Best-of-Both-Worlds Fair-Share Allocations

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Abstract

We consider the problem of fair allocation of indivisible items among n agents with additive valuations, when agents have equal entitlements to the goods, and there are no transfers. Best-of-Both-Worlds (BoBW) fairness mechanisms aim to give all agents both an ex-ante guarantee (such as getting the proportional share in expectation) and an ex-post guarantee. Prior BoBW results have focused on ex-post guarantees that are based on the "up to one item" paradigm, such as envy-free up to one item (EF1). In this work we attempt to give every agent a high *value* ex-post, and specifically, a constant fraction of her maximin share (MMS). The up to one item paradigm fails to give such a guarantee, and it is not difficult to present examples in which previous BoBW mechanisms give some agent only a $\frac{1}{n}$ fraction of her MMS.

Our main result is a deterministic polynomial-time algorithm that computes a distribution over allocations that is ex-ante proportional, and ex-post, every allocation gives every agent at least her proportional share up to one item, and more importantly, at least half of her MMS. Moreover, this last ex-post guarantee holds even with respect to a more demanding notion of a share, introduced in this paper, that we refer to as the *truncated proportional share* (TPS). Our guarantees are nearly best possible, in the sense that one cannot guarantee agents more than their proportional share ex-ante, and one cannot guarantee agents more than a $\frac{n}{2n-1}$ fraction of their TPS ex-post.

The full version of this paper can be found in `https://arxiv.org/abs/2102.04909`.

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