

COST Action IC1205 on Computational Social Choice: STSM Report

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I was hosted for six weeks by Dr David Manlove at the University of Glasgow. During this period, we worked on several open questions belonging to the framework matching mechanisms (WG4). The two main topics of joint research were the following:

- **Stable matchings with restricted edges.** Our first objective was to tackle various problems involving restricted pairs defined by Dias et al. [1]. This problem extends the classical stable matching problem by providing three possible roles for every edge: forced, forbidden and regular. While optimal solutions must contain all forced edges, forbidden edges must be avoided. Dias et al. show that it can be decided in polynomial time whether a matching containing all forced and avoiding all forbidden edges exists in the bipartite one-to-one matching case.

We have solved several additional cases, including generalizations to the stable roommates and flow problems. For the bipartite matching case, we have shown that approximation in terms of smallest number of blocking edges –also known as almost stable matching– is hard, even in the presence of strong restrictions on the degree of vertices. Moreover, we investigated another intuitive approximation concept: minimizing the number of violated constraints on restricted edges. While this latter concept can be tackled efficiently on bipartite instances, it becomes NP-complete in the roommates case.

- **Stable matchings with general preferences.** A recent paper [2] accepted at SAGT'14 gave us the idea of the second topic tackled during my research stay. Consider a bipartite matching instance, where the preferences of a vertex are expressed via pairwise comparison instead of lists. In such a general setting, preferences are not transitive and they may form cycles. The concept of weak, strong and super-stability remains intact though.

We showed that the existence of stable matchings is an NP-complete decision problem in all three cases. This changes to polynomial solvability as soon as certain restrictions –strictly or partially ordered lists– are introduced even for only one color class. We also observed that the Gale-Shapley algorithm can be generalized for weakly and super-stable matchings. This latter concept and result can also be seen as an alternative proof for some of the main results in [2]. At the same time, as already established for partially ordered lists, the strongly stable case remains NP-complete. Our goal is to give a comprehensive analysis of the topic from the algorithmic point of view.

As agreed at the end of this research stay, our plan now is to publish our results on these two topics as two separate full papers. While the problem tackled first was almost completely analyzed during my stay, the second topic still requires some further investigation. However, the guidelines of these forthcoming steps already became clear during the visit.

- [1] V. M. Dias, G. D. da Fonseca, C. M. de Figueiredo, and J. L. Szwarcfiter. The stable marriage problem with restricted pairs. *Theoretical Computer Science*, 306(1-3):391 – 405, 2003.
- [2] L. Farczadi, J. Koenemann, and K. Georgiou. Stable marriage with general preferences. In *7th International Symposium on Algorithmic Game Theory, SAGT 2014, Patras, Greece, Proceedings*, 2014.