Not few but all quantifiers can be negated: Towards a referentially transparent semantics of quantified noun phrases

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Abstract

The main concern of this paper is to introduce not as a noun phrase negation operator. It is used in order to compositionally derive structures such as not(many)(bicycles). However, noun phrase negation is part of a much larger account of plural semantics and quantification, guided by the notion of referential transparency. Therefore the paper splits into two parts: the first part gives a minimal background on a referential transparent NP semantics. A foundation of the semantics of plural count nouns in terms of ordered set bipartitions is given. This includes as a side effect a significant reduction in the number of possible quantifier denotations and a derivation of the conservativity universal. The second part deals with not and how it interacts with an negation and anaphoric accessibility.

1 Motivation

In generalised quantifier theory (GQT), quantifiers are interpreted according to their type [18]. The type of a quantifier is given in terms of tuples of natural numbers, $\langle n_1, \ldots, n_k \rangle$, indicating number and kind of the arguments the quantifier takes, where '1' denotes a set, '2' denotes a binary relation, and so on. These quantifier types figure as templates in terms of which the truth conditions of quantified sentences are spelled out. Here we take a quantifier to be a determiner and a generalised quantifier (GQ) to be determiner—noun projection. While the denotation of, say, many is of type $\langle 1, 1 \rangle$, the denotation type of many glasses is $\langle 1 \rangle$.

GQs are very elegant in terms of quantifier logic. However, they exhibit several features which make it difficult to embed them into compositional and more crucially incremental natural language grammar and semantics:

- In virtually every grammatical theory, the sentential head is the main verb and the verb phrase *predicates* of the subject. In GQT, however, the quantifier is treated as the semantic head of a sentence and expresses a *relation* (the arity of the relation is determined by the quantifier type);
- It is commonly assumed that meaning are derived in a compositional way. However, QGs are typically lexicalised holistically, violating compositionality, e.g., every _ ... a different _ (though see e.g. [12] for an adjectival analysis of 'different');
- An even stronger constraint than compositionality is incrementality: natural language input is not only processed in a systematic manner, but also word for word (and indeed at a higher, sub-lexical latency). Quantified noun phrases (QNPs) are no exception, when used in pragmatically supporting, comprehension-oriented contexts [26], in particular those in spoken conversation [6]. Holistic and relational quantifiers seem to be a serious obstacle to this well-established empirical insight, in particular wrt. quantifier floating.

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NP_{sem} \coloneqq \begin{bmatrix} \max & Set(Ind) \\ c1 & \overrightarrow{PType}(\max set) \\ refset & Set(Ind) \\ compset : Set(Ind) \\ compset : Set(Ind) \\ c2 & union(\max set, compset, refset) \end{bmatrix}
q\text{-cond} & : Rel(q\text{-params.refset}, q\text{-params.compset})
q\text{-persp} & : \text{refset} = \emptyset \lor \text{refset} \neq \emptyset \lor \text{none}
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Figure 1: Semantic anatomy of a quantified noun phrase (NP_{sem})

Here we show how associating a significantly distinct and independently motivated type system with (Q)NPs, formulated in *Type Theory with Records* [4], resolves the compositionality and incrementality issues mentioned above and provides a predicational semantics for quantified subjects. A brief introduction into the methodological guidelines and the type-theoretical setup are therefore necessary (Sec. 2; they have been discussed in more detail elsewhere¹). Sec. 3 relates the type-theoretical set-up to plural denotations. Noun phrase negation is introduced in Sec. 4 and a brief note on incrementality is given in Sec. 5.

2 Background: Referentially transparent QNPs

To blurt things out, the type we associate with NPs is given in Fig. 1. The motivation for such contents derives from the need to attain *referential transparency*, a dialogical desideratum incorporating (i) anaphoric potential, clarificational potential, co-verbal gesture; and (ii) several key recent psycholinguistic results on processing GQs mentioned below (Sec. 5). So far, we recognise four sources of referential transparency:

(1) **Referential transparency**: an NP is referentially transparent if (a) it provides the semantic type required by a clarification request, (b) it provides antecedents for pronominal anaphora, (c) it provides an attachment site for co-verbal gestures, (d) its content parts can be identified and addressed.

Wrt. (1a), [21] show that clarificational potential provides data against higher order denotations (as postulated by GQT) in that answers to reprise fragment clarifications always provide individuals or sets, but not sets of sets. This finding carries particular weight since reprise fragments query (exactly) the semantic (not the pragmatic) content of the fragments being reprised. For such reasons, [21] argue in favour of a more transparent NP denotation in terms of witness sets. We refine and generalise their proposal to all NPs.

As is widely accepted, anaphoric potential covers two kinds of antecedents, a so-called maximal set and a reference set. Both are exemplified in (2), where the plural pronoun in (2a) refers back to demonstrators that actually took part in the rally (the reference set, or refset), and the plural pronoun in (2b) picks up the antecedent which denotes the whole of demonstrators that potentially could have come to the rally (the maximal set, or maxset). When the antecedent NP is formed with a quantifier which is downward monotone on the restriction, like few, even a further witness set can be picked out, namely the complement set (compset) [17]; in (2c) the potential demonstrators that did not come to the rally.

¹Lücking, Cooper and Ginzburg, manuscript under review.

- (2) Few demonstrators were allowed in near the Place d'Italie.
 - a. They raised their signs defiantly.
 - b. But they had all received an invitation.
 - c. They went to a football game instead.

The eponymous contents within the q-params field in Fig. 1 correspond to these three antecedent sets. The 'arrow type', \overline{PType} , is the plural variant of a singular predicate and provides the group-constituting property of the plurality (there are collective and distributive subtypes, but this is yet another story). For instance, a group of students is represented as 'student (maxset)', where a single student is represented as 'in(refind,refset)'. The relation between maxset, refset and refind will become clear shortly in Sec. 3.

The attribute q-persp encodes a quantifier's perspective, that is, the expectancy whether the refset may be empty (q-persp= \emptyset) or not (q-persp $\neq \emptyset$) (following work on GQ processing by [24]). On our account, a denotational underpinning for q-persp's value is provided, namely whether an empty refset is part of a QNP's denotation (Sec. 3). That is, other things being equal (e.g. a non-negative context, to which we turn below in Sec. 4), 'q-persp = \emptyset ' labels downward monotonicity. The q-persp value is also the distinguishing semantic feature between the quantificational minimal pair few vs. a few, which are alike in terms of their q-conds. If a perspective is not applicable, as with proper names, the feature is 'none'. Given this set-up, anaphoric accessibility is simply governed by the rule that refset and maxset are, ceteris paribus, available as antecedents while the compset is only available when the 'refset = \emptyset ' flag is on.

The descriptive quantifier condition (q-cond) provides the relation of a QNP in terms of a cardinality comparison between its refset and the compset (readings which draw on a contextually given standard of comparison [16] can always be derived when a threshold θ is instantiated in context). That is, q-cond is only defined in terms of NP-internal denotations and does not make reference to a so-called scope set. A closely related move has also been made in GQT: it is known that the semantics of conservative quantifiers of type $\langle 1,1\rangle$ can be reduced to unary functions which only take the restrictor set as argument [7, 11]. On the account in [11], the denotation of a QNP is a pair $\langle R, W \rangle$ consisting of a restrictor set R and a set of witnesses R, where R is the set of subsets of R. A quantifier denotes a unary function from subsets of R onto R. For a universe of two objects there are 512 possible quantifier denotations [11]; for comparison: there are R and R quantifier denotations according to GQT [9, p. 632].

While this is a step in the (right) direction of a referentially transparent QNP semantics, W still faces the clarification request argument of [21]. Therefore, yet another denotational and conceptual underpinning is required.

3 Denotational construal

Our starting point to a denotational approach to QNPs is the simple consideration that quantification should rest on a plural semantics. After all, most quantifiers combine with plural nouns.² Plural semantics models the extension of a plural count noun in terms of the power set (or something equivalent such as a join semi-lattice) of the extension of the base noun [13]. We argue that referential transparency offers a more detailed picture on plural denotations, namely the need for sets of ordered set bipartitions.

²Exceptions being *every* and *each*, whose head nouns are syntactically singular (though semantically plural), and—if one wants to subsume it to quantificational determiners—the indefinite article (the definite is ambiguous in this respect, as is 'no'). And there is no dearth of cross-linguistic variation.

(3) Ordered set bipartition

An ordered set bipartition p of a set S is a 2-tuple of pairwise disjoint subsets of S including the empty set such that the union of these subsets is S.

(An example follows shortly in (4).) The extensions of count nouns are computed as follows (we give both the type denotations and interpreted language denotations (i.e. ' $\lceil \cdot \rceil$ ')):

- Extension of a type: $[{}^{\vee}T] = \{a \mid a : T\}.$
- Q-extension of a plural predicate: $[\downarrow \overrightarrow{P}] = p([\downarrow P])$ (where p is the ordered set bipartition operation from (3)). This corresponds to $p(\llbracket P \rrbracket)$.

A simple example should illustrate how Q-extensions look like:

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(4) Let [\downarrow \text{ bicycle}] = \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet} \} (i.e. there are three differently coloured bicycles in our universe U). Then [\downarrow \text{ bicycle}] = p([\downarrow \text{ bicycle}]) = \{\langle \emptyset, \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet} \} \rangle, \quad \langle \{ \stackrel{\bullet \bullet}{\bullet} \}, \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet} \} \rangle, \quad \langle \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet} \} \rangle, \quad \langle \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet} \} \rangle, \quad \langle \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet}{\bullet} \} \rangle, \quad \langle \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet}{\bullet} \} \rangle, \quad \langle \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet}{\bullet} \} \rangle, \quad \langle \{ \stackrel{\bullet \bullet}{\bullet}, \stackrel{\bullet 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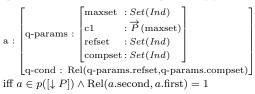
Each ordered set bipartition in the set of ordered bipartitions is structured in the form '(compset, refset)'. That is, the first ordered set bipartition in (4), the one with an empty compset, is the denotation of every bicycle in the sample universe. Note that it is just a structured set of bicycles rather than a set of bicycles which is a subset of all other sets, as assumed in GQT. Ordered set bipartitions therefore provide a considerable simplification of quantificational complexities and are therefore preferable in terms of processing and comprehension. To exemplify: For $U = \{a, b\}$ there are four ordered set bipartitions: $p(\{a, b\}) =$ $\{\langle\emptyset,\{a,b\}\rangle,\langle\{a\},\{b\}\rangle,\langle\{b\},\{a\}\rangle,\langle\{a,b\},\emptyset\rangle\}$. The middle two (those without empty sets) are indistinguishable for a quantifier, since a quantifier just looks at cardinalities. Virtually collapsing the two middle bipartitions, there are seven combinatorially possible quantifier denotations left (2^n-1) , where n=3 (due to collapsing) and -1 is due to skipping \emptyset which is already part of the bipartitions). Thus, for two objects there are seven possible quantifier denotations. This is a significant reduction even compared to the already reduced 512 quantifier denotations of the approach of [11], cf. Sec. 2. Note further that our quantifiers 'live on' [1] the extensions of their head noun, a property which is equivalent to conservativity [10], as recently argued by [27]. That is, if quantification rests on a plural semantics as envisaged by referential transparency, then the semantic universal that all natural language quantifiers are conservative [10] will be a sequitur (cf. also [11]).

Now the bipartitions of a Q-extension are mutually exclusive: they cannot be realised simultaneously (nor can just a pair of them). They define the logical space of possible plural extensions. That is, each denotation corresponds to a separate type-theoretical model. Quantificational determiners impose constraints on the logical space of the plural extension of their head noun. The constraining component is the quantifier condition (q-cond) by dint of which subsets of plural denotations are excised: $[\downarrow \text{DET N}] = \{p \in p([\downarrow \text{N}]) \mid \text{Rel}(p.\text{second}, p.\text{first}) = 1\}$ (where 'Rel' is DET's q-cond value). Since an individual is always an individual within a refset,

singular is as much a special case of plural as definite is of indefinite. We model this intuitively appealing pattern in exactly this way, where a singular count noun introduces a refind ('reference individual') which is an element of a refset. The maxset, finally, is simply the union of the refset and the compset which constitute an ordered set bipartition.

In order to distinguish between referential and quantificational uses of singular and plural QNPs, we employ the mechanism developed by [21]: extensional argument roles can either be referentially grounded (when coerced into contextual parameters (c-params)) or existentially quantified away (when coerced into quantificational parameters (q-params)). Depending on which element goes where, referential and quantificational uses are distinguished in terms of their witnessing conditions:

• quantificational: maxset is part of *q-params*. • indefinite: an individual is part of *q-params*.



```
maxset : Set(Ind)
                                          : \overrightarrow{P} \text{ (maxset)}
                            c1
        q-params:
                            compset : Set(Ind)
                            {\it refind} \quad : Ind
                                          : in(refind,refset)_
iff a \in p([\downarrow P]) \land \text{Rel}(a.\text{second}, a.\text{first}) = \overline{1} \land \exists x [x \in A]
a.second
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• plural reference: maxset is part of *c-params*.

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\lceil \text{maxset} : Set(Ind) \rceil
                              : \overrightarrow{P}(\text{maxset})
                        refset : Set(Ind)
     [q-cond : Rel(q-params.refset,q-params.compset)]
iff a = \iota x[x \in p([\downarrow P]) \land \text{Rel}(x.\text{second}, x.\text{first}) =
1 \land x \in \text{common-ground(spkr, addr)}
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• singular reference: refind is part of *c-params*.

```
Tmaxset : Set(Ind)
                       c1
                                   : \overrightarrow{P} \text{ (maxset)}
                       refset
                                  : Set(Ind)
      c-params :
                       compset : Set(Ind)
                       refind : Ind
                                  : in(refind,refset)
iff a \in p([\downarrow P]) \land \text{Rel}(a.\text{second}, a.\text{first}) = 1 \land \iota x[x \in
a.second \land x \in common-ground(spkr, addr)
```

An NP_{sem} witness is a triplet of sets—maxset, refset, and compset—such that the maxset is the union of refset and compset and the cardinalities of refset and compset comply with the quantifier condition (q-cond). Singular involves a refind on top. Reference is accounted for in terms of common ground membership, which is compatible with various approaches (e.g. with [19]) but not further developed here. Most importantly, structure and type of NP_{sem} is in accordance with the demands of referential transparency as required in (1).

Predication, 'anti-predication' and not_O 4

Visual appearance this time is not deceptive: as the record type representation format suggests, our QNP approach is implemented in a TTR-based variant of a constraint-based grammar (i.e. HPSG [20]). HPSG_{TTR} has been developed and motivated in [2, 8] and further refined in [14]. An example for a $HPSG_{TTR}$ structure is given in Fig. 2, more details can be found in the justgiven sources. The basic idea underlying subject predication is that the verb makes use of the ordered set bipartitions of its argument: A verb phrase, a plural predicate, \overline{PType} , predicates of the refset of its syntactic subject (feature 'nucl') and exerts an 'anti-predication' on the compset ('anti-nucl'). In particular the compset and 'anti-predication' provide the prerequisites for integrating noun phrase negation.

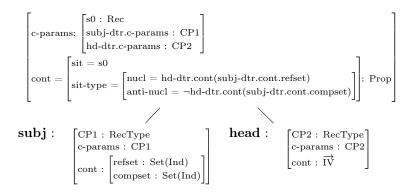


Figure 2: Declarative plural head-subject rule (where \overrightarrow{IV} labels the type of a plural intranstive verb and CPx the c-params values that get inherited to the mother node)

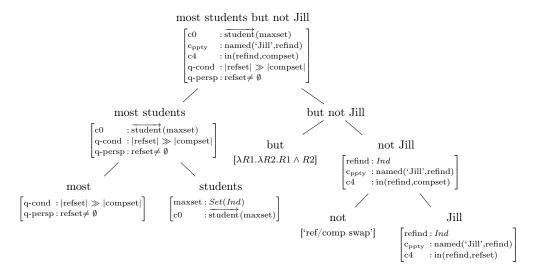


Figure 3: Derivation of the QNP most students but not Jill

Not when applied to a left decreasing quantifier makes the compset available as an antecedent for anaphora:

- (5) a. Many music lovers admire Mozart. #They [= music lovers not admiring Mozart] prefer Reger instead.
 - b. Not many music lovers admire Reger. They [= music lovers not admiring Reger] prefer Mozart instead.

The pattern in (5) might be explained in terms of the QNP's monotonicity. Compset anaphora is licensed by downward monotone quantifiers [17]. Negation reverses the direction of the upward monotone quantifier many, giving rise to (5b). However, compset anaphora under negation is also obtained from non-monotone quantifiers, as shown in (6):

- (6) a. All music lovers admire Mozart. #They [= music lovers not admiring Mozart] prefer Reger instead.
 - b. Not all music lovers admire Reger. They [= music lovers not admiring Reger] prefer Mozart instead.

How to model the semantics of the b-sentences in (5) and (6)? Lexical decomposition of the form $\neg many.x(\phi)(\psi) \leftrightarrow many.x(\phi)(\neg \psi)$ [23], for instance, cannot generally be applied, since for the universal quantifier the inequivalence $\neg all.x(\phi)(\psi) \not\leftrightarrow all.x(\phi)(\neg \psi)$ holds. The 'discourse referent insertion' account of [25] can derive negated universal quantifiers, but only with a collective scope reading ('not all P Q' becomes $\exists X : *P'(X) \land \neg Q'(X)$). Thus, a full compositional analysis of QNPs including negation is still a semantic desideratum, which we address in terms of a novel negation operation, namely NP negation. NP negation, 'not_Q', in essence moves its argument to the 'anti-witnesses' contributed by the compset. The basic semantics of 'not_Q' is given by the recursive rule in (7):

(7) $Not_{\mathbf{Q}}$

- a. 'refset' and 'compset' labels are swapped.
- b. The relation symbol used in q-cond and q-persp is reversed (= \rightarrow \neq , \geq \rightarrow <, < \rightarrow \geq , $> \rightarrow$ \leq , \leq \rightarrow >, , \gg \rightarrow \gg , \gg \rightarrow \gg , \ll \rightarrow \ll , \ll \rightarrow \ll , \neq \rightarrow =).

Instead of lexicalising the QNP most NP_{pl} but not α (α being some NP including proper names) as a holistic quantifier of type (1,1,1) (e.g., most NP_{pl} but not $\alpha' = \{(X,Y,Z):$ $|X \cap Y| > |X \setminus Y| \land Z \not\subseteq Y$), its semantic content is compositionally derived in a way indicated in Fig. 3. Recall that the tree is licensed by grammar rules of the sort exemplified in Fig. 2. The top node in Fig. 3 is the result of two composition methods following the syntactic structure of the NP: firstly, merge (e.g., for combining determiners and head nouns, corresponding to unification), secondly, functional application (not acting on its modifiee). The witnessing conditions for the semantic representation of the full QNP follow the pattern given in Sec. 3. Any verb phrase that predicates of this QNP will predicate of the refset but exclude the compset, including Jill. The right witnessing conditions for the sentence are obtained since noun phrase negation is related to the incompatibility semantics of type negation [22, 4]: $a: \neg T$ iff a:T' and T' precludes T. Now refset and compset are necessarily disjunct. If $a \in b$ second (= refset) for some $b \in p([\downarrow P])$, then $a \notin b$.first (= compset), for all $a \in b$.second $\cup b$.first (= maxset). It follows that $[\overset{\vee}{P}(\text{refset})] \cap [\overset{\vee}{P}(\text{compset})] = \emptyset$. For all sets of objects A, if \overrightarrow{P} (refset=A) is of a non-emtry (now higher-order) type \mathcal{T} , then it follows that \overrightarrow{P} (compset=A) is of an empty type \mathcal{T}' — \mathcal{T} and \mathcal{T}' preclude each other. From incompatibility we get for any predicate type PT: if $PT : \mathcal{T}$ then $PT : \neg \mathcal{T}'$ and $vice \ versa$.

The QNP anatomy in addition to noun phrase negation enables the right witnessing conditions for examples like (8a) in a similarly straightfoward manner, which eludes a deletion based account in light of (8b) [8]:

- (8) a. Who supports Corbyn? Not Boris.
 - b. *Not Boris supports Corbyn. (Following [15])

Our analysis proceeds as follows: The wh-question introduces a propositional abstract, which abstracts from a record of type [refind : Ind] onto the type of supporting Corbyn [8], see (9a). The answer supplies such an individual (which is represented like the *not Jill* node in Fig. 3), and the propositional abstract applies to *not Boris*, resulting in the structure in (9b). Note

that obvious relabelling [3] is employed in order to keep subject ('-sbj') contributions apart from object ('-obj') ones and that structures are shortened to the essential components.

(9) a. Who supports Corbyn? \mapsto

```
\begin{bmatrix} \text{sit} = \mathbf{s} \\ \text{abstr} = \lambda r : \begin{bmatrix} \text{reset-sbj} : Set(Ind) \\ \text{compset-sbj} : Set(Ind) \\ \text{co} : \overrightarrow{\text{person}}(\text{refind-sbj}) \end{bmatrix} ( \begin{bmatrix} \text{c4} : \overrightarrow{\text{support}}(r.\text{refset-sbj,c}) \\ \text{c5} : \neg \overrightarrow{\text{support}}(r.\text{compset-sbj,c}) \end{bmatrix} ) \end{bmatrix}
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b. Applying Who supports Corbyn? to Not Boris \mapsto

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 \begin{bmatrix} \text{sit} = \mathbf{s} \\ \text{q-params} = \begin{bmatrix} \text{compset-sbj} : Set(Ind) \end{bmatrix} \\ \text{c-params} = \begin{bmatrix} \text{refind-sbj} : Ind \\ \text{c1} : \text{in}(\text{refind-sbj,q-params.compset-sbj}) \\ \text{c6} : \text{named}(\text{refind-sbj, 'Boris'}) \end{bmatrix} \\ \text{sit-type} = \begin{bmatrix} \text{c4} : \overrightarrow{\text{support}}(\text{c-params.refset-sbj,c}) \\ \text{c5} : \neg \overrightarrow{\text{support}}(\text{q-params.compset-sbj,c}) \end{bmatrix}
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The question-answer pair in (9b) provides the information that whoever supports Corbyn (i.e. is member of the refset), Boris is part of the people that cannot (the compset).

Since NP_{sem} structures, including singular ones, involve both sets and refinds, anaphora in cases such as (10), which have long been tricky for dynamic semantics [5], can easily be explained:

(10) A: Go get a bike from the vélib station. B: Oh, but I don't see any bike that works there.

a. They are probably rented out.

[refset]

b. It is probably rented out.

[refind]

The negative polarity item *any* meaning 'something' or 'a random one out of many' has two facets. One facet is the existential interpretation which corresponds to a refind analysis. The other facet is the collection of things out of which *any* is picked, which is given by the refset. Both aspects are referred to by the singular and plural pronouns in (10a) and (10b).

The proper name *Peter* refers to an individual from a set of individuals which share the property of being named Peter. This is evidenced in (11a) and (11b):

(11) A: Where is Peter ... I can't read the surname? B: There is no Peter in class today.

a. He is at home with flu.

[refind]

b. They are at home with flu.

[compset]

Since they refers back to the Peters not in class, (11b) is an instance of compset anaphora. Likewise, no shifts the refind in (11a) into the compset. Thus, the compset has at least one Peter in it. In obvious contexts in which (11) could have been uttered some kind of familiarity with the people named Peter can be assumed (cf. the common ground condition in Sec. 3). That is, additional context knowledge can contribute in licensing plural compset anaphora: in case of (11b) there is apparently more than one Peter missing from class.

5 A note on processing and incrementality

Our account is additionally motivated by a variety of experimental evidence. We mention two here. First, the extensive work on the refset/compset partition [24] which in particular shows that compset and maxset are not constructed as fallback interpretation options, but, when available, have the same processing status as default antecedents. Second, experimental evidence for incremental interpretation of QNPs in pragmatically supporting contexts [26]—which our account that assigns a QNP an 'in situ', 'internal' (in the sense of not incorporating a projected verbal argument) meaning, is well placed to capture. We achieve this by a mixture of unification (type merge) and functional application as semantic composition techniques. All semantic bits can be addressed by labels, which secures correct semantic composition as well as anaphoric accessibility.

6 Conclusion

We presented the main ideas of a 'referential transparent noun phrase semantics'. Given space considerations, obviously not all complex quantifiers have been discussed here (and the ones discussed deserve a more detailed discussion). However, the motivation and potential of the approach should we hope become apparent. With regards to the range of QNPs which can be analysed in this way, there seems to be only one principled restriction: the NP-internal architecture precludes non-conservative quantifiers. However, one can always resort to a contextually instantiated threshold θ in order to account for non-conservative effects.

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