A Qualitative Vickrey Auction

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Vickrey versus Qualitative Vickrey

Vickrey's sealed-bid second-price single item auction

- bids are prices
- outcome: winner has highest bid, price of second-highest bid
- bidding private value is a dominant strategy

Qualitative Vickrey auction

- bids are alternatives
- outcome: winner has highest ranked bid, alternative at least as high as second-highest
- bidding highest acceptable alternative is a dominant strategy

Motivating Example: Buy a Super-computer

Limited budget (e.g. from a project) to buy a super-computer

1. Announce ranking of alternatives (including budget) to suppliers
2. Request one (sealed) proposal from each supplier
3. Select winner: supplier with most preferred proposal
4. Select deal (by supplier): higher preferred than second-ranked proposal

Outline

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   - Summary
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Definitions and Assumptions

Notation and Definitions

- An *outcome* is an alternative and a winner: $(a, i) \in A \times N$.
- Center's order over $A \times N$ is given by a linear order $\geq$.
- Bidder i's preferences over $A \times N$ are given by a weak order $\succsim_i$.

Assumptions

- Bidder i can only bid from $A \times \{i\}$.
- Bidder i is indifferent between outcomes where winner is not i.
- Assume each bidder has at least one *acceptable* outcome, where an outcome (a, i) is acceptable to i if $(a, i) \succsim_i (x, j)$ for $j \neq i$.

The Qualitative Vickrey Auction

The *qualitative Vickrey auction* follows the following protocol:

1. The order $\geq$ of the center is publicly announced.
2. Each bidder i submits a sealed bid $(a, i) \in A \times \{i\}$.
3. The bidder i^* who submitted the bid ranked highest in $\geq$ is the winner.
4. The winner i^* may choose from $A \times \{i^*\}$ any outcome ranked at least as high as *second-highest* bid in $\geq$.

Example of a Qualitative Vickrey Auction

$$(a,1) > (a,2) > (a,3) > (b,1) > (b,2) > \cdots > (c,1) > \ldots > (d,3)$$

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Adequate Strategies

A strategy for i is *adequate* if

1. i bids acceptable outcome ranked highest in $\geq$, and
2. if i wins the auction, i selects outcome she prefers most (in $\succsim_i$) from those ranked higher in $\geq$ than the second-highest bid.

Example of Using an Adequate Strategy

$$(a,1) > (a,2) > (a,3) > (b,1) > (b,2) > \cdots > (c,1) > \ldots > (d,3)$$

$\succsim_1$	$\succsim_2$	$\succsim_3$
$(c,1)$	$(d,2)$	$(d,3)$
$(d,1)$	$(b,2)$	$(x,i) \notin A \times \{3\}$
$(x,i) \notin A \times \{1\}$	$(a,2)$	$(a,3)$
$(b,1)$	$(x,i) \notin A \times \{2\}$	$(c,3)$
$(a,1)$	$(c,2)$	$(b,3)$

Example of Using an Adequate Strategy

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Example of Using an Adequate Strategy

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$(x,i) \notin A \times \{1\}$	$(a,2)$	$(a,3)$
$(b,1)$	$(x,i) \notin A \times \{2\}$	$(c,3)$
$(a,1)$	$(c,2)$	$(b,3)$

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Adequate Strategies are Dominant

Theorem

Adequate strategies are dominant.

Proof.

(sketch)

- Let (a, i) be acceptable outcome (to i) ranked highest in $\geq$.
- Let (a', j) be highest-ranked bid by $j \neq i$.
- Two cases:
  1. $(a', j) > (a, i)$: i should bid below (a', j) in $\geq$, because if i wins, she can only select unacceptable outcomes, and
  2. $(a, i) > (a', j)$: i should bid above (a', j) in $\geq$, because then outcome can be highest in $\succsim_i$ which is above (a', j).
- In both cases, optimal strategy for i is to bid (a, i).

$\square$

DSE is Not Strongly Pareto Efficient

$$(a,1) > (a,2) > (a,3) > (b,1) > (b,2) > \cdots > (c,1) > \ldots > (d,3)$$

$\succsim_1$	$\succsim_2$	$\succsim_3$
$(b,1)$	$(b,2)$	$(d,3)$
$(x,i) \notin A \times \{1\}$	$(x,i) \notin A \times \{2\}$	$(a,3)$
$\vdots$	$\vdots$	$(x,i) \notin A \times \{3\}$
		$\vdots$

Bidder 3 will win with outcome $(a,3)$, while

1. $(d,3)$ is strictly higher preferred by bidder 3, and
2. all other bidders are indifferent.

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Other Properties

The dominant strategy equilibrium is

- Weakly Pareto efficient: no outcome is *strictly* preferred by *all* bidders.
- Strongly Pareto efficient when center is also considered: other outcome is either worse for center, or for winner.
- Weakly monotonic: if a bidder moves the equilibrium outcome (a^*, i^*) up in its order, the outcome of the mechanism stays the same.

Summary

- A class of auctions without money, similar to Vickrey's second-price auction
- A dominant strategy equilibrium that is
  - weakly Pareto efficient (but not strongly),
  - strongly Pareto efficient when center is also considered, and
  - weakly monotonic.
- In paper:
  - Escape Gibbard-Satterthwaite by restricting bidders' preferences (distinct acceptable outcomes and indifferent among non-winning)
  - Drop assumption that each bidder has an acceptable outcome

Future Work

- Prove that the Vickrey auction with money is a special case (where $\geq$ is the standard order over prices)
- Show relation to multi-attribute auctions
- Study other qualitative auctions (e.g. English, multi-unit, online)
- Characterise instances of these mechanisms (parameterised by $\geq$)
- Find more interesting applications without money transfers (e.g. grids)