Coalitional Manipulation Under Realistic Assumptions

(based on joint work with Shaun White)

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- to come last
- to know how others voted

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- a manipulating coalition must be somehow formed. Given its size, the process must be complex with a lot of private communication. Opinion polls tell you that there are your potential coalition partners but they do not tell you who they are.
- this group must include a coordination centre who calculates who should submit which linear order and then privately communicates those to coalition members.
- all the coalition members must obey the instructions of the centre but there does not seem to be obvious ways to reinforce the discipline.

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If the value of the social choice function may not drop below the status quo, then we say that such call is safe.

Example 1

Suppose the Borda rule is used.

17	15	18	16	14	14
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

Then $\text{Sc}(A) = 96$, $\text{Sc}(B) = 99$, $\text{Sc}(C) = 87$. So

$$F(R) = B.$$

This profile is not manipulable from GS Theorem point of view but incentives to vote strategically exist.

Example 1 continued

ACB types are unhappy.

17	15	18	16	14	14
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

$$\begin{bmatrix} A \\ C \\ B \end{bmatrix} \xrightarrow{13} \begin{bmatrix} C \\ A \\ B \end{bmatrix}$$

makes $\text{Sc}(A) = 83$, $\text{Sc}(B) = 99$, $\text{Sc}(C) = 100$. So

$$F(R') = C.$$

If a smaller number of *ACB* types switch, nothing happens. The call is safe.

Example 1 continued

ABC types are not completely happy.

17	15	18	16	14	14
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} \xrightarrow{4-8} \begin{bmatrix} A \\ C \\ B \end{bmatrix} \text{ makes } F(R') = A.$$

But

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} \xrightarrow{>8} \begin{bmatrix} A \\ C \\ B \end{bmatrix} \quad \text{makes} \quad F(R'') = C.$$

The call is unsafe.

The Geometry of Example 1

Given weights $w_1 \geq w_2 \geq \ldots \geq w_m = 0$ and a profile $R = (R_1, \ldots, R_n)$, every alternative a gets a positional score $sc(a)$.

Then the normalised positional score of the alternative a is given by:

$$scn(a) = \frac{sc(a)}{sc(a_1) + \ldots + sc(a_m)}.$$

After this normalisation we have

$$scn(a_1) + scn(a_2) + \ldots + scn(a_m) = 1.$$

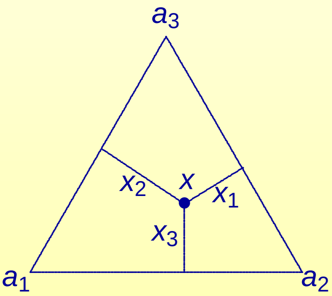
Geometric representation of scores

A normalised vector of scores $scn(a)$ can be represented as a point $\mathbf{x}$ of the m-dimensional simplex S^{m-1}:

$$\mathbf{x} = (x_1, \ldots, x_m), \quad x_1 + \ldots + x_m = 1,$$

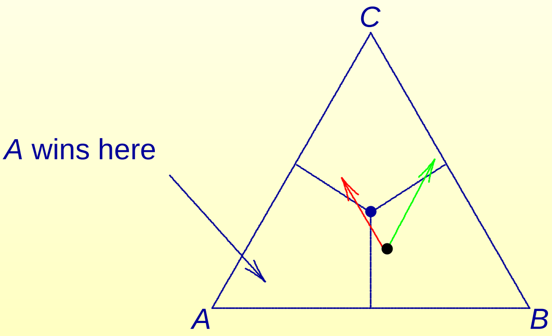
where $x_i = scn(a_i)$ is the normalised score of the ith alternative.

We treat $x_1, \ldots, x_n$ as the homogeneous barycentric coordinates of $\mathbf{x}$.



Winning Areas

The simplex S^{m-1} is divided into three zones: where the candidates A, B and C win, respectively.



The green arrow is the safe manipulation and the red arrow is the unsafe one.

Example 2

Suppose the $(3, 1, 0)$ scoring rule is used.

30	0	20	0	0	30
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

$F(R) = B$ since $\text{Sc}(A) = 110, \quad \text{Sc}(B) = 120, \quad \text{Sc}(C) = 90.$
But

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} \xrightarrow{10<k<20} \begin{bmatrix} A \\ C \\ B \end{bmatrix} \quad \text{makes} \quad F(R') = A,$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} \xrightarrow{k>20} \begin{bmatrix} A \\ C \\ B \end{bmatrix} \quad \text{makes} \quad F(R'') = C.$$

Only unsafe strategic votes exist!

Main Results

Theorem

Suppose that the number of alternatives is at least three. Let F be any onto and non-dictatorial social choice function. Then there is a profile R at which a voter can make a safe strategic call.

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Theorem (Extention of the GS Theorem)

Suppose that the number of alternatives is at least three. Then any onto and non-dictatorial social choice rule is safely manipulable by a single voter.

Sample Questions

1. How to evaluate the real complexity of forming a coalition of manipulators?

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2. What is the complexity of deciding if it possible for someone to make a safe strategic call?