COST Action IC1205 on Computational Social Choice: STSM Report
Applicant: Aris Filos-Ratsikas
Home institution: Aarhus University
Home country: Denmark
Host: Ioannis Caragiannis
Host institution: University of Patras
Host country: Greece
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During my visit to the University of Patras, I worked together with Ioannis Caragiannis, Simina Branzei and Panagiotis Kanellopoulos on problems involving fair division in markets, specifically envy-free pricing of markets with indivisible items. Our discussions have resulted in a solid problem description with clear questions to be answered and some preliminary results. Our goal is to continue working on the subject and write a paper on it in the near future.

In more detail, the problem is the following. There is a set of n agents and a set of m indivisible items. Each agent has a valuation function over subsets of items, which in general maps every possible subset of items to a real-value. Each item is given a non-negative price. In an assignment, each agent receives a set (a bundle) of items and her utility is her value for the set minus the sum of prices for the items in that bundle. An envy-free pricing solution is an assignment and prices such that no agent would prefer to buy a bundle assigned to another agent at the given prices. We are looking for an envy-free solution that approximately maximizes the social welfare, i.e. the sum of valuations of agents for their assigned bundles.

This problem is related to the the well-known notion of market equilibrium in economics; a market equilibrium is an envy-free pricing solution where all items are sold (market clearing). When items are divisible, it has been proved that a market equilibrium exists; this is not the case however for indivisible items. By relaxing the market clearing condition, envy-free pricing solutions always exist, but they might not be very efficient (in terms of social welfare). We are looking for algorithms that produce approximately efficient solutions.

For general valuation functions, we came up with an algorithm that finds an envy-free pricing solution that is min (n,m) - approximate, which is (probably) the best that we can hope for in the general case. Then, we consider some natural restricted classes of valuation functions, namely single-minded valuations. An agent is single-minded if there is a subset of items S such that her value for S or any superset of S is v_i and 0 otherwise. As a special case, we consider the case of uniform single-minded valuations, where v_i = v for all agents i.

For single-minded valuations, there exists a lower bound of sqrt(m) on the approximation ratio of any envy-free pricing solution, with respect to the social welfare. Interestingly, this lower bound does not apply to the case of uniform single-minded valuations. For that case, we have proved that no envy-free pricing solution can be better than 2-approximate.

There are many questions that we would like to answer. First, for uniform single-minded valuations, is there an algorithm that achieves a 2 approximation (given that our lower bound is 2)? Can that algorithm run in polynomial time? Alternatively, given an optimal solution to the problem (one that maximizes social welfare) is there a polynomial-time algorithm that outputs an envy-free pricing 2-approximate solution? For the case of (non-uniform) single-minded valuations, is there an algorithm that produces an envy-free solution with a sqrt(m) approximation ratio? Is that algorithm polynomial time? What other classes of natural valuation functions can we consider and what kind of bounds can we prove?