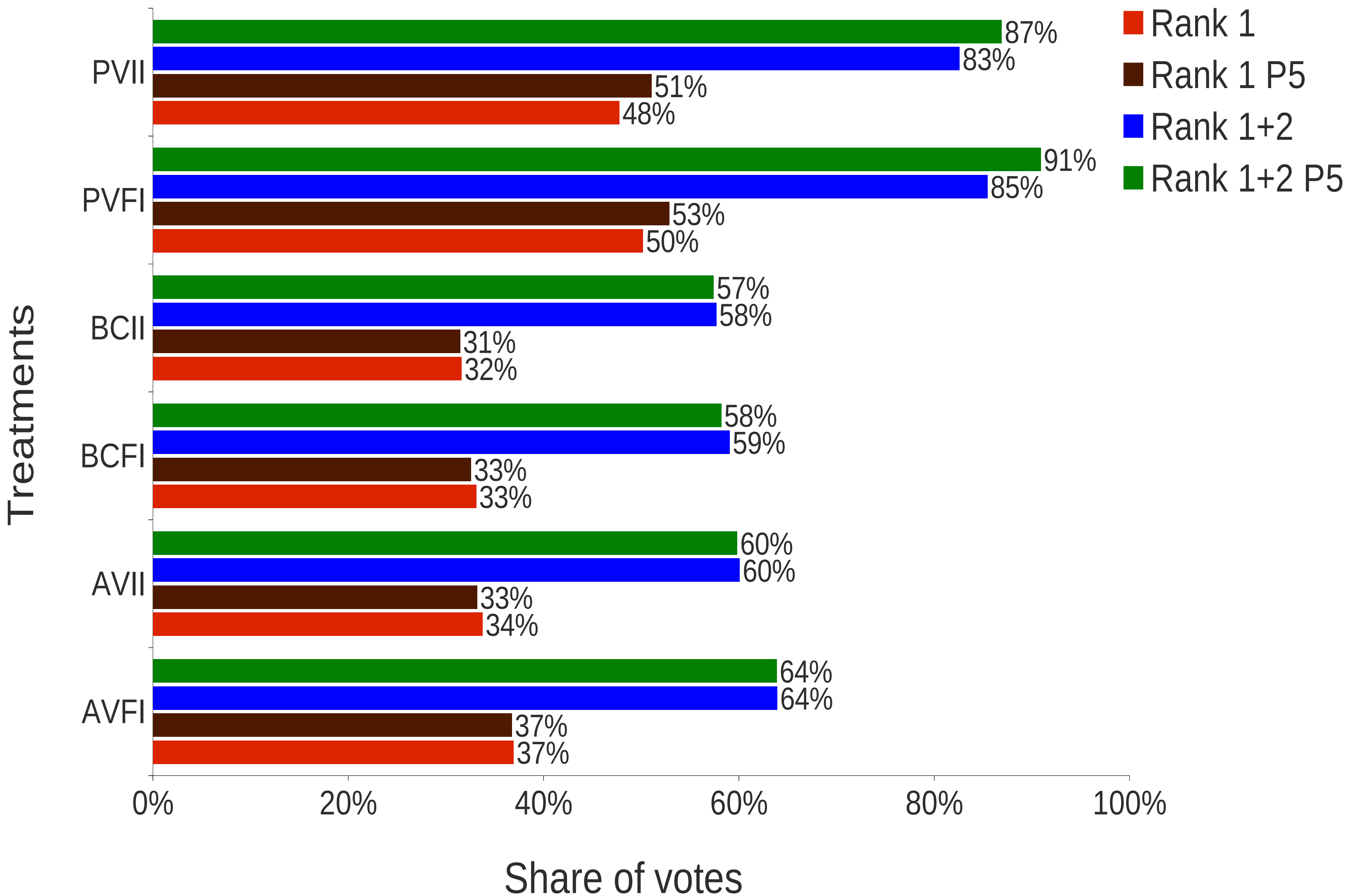
(f) PVII

Ties, Close Races, Duverger's Law

	No Ties	Two-Way Ties	Three-Way Tie	Four-Way Tie
AVFI	139	39	11	3
AVII	124	45	20	3
BCFI	159	20	11	2
BCII	159	27	6	0
PVFI	118	38	4	0
PVII	132	55	5	0

- AV creates more ties than BC and PV (Kruskal-Wallis, weakly significant for FI, p-value=0.082, highly significant for NI, p-value=0.001)
- Change from FI to II increases Ties for AV (WRS, p-value=0.087)

Ties, Close Races, Duverger's Law



Individual Voting Behaviour

- AV does not degenerate to PV: irrespective of information treatment, average approvals » 1
- Strategic voting:
  - Under FI, fraction of sincere ballots cast under AV: 83.26%. Under PV: 51.30%. Under BC: 41.96%
  - Under NI, fraction of sincere ballots cast under AV: 93.01%. Under PV: 75.82%. Under BC: 46.5%
- No impact on information structure on sincere voting for AV and BC. As in other studies, under PV and uncertainty sincerity increases

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Conclusion

- Multi-votes methods ('One Man, many Votes') like AV and BC facilitate coordination among the divided majority groups
- Coordination failures are not only reduced effectively, multi-votes methods also increase coordination efficiently as indicated by the corresponding large winning frequencies of the Condorcet-Winner
- Coordination on the Condorcet-Winner is much harder to establish under a single-vote method than under a multiple-vote method. The limited amount of information that is transmitted through a Plurality Voting ballot hinders coordination
- Informational structure (i.e., responsiveness towards it) may serve as another dimension to evaluate the merits of voting methods

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Thank you for your attention

0.1 Bibliography

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Approval Voting

- Approval Voting (AV): Proposed by Steven J. Brams and Peter C. Fishburn (1977)
- Each voter can assign 1 or 0 votes to each candidate. That is, “approve of” as many candidates as wished. The candidate with the most approvals wins
- Arguments in the literature: AV provides an accurate reflection of voters’ wishes and is not vulnerable to voter manipulation (see Brams and Fishburn, 1978; Fishburn, 1978a,b; Brams and Fishburn, 2005; Wolitzky, 2009)

Preliminary Work: Field Experiments

- Get permission from State and Federal Authorities *This was funny.*
- Inform all involved registered voters per mail prior to the election, explain the method. *This was expensive*
- Election day: established one experimental polling station in each of the preselected constituencies (same building, different room). *This was a lot of work*

  Use official ballot boxes and voting urns.
- After casting a ballot in the official polling stations, a “certificate“ was handed over to the voters by the polling clerks which qualified them for participation in the experiment

  Guarantees undisturbed official election and that we only got actual voters; but allows for a serious drop-off and maybe self-selection effects

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2008 State election in Hesse

1909 eligible voters went to the polls, of which, in turn, 967 participated in our experiment (participation rate 50.7%). With 6 invalid votes, the data set consists of 961 AV ballots.

2008 State election in Hesse

Party	Approvals	AV Rank	Official Votes	PV Rank
SPD	53,8 %	1	38,9 %	1
CDU	44,6 %	2	36,0 %	2
The Greens	36,1 %	3	7,0 %	4
FDP	32,6 %	4	9,0 %	3
The Left	12,3 %	5	4,9 %	5
Animal Protection Party	9,6 %	6	0,8 %	7
The Family Party	9,6 %	6	0,2 %	12
The Free Voters	7,1 %	8	0,5 %	9
The Republicans	3,3 %	9	1,0 %	6
The Popular Vote	2,9 %	10	0,2 %	13
NPD	2,8 %	11	0,8 %	7
The Hessian Pirates	2,8 %	11	0,3 %	10
The Grey Party	2,5 %	13	0,2 %	13
UB	2,1 %	14	0,1 %	15
The Violet Party	1,0 %	15	0,3 %	11
PSG	0,9 %	16	0,1 %	15
Civil Liberties Party	0,9 %	16	0,1 %	15
Total	225,0 %		100,0 %	

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