Locating Two Public Bads in an Interval

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1. Problem

- We consider a region, which can be modelled by a line segment. For example, consider the coast line of Bay of Biscay in Basque country.

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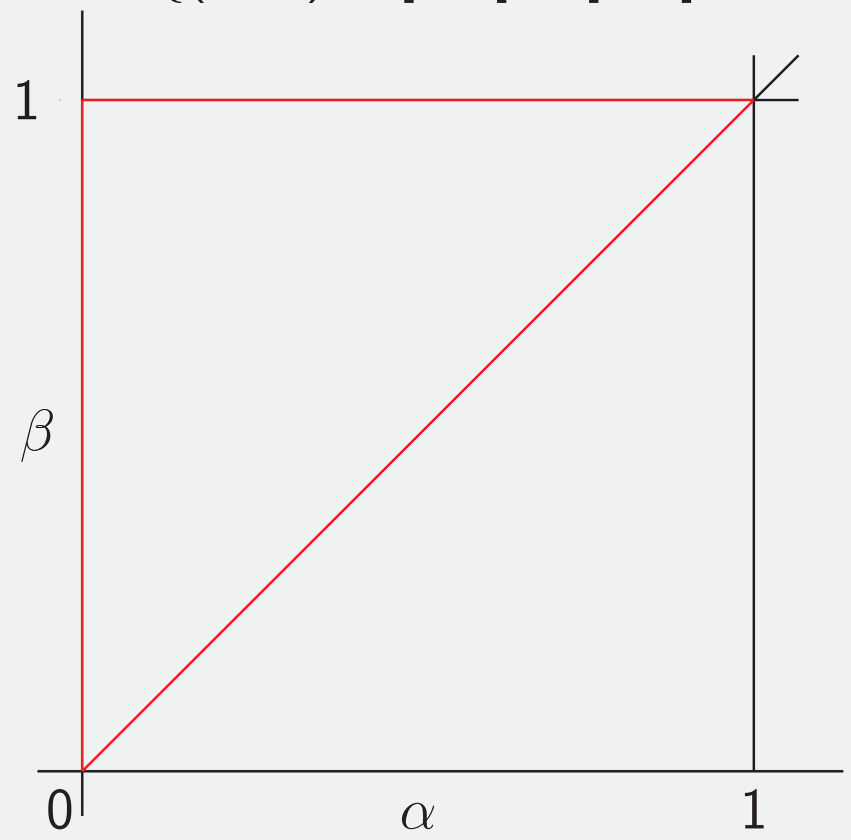
Set of residents N, with cardinality n

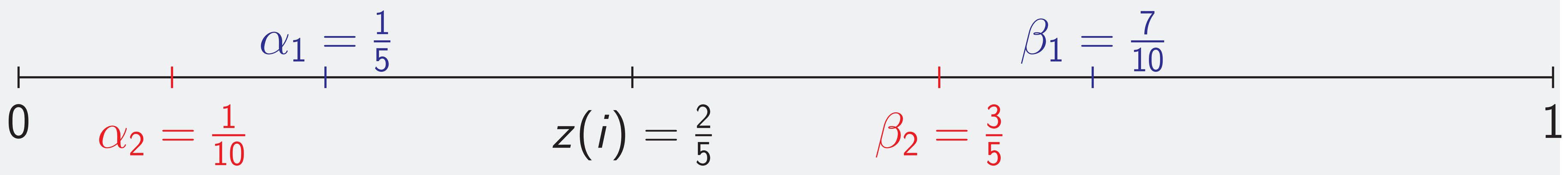
- The objective is to locate two public bads (noxious facilities) along this line.
Set of alternatives : $\mathcal{A} = \{(\alpha, \beta) \in [0,1] \times [0,1] : \alpha \leq \beta\}$.



- Residents' preferences are single dipped.
- We characterise all possible solutions of this location problem which satisfy certain desirable properties.

2. Preferences (Lexicographic single dipped)

- Each agent i has a preference characterised with a dip $z(i)$ such that i prefers (α_1, β_1) to (α_2, β_2)
  If the closest bad in $\{\alpha_1, \beta_1\}$ w.r.t. $z(i)$ is more remote to $z(i)$ than that of $\{\alpha_2, \beta_2\}$.
  Or if these are on equal distance to $z(i)$, but the other bad of $\{\alpha_1, \beta_1\}$ is more remote to $z(i)$ than the other bad of $\{\alpha_2, \beta_2\}$.



- $R_{z(i)}$, $P_{z(i)}$ and $I_{z(i)}$ denotes the weak, strict and indifference part of the preference.

3. Objective

In this context,

- A profile $z := (z(1), z(2), \ldots, z(n))$.
- A social choice function $f(z) = (\alpha(z), \beta(z))$.

We characterise the class of social choice functions that satisfies following properties.

4. Strategy-Proofness

No agent can benefit by unilaterally misreporting his dip.
Consider two profile z and z' which differ only in agent i's dip.
$f(z) R_{z(i)} f(z')$ and $f(z') R_{z'(i)} f(z)$.

5. Pareto Optimality

A social choice function f is Pareto optimal if for every profile z there is no alternative $a \in \mathcal{A} \backslash \{f(z)\}$ such that $a R_{z(i)} f(z)$ holds for all $i \in N$ with $a P_{z(j)} f(z)$ for atleast one $j \in N$.

Theorem 1 : No internal solutions

If f satisfies strategy-proofness and Pareto optimality, then $f(z) \in \{(0,0), (1,1), (0,1)\} := \mathcal{B}$ for all profiles z.

Characterisation

For any profile z, define $S(z) = \{i \in N : z(i) < \frac{1}{2}\}$ and $T(z) = \{i \in N : z(i) = \frac{1}{2}\}$.

Strategy-proof but not Pareto optimal rule

Consider a rule $h(z) = (\alpha_1(z), \beta_1(z))$ as follows

$$\alpha_1(z) = \begin{cases} 1 & \text{if } |S(z) \cup T(z)| \geq \frac{3}{4}n \\ 0 & \text{otherwise.} \end{cases}$$

$$\beta_1(z) = \begin{cases} 1 & \text{if } |S(z) \cup T(z)| \geq \frac{n}{4} \\ 0 & \text{otherwise.} \end{cases}$$

Strategy-proof and Pareto optimal rule

Consider another rule $g(z) = (\alpha_2(z), \beta_2(z))$ as follows

$$\alpha_2(z) = \begin{cases} 1 & \text{if } S(z) \cup T(z) = N \text{ and } S(z) \neq \emptyset \\ 1 & \text{if } |S(z)| \geq \frac{3}{4}n \text{ and } T(z) \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

$$\beta_2(z) = \begin{cases} 1 & \text{if } S(z) \cup T(z) = N \text{ and } S(z) \neq \emptyset \\ 1 & \text{if } |S(z)| \geq \frac{n}{4} \text{ and } T(z) \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

Strategy-proof, Unanimous but not Pareto optimal rule

$$N = \{i, j\}$$

$$h^{(\alpha,\beta)}_{(1,1)}(z) = \begin{cases} (1,1) & \text{if } z(k) \leq \frac{1}{2} \text{ for all } k \in N \\ (0,0) & \text{if } z(k) \geq \frac{1}{2} \text{ for all } k \in N \\ & \quad \text{with } z(l) > \frac{1}{2} \text{ for atleast one } l \in N \\ (\alpha, \beta) & \text{if } 2\min\{z(i), z(j)\} < \alpha \\ & \quad \text{and } \beta < 2\max\{z(i), z(j)\} - 1 \\ (0,1) & \text{otherwise} \end{cases}$$

References

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