COST Action IC1205 on Computational Social Choice: STSM Report

Applicant: Angelo Fanelli
Home institution: CNRS, University of Caen
Home country: France
Host: Edith Elkind
Host institution: Department of Computer Science, University of Oxford
Host country: UK
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The Price of Pareto Optimality in Fractional Hedonic Games

We primarily define the new concept of price of Pareto optimality, as the ratio of social optimum, (i.e., an assigment of strategies to players achieving optimal social welfare) to that of the social welfare of the Pareto optimal. This concept, as opposed to the standard notion of price of anarchy, may be thought of as a measure of the potential efficiency loss due to lack of individual or group selfishness, when players are concerned always with the overall social welfare, making sure that their strategy selections do not make any other individual worse off. We investigate the price of Pareto optimality in fractional hedonic games. Fraction hedonic games belong to the succinct class of hedonic games representable by a weighted directed graph where the weight for edge (i,j) denotes the value agent i has for agent j. The utility of an agent i in a coalition C, is given by the sum of the values that i has for any player in C, divided by |C|. We say that a fraction hedonic game is simple if the game is the weights of the edges are 1. Furthermore a fractional hedonic game is symmetric if the graph is undirected. The study of the efficiency of Pareto optimality in fractional hedonic games is motivated by the observation that Nash stable solutions may perform very badly, as observed in Bilo', Fanelli, Flammini, Monaco, Moscardelli, "Nash Stability in Fractional Hedonic Games" (WINE '14).

We observed that Pareto optimality in a fraction hedonic games may be sub-optimal, differently from standard hedonic games in which the ground coalition is both the social optimum and the unique Pareto optimal. In particular we show that for symmetric and simple fraction hedonic games when the underlying graph is a tree, the price of Pareto optimality is bounded by d+2+1/d, where d is the maximum degree of any node in the graph. Conversely, in general graphs the bound grows to d(d+1).