Inequality Indices

in Multi-Agent Resource Allocation - A Distributed Approach

(Summer School on Computational Social Choice, San Sebastian 2016)

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[a]joint work with Britta Dorn & Ulle Endriss

Setting

Definition. *(MARA-framework:) We consider a finite set of agents* $\mathcal{N} = \{1, \dots, n\}$ *and a finite set* $\mathcal{G}$ *of goods, where every agent* $i \in \mathcal{N}$ *has preferences over all possible bundles of goods* $B \in 2^{\mathcal{G}}$ *given by* utility functions *from the set* $\mathcal{U} = \{u_i : 2^{\mathcal{G}} \to \mathbb{R}^+ : i \in \mathcal{N}\}$.

Definition. *(Fairness I) Maximising* social welfare

- $sw_{util}(A) = \sum_{i=1}^{n} u_i(A(i))$
- $sw_{nash}(A) = \prod_{i=1}^{n} u_i(A(i))$

Definition. *(Fairness II) Minimizing* inequality

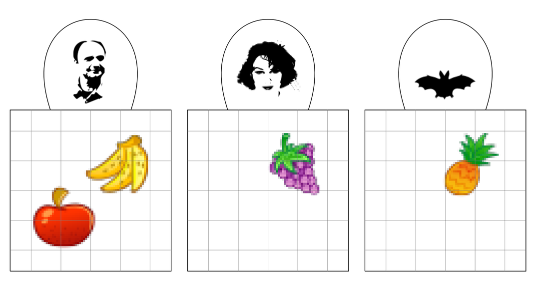
$$I_{nash}(A) = 1 - \frac{\sqrt[n]{\prod_{i=1}^{n} u_i(A)}}{\frac{1}{n}\sum_{i=1}^{n} u_i(A(i))}$$

Consider the two scenarios $\langle \mathcal{N}, \mathcal{G}, \mathcal{U}^1 \rangle$ and $\langle \mathcal{N}, \mathcal{G}, \mathcal{U}^2 \rangle$ with $\mathcal{N} = \{$Alfred, Rachel, Bruce$\}$, $\mathcal{G} = \{$🍎, 🍌, 🍇, 🍍$\}^*$ and the two sets $\mathcal{U}^1$ and $\mathcal{U}^2$ of additive utility functions :

$\mathcal{U}^1$:	🍎	🍌	🍇	🍍	$\mathcal{U}^2$:	🍎	🍌	🍇	🍍
Alfred :	2	1	3	4	Alfred :	2	1	3	4
Rachel :	2	5	2	1	Rachel :	2	5	2	1
Bruce :	1	2	1	6	Bruce :	3	2	3	2

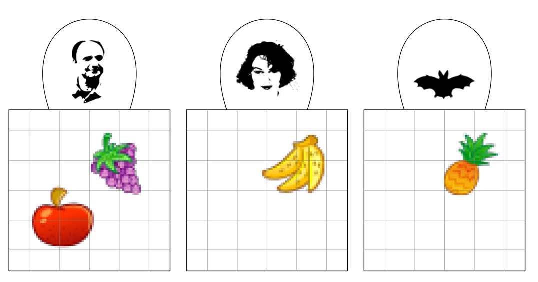
Now we will have a look at the allocations

A°



and

A^*



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	$\langle \mathcal{N}, \mathcal{G}, \mathcal{U}^1 \rangle$			$\langle \mathcal{N}, \mathcal{G}, \mathcal{U}^2 \rangle$		
	A°		A^*	A°		A^*
sw_{util}	11	<	16	7	<	12
sw_{nash}	36	<	150	12	<	50
I_{nash}	0.1	>	0.004	0.02	<	0.08

A° is fairer with respect to sw_{util}/sw_{nash} in both scenarios.
With respect to I_{nash}, A° is fairer in $\langle \mathcal{N}, \mathcal{G}, \mathcal{U}^1 \rangle$, but A^* is fairer in $\langle \mathcal{N}, \mathcal{G}, \mathcal{U}^2 \rangle$.

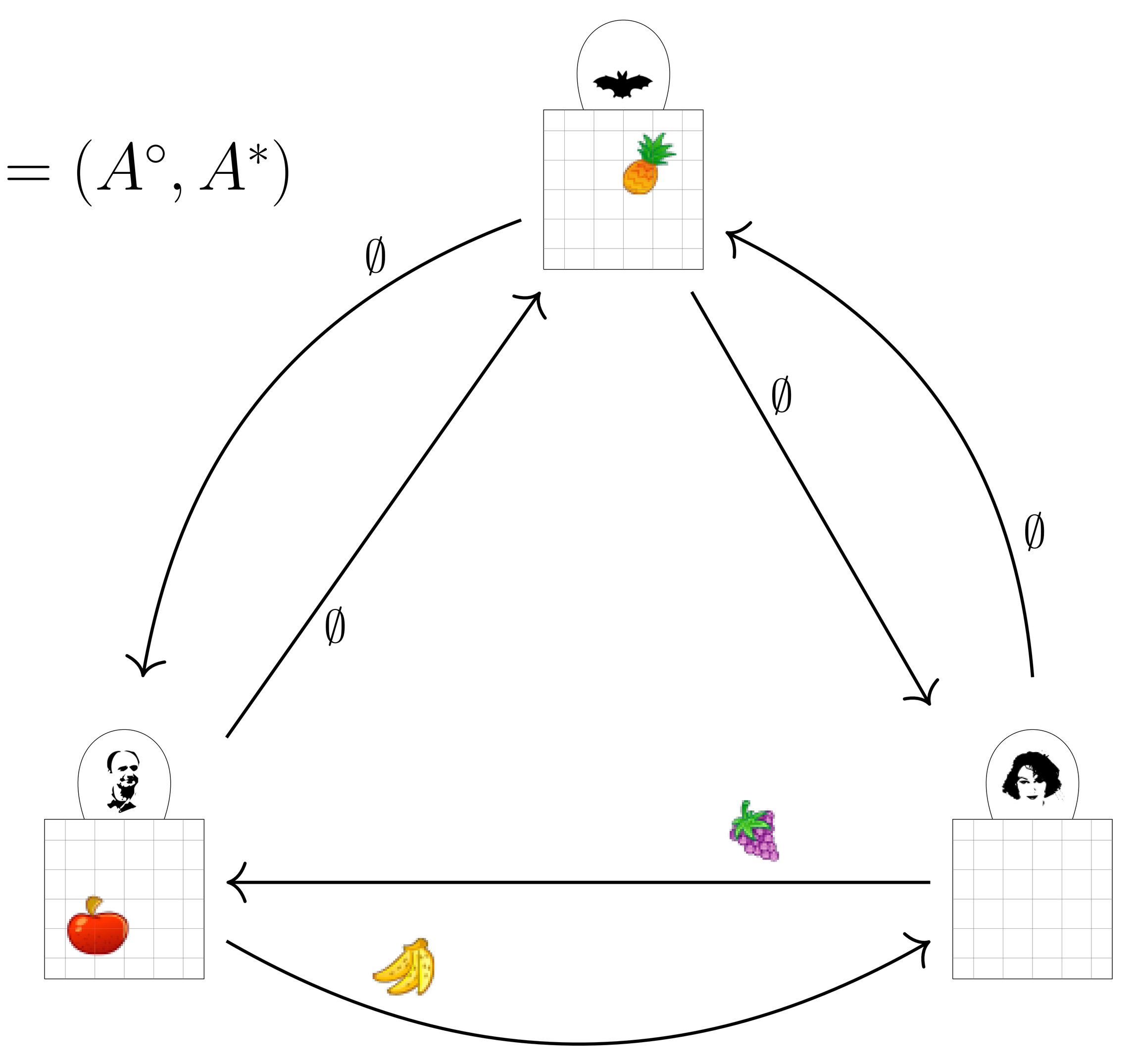
1 Distributed Approach

Idea: calculate an optimum not at once, but with a lot of "small" improvements, using only local data.

1.1 Deals

A *deal* δ is a tuple of two (distinct) allocations A and A'. The set of agents involved in a deal is denoted by $\mathcal{N}^\delta$.

$\delta = (A^\circ, A^*)$



Deal-sq $A^{(1)} \overset{\delta}{\rightsquigarrow} A^{(2)} \overset{\delta'}{\rightsquigarrow} A^{(3)} \overset{\delta''}{\rightsquigarrow} \dots$
with $sw(A^{(1)}) < sw(A^{(2)}) < sw(A^{(3)}) < \dots$
or $I(A^{(1)}) > I(A^{(2)}) > I(A^{(3)}) > \dots$

Definition. *A deal* $\delta = (A, A')$ *is called* nash rational *iff*

$$\prod_{i \in \mathcal{N}^\delta} u_i(A) < \prod_{i \in \mathcal{N}^\delta} u_i(A').$$

Theorem. *Any sequence of nash rational deals will eventually terminate in an allocation with max* sw_{nash}.[†]

Problem for inequality indices: there is no local rationality criterion in the classical sense.

⇝Trick: calculate $\sum_{i \in \mathcal{N}} u_i(A(i))$ with local information.

$$M(A) = \sum_{i \in \mathcal{N}} u_i(A(i))$$
$$M(A') = M(A) + \sum_{i \in \mathcal{N}^\delta} \big(u_i(A') - u_i(A)\big)$$

Definition. *A deal* $\delta = (A, A')$ *is called* Atkinson index rational *(AIR) iff*

$$\frac{\sqrt[n]{\prod_{i \in \mathcal{N}^\delta} u_i(A)}}{M(A)} > \frac{\sqrt[n]{\prod_{i \in \mathcal{N}^\delta} u_i(A')}}{M(A')}.$$

Theorem. *Any sequence of* AIR*-deals will eventually terminate in an allocation with min* I_{nash}.

2 Results

2.1 Necessary Deals

Theorem. *For every deal* $\delta = (A, A')$ *there exist utility functions* $(u_i)_{i \in \mathcal{N}}$ *and a starting allocation, such that the deal* δ *is necessary for reaching an allocation with a minimal possible value of* I_{nash}.

2.2 Communication Complexity

Theorem. *A sequence of* AIR *deals can consist of at most* $|\mathcal{N}|^{|\mathcal{G}|} - 1$ *deals.*

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[†]S. Ramezani and U. Endriss. Nash social welfare in multiagent resource allocation. In *Proceedings of the 11th International Workshop on Agent-Mediated Electronic Commerce (AMEC-2009)*, May 2009.