On informational requirements of social choice rules

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Informational requirements

- “Minimal informational reuiqurements” as an axiom.

Cost of information processing

- Without some electronic device, processing a large amount of information is not an easy task.
- The amont of information is positively associated with
  - Time
  - Human resources
  - Risk of making an error
- The designer wants to minimize such a cost provided that a rule satisfies some desirable properties.

Central question and results

Question

Given a group of social choice rules satisfying some *reasonable* properties, which of them operates on the minimal amount of information?

Results

Characterizations of the plurality rule (and the antiplurality rule).

- $N = \{1, \ldots, n\}$: a finite set of agents.
- X: a finite set of alternatives. (Candidates)
- $\mathcal{L}$: the set of rankings over X without indifferences. (Preferences)
- $R_N = (R_1, \ldots, R_n) \in \mathcal{L}^N$: a preference profile.

Definition (Rule)

A pair $(\mathcal{M}_N, f)$ is called a *rule* if

- $\mathcal{M}_i$: a partition of $\mathcal{L}$. (Message space)
- $\mathcal{M}_N = \prod_{i \in N} \mathcal{M}_i$.
- f: a correspondence of $\mathcal{M}_N$ into X. (Social choice rule)

Scenario:

- The designer sets $\mathcal{M}_N$ and f.
- He tells each agent i to sincerely report the message containing agent i's preference.
- f makes a decision based on reported messages.

The plurality rule $(\mathcal{M}_N^p, f^p)$

"Choose the alternatives top ranked by the largest number of agents."

- $M(x) = \{R \in \mathcal{L} \mid x \text{ is top ranked at } R\}$.
- $\mathcal{M}_i^p = \{M(x) \mid x \in X\}$, and $\mathcal{M}_N^p = \prod_{i \in N} \mathcal{M}_i^p$.
- f^p is defined by for each $M_N = (M_1, \dots, M_n) \in \mathcal{M}_N^p$,

$$f^p(M_N) = \{x \in X \mid |\{i \mid M_i = M(x)\}| \geq |\{i \mid M_i = M(y)\}| \; \forall y \in X\}.$$

The antiplurality rule $(\mathcal{M}_N^a, f^a)$

"Choose the alternatives bottom ranked by the smallest number of agents."

- $M(x) = \{R \in \mathcal{L} \mid x \text{ is bottom ranked at } R\}$.
- $\mathcal{M}_i^a = \{M(x) \mid x \in X\}$, and $\mathcal{M}_N^a = \prod_{i \in N} \mathcal{M}_i^a$.
- f^a is defined by for each $M_N = (M_1, \ldots, M_n) \in \mathcal{M}_N^a$,

$$f^a(M_N) = \{x \in X \mid |\{i \mid M_i = M(x)\}| \leq |\{i \mid M_i = M(y)\}| \ \forall y \in X\}.$$

- $\mathcal{M}_i$: fine $\Rightarrow$ a lot of information
  - $\mathcal{M}_i = \{\{R\} \mid R \in \mathcal{L}\}$: full information
- $\mathcal{M}_i$: coarse $\Rightarrow$ scanty information
  - $\mathcal{M}_i = \{\mathcal{L}\}$: no information

Definition

Given a rule $(\mathcal{M}_N, f)$, the number $\sum_{i=1}^{n} |\mathcal{M}_i|$ is called the **informational size of** $(\mathcal{M}_N, f)$.

A link between preferences and social outcomes

Definition

Given a rule $(\mathcal{M}_N, f)$, for each preference profile R_N, let

$$\varphi_N(R_N) \in \mathcal{M}_N$$

be the message profile reported to f when agents have a preference profile R_N.

- A rule: message profile $\mapsto$ social outcome
- The composite correspondence $f \circ \varphi_N$: preference profile $\mapsto$ social outcome.

Anonymity, neutrality, monotonicity, Pareto efficiency

A rule $(\mathcal{M}_N, f)$ is said to satisfy

- monotonicity if

$$\begin{bmatrix} x \text{ belongs to the social outcome at } R_N \\ \text{At } R'_N\text{, the position of } x \text{ improves.} \end{bmatrix}$$
$$\Rightarrow x \text{ still belongs to the social outcome at } R'_N.$$

- Pareto efficiency if [x is strictly preferred to y by all agents] $\Rightarrow$ [y is not socially chosen].
- anonymity if the agents are treated symmetrically.
- neutrality if the alternatives are treated symmetrically.

Classes of rules

- Let $\mathcal{ANM}$ denote the set of nonconstant rules satisfying anonymity, neutrality, and monotonicity,
- Let $\mathcal{ANMP}$ denote the set of rules satisfying anonymity, neutrality, monotonicity, and Pareto efficiency.

Definition

Given a set of rules $\mathcal{F}$,

- a rule $(\mathcal{M}_N, f) \in \mathcal{F}$ is said to **operate on minimal informational requirements in** $\mathcal{F}$ if the informational size of $(\mathcal{M}_N, f)$ is smallest among the rules in $\mathcal{F}$.

Theorems

Theorem

If a rule $(\mathcal{M}_N, f)$ operates on minimal informational requirements in $\mathcal{ANM}$, then either

- $\mathcal{M}_N = \mathcal{M}_N^p$, *or*
- $\mathcal{M}_N = \mathcal{M}_N^a$.

Theorems

Definition

A rule $(\mathcal{M}_N, f)$ is called a *supercorrespondence* of a rule $(\mathcal{M}'_N, f')$ if for every $R_N \in \mathcal{L}^N$,

$$f'(\varphi'_N(R_N)) \subset f(\varphi_N(R_N)).$$

Theorem

If a rule $(\mathcal{M}_N, f)$ operates on minimal informational requirements in $\mathcal{ANM}$, then it is a supercorrespondence of either the plurality rule or the antiplurality rule.

Theorem

If a rule $(\mathcal{M}_N, f)$ operates on minimal informational requirements in $\mathcal{ANMP}$, then it is a supercorrespondence of the plurality rule.

Concluding remarks

Main result

The plurality rule (and the antiplurality rule) is (are) the answer to the following sequence of requirements:

1. First, you want a rule to satisfy some desirable properties,
2. Next, you want the cost of information processing to be as low as possible, and
3. Finally, you want the value (set of alternatives) of a social choice rule to be as small as possible, i.e., you want a selective rule.

Related literature

- Conitzer and Sandholm (2005) "Communication complexity of common voting rules", *Proceedings of ACM-EC*.
  Main difference: axiomatic or not.