Voting on combinatorial domains

Jérôme Lang LAMSADE, CNRS – Université Paris-Dauphine A key question: *structure* of the set *x* of candidates?

Example 1 choosing a common menu:

Example 2 *multiple referendum*: a local community has to decide on several interrelated issues (should we build a swimming pool or not? should we build a tennis court or not?)

Example 3 *choosing a joint plan.* A group of friends has to travel together to a sequence of possible locations, given some constraints on the possible sequences.

Example 4 committee election; choose three representatives out of 6 candidates.

$$X = \{A \mid A \subseteq \{a, b, c, d, e, f\}, |A| \le 3\}$$

Example 1 common menu

Example 2 multiple referendum

$$X = \{\text{swimming pool, no swimming pool}\} \times \{\text{tennis, no tennis}\}$$

Example 3 joint plan / group traveling

X = set of all possible allowed paths in the graph

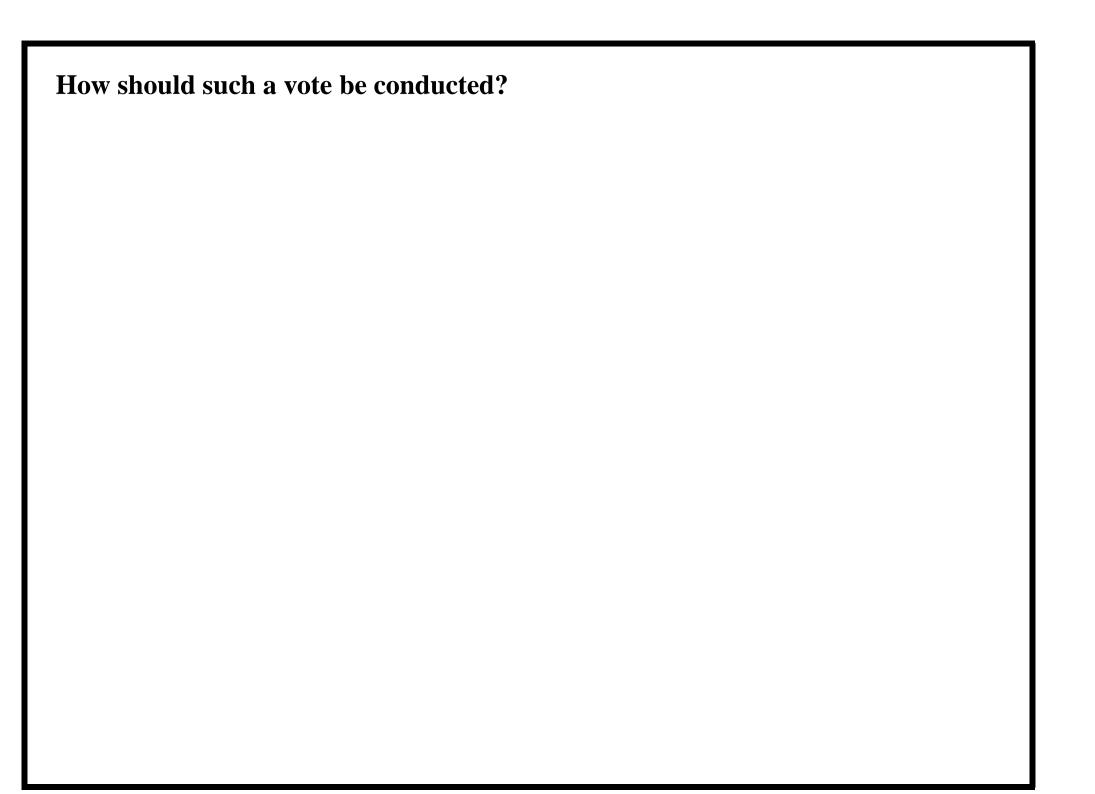
Example 4 committee election

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Examples 1-4: voting on a combinatorial domain.

Set of alternatives: $X = D_1 \times ... \times D_p$ where

- $\mathcal{V} = \{X_1, \dots, X_p\}$ set of *variables*, or *issues*;
- D_i is a finite value domain for variable X_i)



1. don't bother and vote simultaneously on each variable

Example

2 binary variables S (build a new swimming pool), T (build a new tennis court)

voters 1 and 2
$$S\bar{T} \succ \bar{S}T \succ \bar{S}\bar{T} \succ ST$$

voters 3 and 4
$$\bar{S}T > S\bar{T} > \bar{S}\bar{T} > ST$$

voter 5
$$ST \succ S\overline{T} \succ \overline{S}T \succ \overline{S}\overline{T}$$

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voter 5 $ST \succ \bar{S}\bar{T} \succ \bar{S}\bar{T} \succ \bar{S}\bar{T}$

Problem 1: voters 1-4 feel ill at ease reporting a preference on $\{S, \bar{S}\}$ and $\{T, \bar{T}\}$

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Problem 1: voters 1-4 feel ill at ease reporting a preference on $\{S, \bar{S}\}$ and $\{T, \bar{T}\}$

Problem 2: suppose they do so by an "optimistic" projection

- voters 1, 2 and 5: S; voters 3 and 4: $\bar{S} \Rightarrow \text{decision} = S$;
- voters 3,4 and 5: T; voters 1 and 2: $\bar{T} \Rightarrow \text{decision} = T$.

Alternative ST is chosen although it is the worst alternative for all but one voter.

Multiple election paradoxes arise as soon as some voters have nonseparable preferences

- 1. don't bother and vote simultaneously on each variable.
- 2. ask voters to specify their preference relation by ranking all alternatives *explicitly*.

$$\mathcal{V} = \{X_1, \dots, X_p\}; \mathcal{X} = D_1 \times \dots \times D_p$$

There are $\Pi_{1 \leq i \leq p} |D_i|$ alternatives.

Example: in a committee election with 15 candidates, there are $2^{10} = 32768$ alternatives.

As soon as there are more than three or four variables, explicit preference elicitation is irrealistic.

- 1. don't bother and vote simlutaneously on each variable.
- 2. ask voters to specify their preference relation by ranking all alternatives *explicitly*.
- 3. ask voters to report only a small part of their preference relation and appply a voting rule that needs this information only, such as plurality.

5 voters, 2⁶ alternatives; rule : plurality

001010: 1 vote; 010111: 1 vote; 011000: 1 vote; 101001: 1 vote; 111000: 1 vote all other candidates : 0 vote.

Results are generally completely nonsignificant as soon as the number of alternatives is much higher than the number of voters $(2^p \gg n)$.

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- 3. ask voters to report only a small part of their preference relation and appply a voting rule that needs this information only, such as plurality.
- 4. ask voters their preferred alternative(s) and complete them automatically using a predefined *distance*.
- the agent specifies only her preferred alternative \vec{x}
- and her preference is completed by $\vec{y} \succ \vec{z}$ if and only if \vec{y} is closer to \vec{x} than \vec{z}

Example: Hamming distance d_H

- $\vec{x} = ab\overline{c}$
- $ab\overline{c} \succ [abc \sim a\overline{b}\overline{c} \sim \overline{a}b\overline{c}] \succ [a\overline{b}c \sim \overline{a}\overline{b}\overline{c} \sim \overline{a}bc] \succ \overline{a}\overline{b}c$

Needs an important domain restriction + can be computationally difficult

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- 4. ask voters their preferred alternative(s) and complete them automatically using a predefined *distance*.
- 5. sequential voting: decide on every variable one after the other, and broadcast the outcome for every variable before eliciting the votes on the next variable.

Sequential voting

voters 1 and 2
$$S\bar{T} \succ \bar{S}T \succ \bar{S}\bar{T} \succ ST$$

voters 3 and 4 $\bar{S}T \succ S\bar{T} \succ \bar{S}\bar{T} \succ ST$
voter 5 $ST \succ \bar{S}\bar{T} \succ \bar{S}\bar{T} \succ \bar{S}\bar{T}$

Fix the order S > T.

Step 1 elicit preferences on $\{S, \bar{S}\}$

if voters report preferences optimistically: $3: S \succ \bar{S}$; $2: \bar{S} \succ S$

Step 2 compute the local outcome and broadcast the result

S

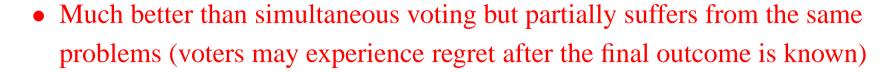
Step 3 elicit preferences on $\{T, \overline{T}\}$ given the outcome on $\{S, \overline{S}\}$

4:
$$S: \overline{T} \succ T$$
; 1: $S: T \succ \overline{T}$

Step 4 compute the final outcome

Sequential voting





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- 4. ask voters their preferred alternative(s) and complete them automatically using a predefined *distance*.
- 5. *sequential voting*: decide on every variable one after the other, and broadcast the outcome for every variable before eliciting the votes on the next variable.
- 6. use a *compact preference representation language* in which the voters' preferences are represented in a concise way.
 - potentially expensive in elicitation and/or computation

Conclusions: we have to make trade-offs between:

- strong domain restrictions
- inefficiency
- high computational cost
- high communication cost
- ⇒ design "efficient" *elicitation protocols*; try to minimize the amount of communication between the voters and the central authority
- ⇒ develop sophisticated algorithms
- ⇒ identify restrictions under which the elicitation cost and/or the complexity cost are reasonable/