Hedonic Games with Graph Restricted Communication

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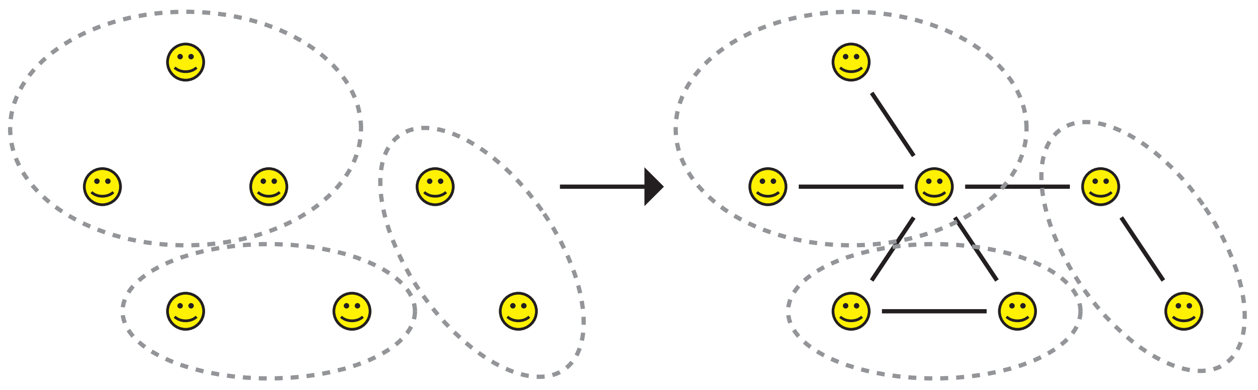
Hedonic Games with Graph Structure

- Players have complete preferences over subsets. (e.g. $\{1,2,3\} \succ_1 \{1,2\} \succ_1 \{1,3\} \succ_1 \{1\}$)
- Question: which partitions are stable?

Standard model $(N, (\succeq_i)_{i \in N})$ **Our model** $(N, (\succeq_i)_{i \in N}, L)$

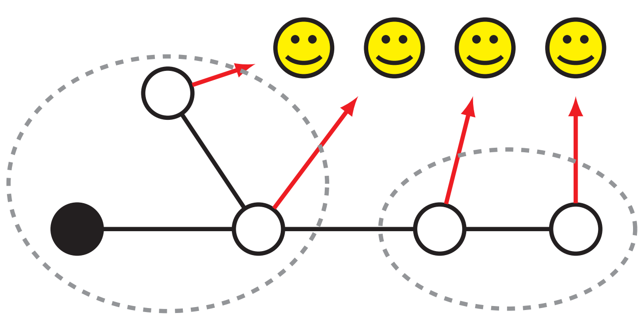


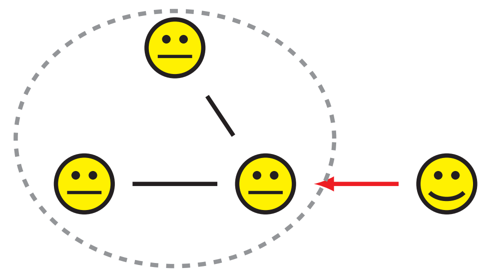
No restrictions on coalition formation

Only connected subsets can form a coalition. ($\mathcal{F}_L$ = the set of connected subsets of a graph)

Stability concepts

Core stability (CR) **Individual stability (IS)**



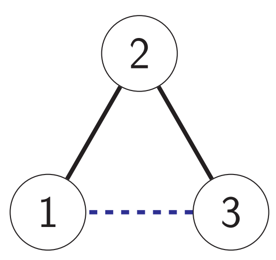


- A partition π of N is core stable if no feasible coalition can profitably deviate: $\nexists S \in \mathcal{F}_L : S \succ_i \pi(i), \forall i \in S$.
- A partition π of N is individually stable if no player can profitably deviate to another group without hurting some members of the group: $\forall i \in N$, $\forall S \in \pi \cup \{\emptyset\}$, $S \cup \{i\} \in \mathcal{F}_L \wedge S \cup \{i\} \succ_i \pi(i) \Rightarrow \exists j \in S : S \succ_j S \cup \{i\}$.

Stable partitions may not exist.

$N = \{1,2,3\}$, $L_a = \{\{1,2\},\{2,3\},\{1,3\}\}$, $L_b = L_a \setminus \{1,3\}$

$1 : \{1,2\} \succ_1 \{1,3\} \succ_1 \{1,2,3\} \succ_1 \{1\}$
$2 : \{2,3\} \succ_2 \{1,2\} \succ_2 \{1,2,3\} \succ_2 \{2\}$
$3 : \{1,3\} \succ_3 \{2,3\} \succ_3 \{1,2,3\} \succ_3 \{3\}$



- The game $(N, (\succeq_i)_{i \in N}, L_a)$ has neither a core stable partition nor an individually stable partition.
- The game $(N, (\succeq_i)_{i \in N}, L_b)$ has a core and individually stable partition $\{\{1\},\{2,3\}\}$.

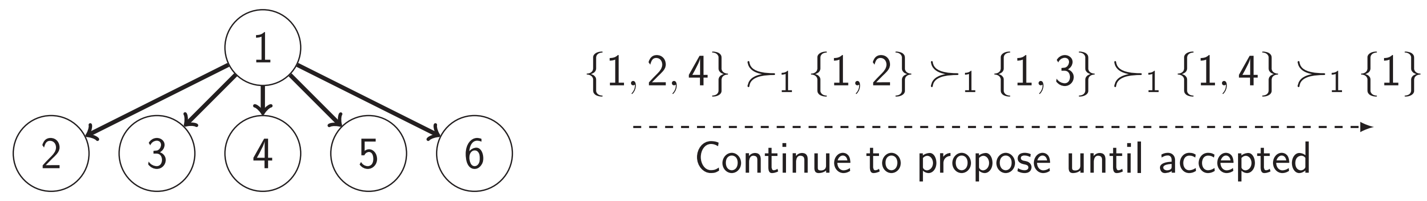
Acyclic games: CR and IS existence

Every game $(N, (\succeq_i)_{i \in N}, L)$ admits a CR and IS partition if and only if (N, L) is acyclic [Igarashi & Elkind, 2016]. (The core version was proven in [Demange, 2004]).

Acyclic games: Algorithm for CR

A core stable partition can be constructed in time poly($|\mathcal{F}_L|$) if (N, L) is acyclic [Demange, 2004].

- Basic idea: Construct a rooted tree and compute a CR partition of each subtree. Each subroot i proposes to a coalition from the most preferred to the least preferred until all the subordinates of the coalition accept it.
- Example: Star graphs

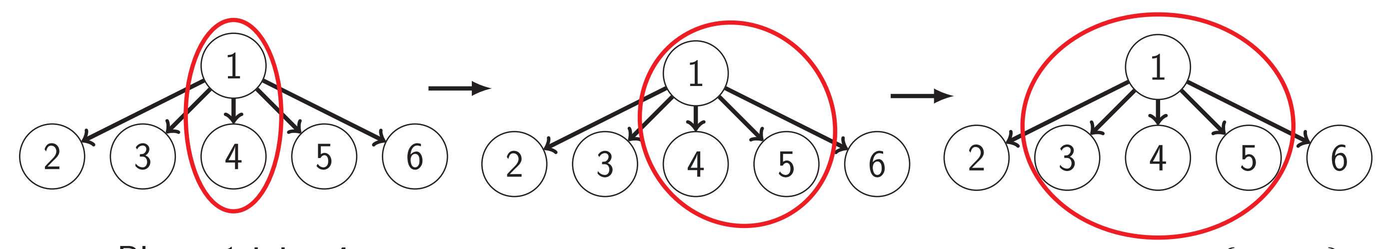


The algorithm may require an exponential number of steps but is perhaps optimal. Indeed, it is NP-hard to compute CR for additive games on stars [Igarashi & Elkind, 2016].

Acyclic games: Algorithm for IS

IS can be constructed in time poly($|N|$) if (N, L) is acyclic [Igarashi & Elkind, 2016].

- Basic idea: Construct a rooted tree and compute an IS partition of each subtree. Each subroot i moves to the most preferred coalition to which i can deviate. Then, keep adding a player outside of the coalition if she can deviate to i's coalition.
- Example: Star graphs



Player 1 joins 4 — Add player 5 to $\{1,4\}$ — Add player 3 to $\{1,4,5\}$

Almost acyclic games: Tractability results

Stable partitions that are resistant to individual deviations can be computed in polynomial time for IRLC and anonymous games whose underlying graph has bounded treewidth [Igarashi & Elkind, 2016].

Summary: computational results

(N, L)	Complete		B-Treewidth		Tree		
$\succeq_i$	Additive	IRLC	Additive	IRLC	Compact	Additive	IRLC
SCR	Σ_2^p-h	NP-c	NP-h	?	NP-h	NP-h	?
CR	Σ_2^p-h	NP-c	NP-h	?	NP-h	NP-h	P
NS	NP-c	NP-c	NP-c	P	NP-c	NP-c	P
INS	NP-c	NP-c	NP-c	P	NP-c	NP-c	P
IS	NP-c	NP-c	NP-c	P	P	P	P

Blue: previous results Red: our results