A fair payoff distribution for myopic rational

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Summary

- How to partition a population of agents?

  (e.g. making multiple teams from a pool of players, groups of students, etc.)
- Each agent has a valuation for a partition
- Preference of agents conflicts

$\longrightarrow$ there may not exist any stable partition.

- Which partition to form?
- How to make it stable?

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- Use side payments to stabilize population
- Agents have incentive to follow our mechanism.

Notation

Population N of n agents.

Definition (Coalition)

A coalition $\mathcal{C}$ is a set of agents: $\mathcal{C} \in 2^N$.

$\mathscr{C}$ is the set of all coalitions.

Definition (Coalition structure)

A coalition structure s is partition of agents into coalitions:
$s = \{\mathcal{C}_1, \ldots, \mathcal{C}_k\}$ where $\cup_{i \in \{1..k\}} \mathcal{C}_i = N$ and $i \neq j \Rightarrow \mathcal{C}_i \cap \mathcal{C}_j = \emptyset$

$\mathscr{S}$ is the set of all coalition structures.
$s(i)$ denotes the coalition of agent i in the coalition structure s

Valuation

- Valuation function $v : N \times \mathscr{S} \mapsto \mathbb{R}$

$\longrightarrow$ private valuation (hedonic coalition formation flavor)

$\longrightarrow$ valuation may depend on other coalition in the population (externalities, endogeneous coalition formation)

$\longrightarrow$ Preference order over CSs $\succsim_i$

Fair payoff distribution for myopic rational agents

Hypothesis

- Self interested agents: agents maximize expected private utility
- Myopic agents: agents only care about immediate reward and do/can not analyze future implication of their actions.
+ no coordinated change of coalition (only individual actions)
+ one agent at a time can change coalition
+ a coalition's member can veto the arrival of a new agent in the coalition (individually stable)

Fairness & efficiency

- Agents should feel that the payoff they obtain corresponds to their abilities
- The coalition chosen should maximize social welfare

Rationality Concept for non transferable utility

Definition ($\succsim_i$ denotes preferences over coalitions)

A coalition structure s is **core stable** iff $\nexists C \subset N \mid \forall i \in C, C \succ_i s(i)$.

A coalition structure s is **Nash stable**
$(\forall i \in N)\ (\forall C \in s \cup \{\emptyset\})\ s(i) \succsim_i C \cup \{i\}$

A coalition structure s is **individually stable** iff
$(\nexists i \in N)\ (\nexists C \in s \cup \{\emptyset\}) \mid (C \cup \{i\} \succ_i s(i))\ \textit{and}\ (\forall j \in C,\ C \cup \{i\} \succsim_j C)$

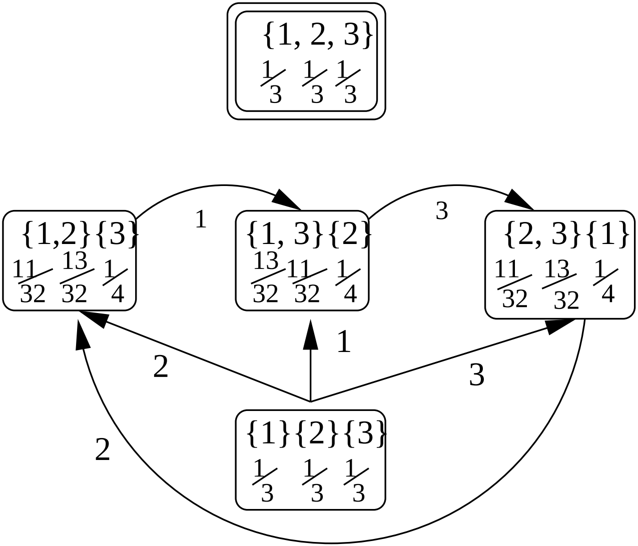
A coalition structure s is **contractually individually stable** iff
$(\nexists i \in N)\ (\nexists C \in s \cup \{\emptyset\}) \mid (C \cup \{i\} \succ_i s(i))$ and
$(\forall j \in C,\ C \cup \{i\} \succsim_j C)\ \textit{and}\ (\forall j \in s(i) \setminus \{i\},\ s(i) \setminus \{i\} \succsim_j s(i))$

Additional criteria

Individual rationality: $\forall i \in N,\ u(i) \geq v(\{i\})$
agent obtains at least its self-value as payoff.

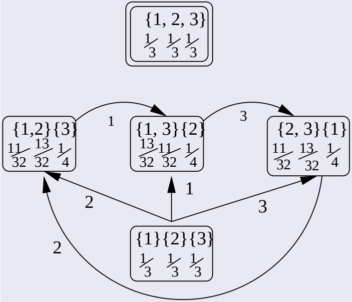
Pareto Optimal: $\nexists y \mid \exists i \in N \mid y_i > u_i$ and $\forall j \neq i,\ y_j \geq u_j$.
no agent can improve its payoff without lowering the payoff of another agent.

Example of a transition function



Markov chains

Transient states: states the chain will eventually leave to never visit again
Ergodic states: states the chain will keep coming back to
Communication class: set of ergodic states where the chain is trapped (sink equilibrium). Which communication class is reached depends on 1) initial state 2) transient states visited



$$
\begin{array}{c} \\ \{1,2,3\} \\ \{1,2\}\{3\} \\ \{1,3\}\{2\} \\ \{2,3\}\{1\} \\ \{1\}\{2\}\{3\} \end{array}
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\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \end{array}
$$

Proposed approach

Provide an incentive to form a social welfare maximizing coalition structure

1. Compute the expected utility of each agent i, $E(v_i)$, when agents are acting as myopic rational agents (exact computation requires the analysis of a Markov chain)
2. Share the value of the social maximizing coalition structure proportionally to the expected value.

$$u_i = \frac{E(v_i)}{\sum_{j \in N} E(v_j)} v(s^\star)$$

Properties

- Guarantees a payoff that is at least the expected utility:
  $u_i = \frac{E(v_i)}{\sum_{j \in N} E(v_j)} v(s^\star) \geq E(v_i)$,
  i.e., the payoff of an agent is at least as good as the expected utility that an agent would get on average if the agents are myopically rational.

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- Exact computation limits usability to small set of agents.
- size of the share is "Fair" in the sense that, on average, assuming equal probability of the initial state, an agent gets $E(v_i)$.

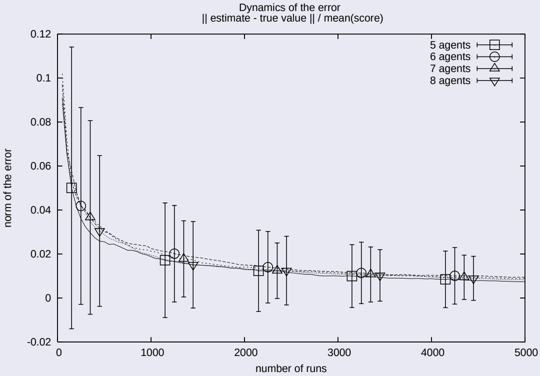
Experimental results

Average payoff over all CSs, expected value, weight and protocol payoff for each agent for a random valuation function in $\mathfrak{D}$

agent	avg	$\overline{v_i}$	w_i	u_i
0	0.50	0.61	0.17	0.96
1	0.49	0.63	0.17	0.99
2	0.50	0.60	0.16	0.93
3	0.51	0.64	0.18	1.00
4	0.56	0.54	0.15	0.85
5	0.50	0.58	0.16	0.90
total	3.06	3.60	1.00	5.63

Experimental results - Approximation

Dynamics of the error of the estimated payoff averaged over 50 instances of the ART problem



Discussion and Conclusion

1. it is possible that, for each coalition $\mathcal{C} \in s^\star$, $\sum_{i \in \mathcal{C}} u_i \neq \sum_{i \in \mathcal{C}} v_i(s^\star)$. (unbalanced inter coalition side payments)

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Future Work

- Analysis of approximations
- Analysis of manipulation
- Complete protocols

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