Frequency Based Analysis of Voting Rules

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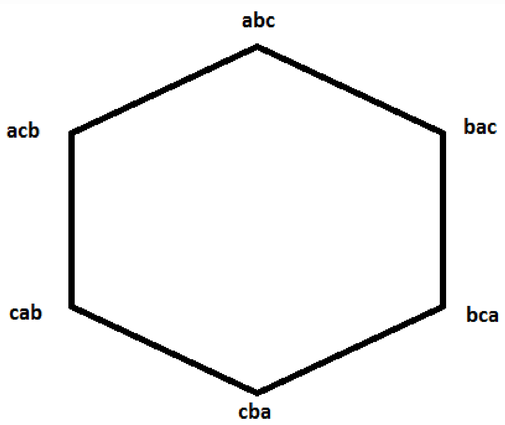
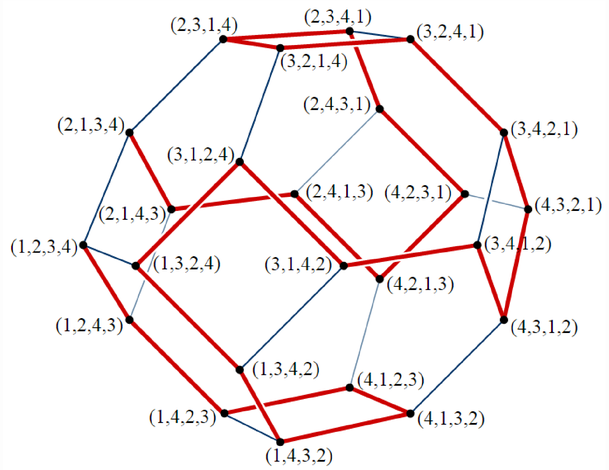
1. Introduction

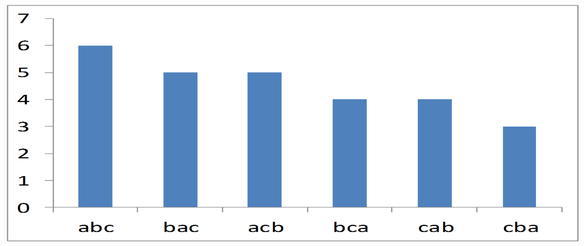
Real life voting problems deal with large electorates. To find the collective decision a realistic way would be to analyze frequency distribution of the preferences. First we consider a particular type of distribution (unimodal), we observe that many well known rules choose mode as the outcome. We find a set of properties of the rules that is sufficient to choose mode under unimodal distribution. Then we allow for perturbation in the tail of the distribution. Interestingly we find that still pairwise rule, Borda rule, plurality rule choose mode as the outcome under certain kind of perturbations in the tail. We also analyze multimodal distributions as a superposition of unimodal distributions.

2. Preference Space





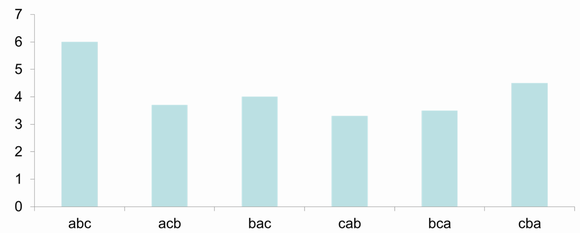
3. Unimodal Distribution



For a unimodal distribution all well known rules like : **Condorcet like rules, Scoring rules, Coombs rule, Kemeny like rules** choose **mode** as the outcome. We can characterise these rules by the following conditions.

- **Monotonicity** means that if in comparison of profiles p and q the preference among the agents increases when going from p to q, then this preference at the outcomes should not decrease.
- **Discrimination** means that at a profile p where for two alternatives x and y every linear ordering R at which x is strictly preferred to y strictly outnumber linear ordering $\tau_{xy}R$ at which y is strictly preferred to x the preference rule cannot be indifferent between x and y.
- It is natural to go one step further and to impose that in those situations x should be strictly preferred to y. This condition is referred to as **positive discrimination**.

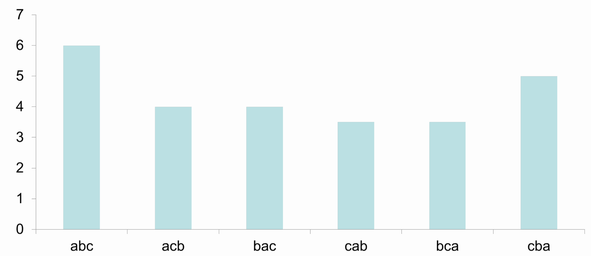
4. Disturbances



Pairwise Rule For a distribution with

- Unimodal structure
- Possibly with unequal frequencies for all the preferences having same distance from the mode.
- Larger frequencies for the first half of the distribution compared to the respective second half of the distribution.

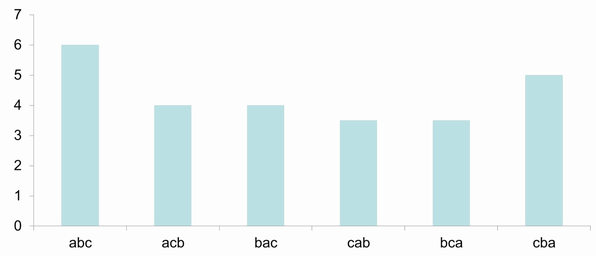
selects *mode* as the outcome.



Borda Rule For a distribution with

- Unimodal structure
- Equal frequency for all the preferences having same distance from the mode.
- Larger frequencies for the first half of the distribution compared to the respective second half of the distribution.

selects *mode* as the outcome.



Plurality Rule For a distribution with

- Unimodal structure
- Equal frequency for all the preferences having same distance from the mode.
- Larger frequencies for the first portion of the distribution compared to the respective second portion of the distribution.
- First portion is till $k^{max} = \lceil \frac{1}{2}\binom{m-1}{2} \rceil + m - 2$.

selects *mode* as the outcome.

4. Multimodal Distribution

- We define multimodal distributions as superimpositions of several unimodal distribution.
- Consider a distribution formed by superimposition of two unimodal distribution. If (x, y) belongs to the intersection of two modes then (x, y) is chosen as the outcome.
- The same holds true for a distribution formed by superimposition of more than two unimodal distribution.
- It is clear to see that if the modes are closely related (w.r.t. Kemeny distance) then it is easier to identify the outcome.
- If the modes are far away from each other then not much can be said about the choices made by the rule. In the extreme case when modes are exactly opposite then nothing can be said.

Contact

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References

The Strategy of Social Choice-By H. Moulin.