# *At least* and Quantity Implicature: Choices and Consequences*

Bernhard Schwarz

McGill University, Montreal, Quebec, Canada
bernhard.schwarz@mcgill.ca

## 1 Introduction

Numerals modified by *at least* lack the upper-bounding implication typically associated with bare numerals ([7, 9]). In contrast to (2), for example, (1) lacks the upper-bounding implication that Al did not hire more than two cooks.

(1) Al hired at least two cooks.
(2) Al hired two cooks.

*At least* also introduces an implication of speaker ignorance ([9, 1, 12, 19]). (1) conveys that the speaker is uncertain about the exact number of cooks Al hired. In one view, pioneered by Büring ([1]), this ignorance implication is a Gricean Quantity implicature. Büring proposes to capitalize on a striking similarity between *at least* and disjunction. Just like 1, 3 below lacks the upper-bounding implication of 2, and 3 also shares the ignorance implication of 1.

(3) Al hired exactly two cooks or Al hired more than two cooks.

Assuming that ignorance implications with *at least* and disjunction alike are Quantity implicatures, [1] proposes that in the two cases these implicatures are also calculated in a parallel fashion, suggesting that statements with *at least* in some sense *are* disjunctions. But [1] stops short of executing this idea within a general theory of Quantity implicature. As a consequence, Büring's proposal remains unclear with regard to precise relation between *at least* and disjunction, and in fact about the logic of ignorance implications in either case.

Building on [19] and [10], I will in the following formulate, and explore the consequences of, a neo-Gricean implementation of Büring's approach. This account attempts to derive ignorance implications with *at least* through a straightforward adaptation of the treatment of disjunction by Sauerland in [15]. Such a derivation is suggested to call for a departure from the standard neo-Gricean algorithm for calculating Quantity implicatures ([15, 4, 5]), requiring the Neo-Gricean correspondent of a notion of implicature maximization ([16, 4, 20]).

## 2 Alternatives and the Standard Recipe

Under the neo-Gricean approach, listeners enrich the semantic meaning of an utterance by reasoning about formally defined *alternatives*, semantic meanings of certain alternative utterances that the speaker could have made but did not. The Neo-Gricean theory of alternatives is the

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topic of subsection 2.2. Section 2.1 spells out a standard elaboration of Gricean Quantity-based reasoning about alternatives, known as the *Standard Recipe* (SR)[1]. In the introduction of SR in subsection 2.1, the relevant alternatives are in the meantime simply assumed as given.

## 2.1 The Standard Recipe

### 2.1.1 Primary and Secondary Implicatures

According to SR, a listener interpreting an utterance infers that the speaker lacks the knowledge to support assertion of semantically stronger alternatives. Such *primary implicatures* take the form $\neg \Box \beta$, where $\beta$ is an alternative and $\Box$ expresses speaker certainty. So under a weak semantics for numerals, 2 semantically conveys that Al hired more than one cook, **[2...)**. According to SR, if **[3...)** is an alternative, a listener infers from 2 that the speaker lacks the knowledge to support the assertion of **[3...)**, and infers the primary implicature $\neg\Box$**[3...)**.

(2) Al hired two cooks.

SR posits that listeners sometimes strengthen a primary implicature $\neg \Box \beta$ to a *secondary implicature*, $\Box\neg\beta$. This happens just in case the listener takes the speaker to be *competent* with regard to the alternative $\beta$, assuming $\Box\beta\vee\Box\neg\beta$. For (2), a listener who assumes $\Box$**[3...)**$\vee\Box\neg$**[3...)** will infer $\Box\neg$**[3...)**, which captures the attested upper-bounding implication of 2.

### 2.1.2 Excludability and Symmetry

SR assumes that the listener takes the speaker to respect Quality, inferring $\Box\alpha$ from an utterance with semantic content $\alpha$. I will refer to the conjunction of the Quality implicature and all primary implicatures as the *implicature base*. Also, if $\neg \Box \beta$ is a primary implicature, let me say that an alternative $\beta$ is *weakly excludable* just in case the potential secondary implicature $\Box\neg\beta$ is consistent with the implicature base. SR posits weak excludability of $\beta$ as a necessary condition for strengthening the primary implicature $\neg \Box \beta$ to the secondary implicature $\Box\neg\beta$.

If an alternative $\beta$ fails to be weakly excludable, this is due to the implicature base entailing an ignorance implication $\neg \Box \beta \wedge \neg \Box \neg\beta$ that is inconsistent with the potential secondary implicature $\Box\neg\beta$ and the competence assumption $\Box\beta \vee \Box\neg\beta$ that would be required to support it. The paradigm case illustrating this is disjunction. Suppose that for 4 below, the listener strengthens the semantic meaning $\mathbf{b}\vee\mathbf{c}$ by reasoning about the stronger alternatives **b** and **c**. Under SR, the listener's reasoning then yields the primary implicatures $\neg\Box\mathbf{b}$ and $\neg\Box\mathbf{c}$.

(4) Al hired Bill or Al hired Carol.

But neither **b** nor **c** is weakly excludable. The reason is that in conjunction with the Quality implicature $\Box\mathbf{b}\vee\mathbf{c}$, $\neg\Box\mathbf{b}$ entails $\neg \Box \neg\mathbf{c}$ and $\neg\Box\mathbf{c}$ entails $\neg \Box \neg\mathbf{b}$. So the implicature base entails the ignorance implications $\neg\Box\mathbf{b}\wedge\neg \Box \neg\mathbf{b}$ and $\neg\Box\mathbf{c}\wedge\neg \Box \neg\mathbf{c}$. These ignorance implications are inconsistent with the competency assumptions $\Box\mathbf{b}\vee \Box \neg\mathbf{b}$ and $\Box\mathbf{c}\vee \Box \neg\mathbf{c}$, and hence with the potential secondary implicatures $\Box\neg\mathbf{b}$ and $\Box\neg\mathbf{c}$. The weak excludability condition predicts, then, that neither secondary implicature will be inferred. All of these predictions about (4) are correct. An utterance of (4) does not support the inferences that either of **b** and **c** is false. Instead, it conveys that the speaker is uncertain about the truth or falsity of both **b** and **c.**

In SR, ignorance implicatures arise when, and only when, for a Quality implicature $\Box\alpha$ and two primary implicatures $\neg \Box \beta$ and $\neg \Box \gamma$, the conjunction of $\Box\alpha$ and $\neg \Box \beta$ entails $\neg \Box \neg\gamma$ and

[1] See, in particular, [15, 4, 5]. The term *Standard Recipe* is due to Geurts ([5]). The terminology introduced below is for the most part taken from, or adapted from, language established in [15, 4, 5].

the conjunction of $\Box\alpha$ and $\neg\Box\gamma$ entails $\neg\Box\neg\beta$. The Quality implicature and the two primary implicatures will relate in this way just in case $\beta$ and $\gamma$ jointly exhaust $\alpha$ in the sense that $\alpha$ entails $\beta \vee \gamma$. In that case, $\beta$ and $\gamma$ are also said to be *symmetric*.

### 2.2 Horn scales and the Substitution Method

In the Neo-Gricean approach, the availability of alternatives is grammatically regulated. Horn in [7] proposes that alternatives to the asserted utterance meaning are generated from sets of alternative lexical items, the *Horn scales*. Alternative meanings are the semantic meanings of syntactic structures obtained from the syntactic structure of the asserted sentence by substituting one or more lexical item with a Horn scale mate. Under this approach, the proposition **[3...)** counts as an alternative to the asserted meaning of (2), **[2...)**, because the numerals *two* and *three* are both members of the Horn scale of numerals in (5), and because substituting *three* for *two* in (1) results in a sentence with the meaning **[3...)**.

(5) Horn scale: {*one*, *two*, *three*, ...}

## 3 *At least* and The Standard Recipe

Recall that (1), repeated below, conveys that the speaker is uncertain about the exact number of cooks Al hired. Büring ([1]) proposes that such ignorance implications are Gricean inferences, hence are not semantically entailed. If so, *at least* in (1) can be analyzed as truth conditionally inert, so that the semantic meaning of (1) is the same as that of (2), namely **[2...)**.

(1) Al hired at least two cooks.

### 3.1 Ignorance via Symmetry

Pursuing the approach pioneered by Büring ([1]), Schwarz and Shimoyama ([19]) and Mayr ([10]) articulate an account of ignorance implication with *at least* under SR by positing suitable alternatives. Suppose that the alternatives to the semantic meaning of (1), **[2...)**, include **[3...)** and also **[2]**, the proposition that Al hired exactly two cooks. Since both of these alternatives are stronger than **[2...)**, SR derives the primary implicatures ¬□**[3...)** and ¬□**[2]**. Neither of these is expected to strengthen into a secondary implicature, since neither **[3...)** nor **[2]** is weakly excludable: **[3...)** and **[2]** are symmetric, jointly exhausting **[2...)**. The implicature base therefore entails the ignorance implications ¬□**[3...)**∧¬□¬**[3...)** and ¬□**[2]**∧¬□¬**[2]**, which are inconsistent with the assumption that speaker is competent with regard to **[3...)** or **[2]**, hence inconsistent with the potential secondary implicatures □¬**[3...)** and □¬**[2]**.

This predicts that a listener will infer from an utterance of (1) that the speaker lacks an opinion on whether Al hired exactly two cooks and on whether Al hired more than two cooks. Note that the implication derived amounts to *partial* ignorance. It is consistent with the speaker having an opinion regarding the number of cooks hired that goes beyond **[2...)**. For example, it is consistent, with the speaker being certain that Al hired no more than ten cooks. All of these these predictions appear consistent with intuitions about the meaning of (1).[2]

[2]Schwarz and Shimoyama ([19]) note that ignorance implications with *at least* are obviated in cases where *at least* is interpreted under a universal quantifier. For example, *Every manager hired at least two cooks.* is consistent with the speaker having full knowledge of the number of cooks hired by each manager. They also note that this ignorance obviation is predicted under SR, since, as [4] discusses, universal quantifiers break the symmetry between alternatives. Ignorance obviation under universals (and other operators, see [17, 10]) is unaccounted for in certain analyses of *at least*, including [6] and [12].

### 3.2 A Scale Mate for *at least*

The proposed application of the SR to *at least* rests on the assumption that the requisite alternatives are generated by the Horn-scale based substitution method. For (1), the substitution method and the Horn scale in 5 deliver the alternative **[3...)**. What is missing is a source for the alternative [**2**]. A straightforward way to fill this gap, is the addition of the Horn scale in 6, which pairs *at least* with *exactly*. Substituting *exactly* for *at least* in (1) results in a structure that has the intended double sided meaning ([17, 10]).

(6) Horn scale: {*at least*, *exactly*}

Note that under the account arrived at, a coherent interpretation emerges of Büring's proposal that *at least* has a meaning that is in some sense disjunctive. What *at least* has in common with disjunction is that both give rise to alternatives that instantiate the configuration of symmetry, jointly exhausting the semantic meaning of the assertion.[3]

## 4 Overgeneration

Assuming the two Horn scales in (5) and (6), the substitution method applied to (1) delivers not just the alternatives [**2**] and **[3...)**. The full set of alternatives generated is **ALT** = {[**n**]: **n≥2**}∪{**[m,...): n≥3**}. Mayr ([10]) notes that the additional alternatives in **ALT** support an unattested secondary implicature.[4]Applied to example (1), Mayr's claim is that SR derives the secondary implicature □¬**[4...)**, so that (1) is predicted to carry the upper bounding implication that Al did not hire more than three students. (1) should then convey that Al hired exactly two or three students. Clearly, (1) cannot be so understood.

While Mayr is pointing to a real problem, a closer look at the predictions under SR leads to a refinement of its characterization. Determining predictions can be facilitated by displaying **ALT** in the way shown in (7), where meanings are aligned to reflect semantic strength.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | **...** | **...** | | |
| | | | [**4**] | **[5** | **...** | ) | |
| (7) | | [**3**] | **[4** | **5** | **...** | ) | |
| | [**2**] | **[3** | **4** | **5** | **...** | ) | alternatives |
| | **[2** | **3** | **4** | **5** | **...** | ) | asserted meaning |

SR delivers the primary implicatures ¬□[**3**] and ¬□**[4...)**. Neither of [**3**] and **[4 ...)** pairs up with the other, or any available alternative, in a configuration of symmetry to exhaust the semantic meaning **[2...)**. Hence SR derives no ignorance implications for these alternatives, which are therefore weakly excludable. So nothing in SR prevents strengthening of either ¬□[**3**] or ¬□**[4...)** to □¬[**3**] and □¬**[4...)**, respectively. According to SR, then, listeners might infer from (1) that Al did not hire exactly three cooks, or that he did not hire more than three.[5] More generally, SR allows for the secondary implicatures □¬[**n**] and □¬**[m...)**, for all n≥3 and m≥4. None of these additional Quantity inferences can be detected in the meaning of (1).

[3] In the case of *at least*, the two alternatives posited happen to be mutually exclusive. But mutual exclusivity is not, as [1] and [8] seem to suggest, a crucial component of a neo-Gricean derivation of ignorance implications.

[4] I first learned about this problem from Brian Buccola (personal communication, commenting on [17]).

[5] In fact, SR as stated does not exclude strengthening of both ¬□[**3**] *and* ¬□**[4...)** together, which would result in an inconsistent set of implicatures. As [4] notes (with regard to different types of examples), SR is not contradiction free.

## 5 Adjusting the Alternative Set?

Under SR, the absence of an unwanted upper-bounding inference $\Box\neg\beta$ can have two possible sources: first, it can be credited to the assumption that $\beta$ is not available as an alternative in the first place; second, it can be credited to the assumption that $\beta$ and some other alternative establish a configuration of symmetry, so that neither alternative is weakly excludable. I will consider both options, starting with the second.

### 5.1 Closure under Disjunction?

One strategy to preempt the unwanted secondary implicatures supported by the alternatives in (7) is to enlarge the alternative set in a way that lets every alternative participate in a configuration of symmetry, thereby ensuring that no alternative is weakly excludable. A principled way of doing this is to follow a suggestion in [20] to close alternative sets under disjunction.

Closure under disjunction will expand **ALT** into **ALT'** = {⋁**S**: **S**⊆{[**n**]:**n**≥**2**}}. In the case at hand, closure under disjunction will, for example, add the alternative **[2,3]**, which is symmetric to **[4,...)**, so that neither is weakly excludable; likewise, **[2,4,...)** will be added, which is symmetric to **[3]**, so that again neither is weakly excludable. This generalizes to all the alternatives in **ALT'**. Closure of the alternative set under disjunction, then, correctly preempts derivation of any secondary implicatures for (1).

However, closure under disjunction eliminates unwanted secondary implicatures by virtue of supporting additional ignorance implications. Thus the particular alternatives considered above will yield the ignorance implications ¬□**[4,...)**∧¬□¬**[4,...)**, ¬□**[2,3]**∧¬□¬**[2,3]**, ¬□**[3]**∧¬□¬**[3]**, and ¬□**[2,4,...)**∧¬□¬**[2,4,...)**. Since in fact every element of **ALT'** participates in a symmetric pair, closure under disjunction predicts (1) to carry an implication of *total* ignorance modulo the asserted meaning. That is, (1) is predicted to convey that, beyond the assertion that Al hired more than one cook, the speaker has no opinion at all regarding the number of cooks hired. As already noted, however, (1) only conveys partial ignorance, concerning the question whether Al hired exactly two cooks or more than two. Intuitions on (1) furnish no evidence for the total ignorance derived by closure under disjunction. Closing the alternative set under disjunction, then, does not appear to be a promising solution to the overgeneration problem.[6]

### 5.2 A one-scale analysis?

What emerges from the above is that in order for SR to apply correctly to (1), it is necessary for Quantity inferences to be drawn on the basis of **[2]** and **[3,...)** alone. The question is then whether a general theory of alternatives can be motivated that delivers the alternative set **ALT"** = {**[2]**, **[3,...)**}. Krifka's ([9]) proposal that *at least* blocks projection of alternatives generated in its scope provides one possible part of such a theory. To be sure, though, along with the unwanted alternatives **[n,...)**, for all **n**≥**4**, this also eliminates the crucial alternative **[3,...)**. But **[3,...)** can conceivably be recovered by adopting the proposal that in generating alternatives, the possible substitutes for *at least* include the comparative operator *more than* (see [3] and [8]). In a Neo-Gricean implementation, this amounts to replacing the Horn scale in (6) above with (8) below.

(8) Horn scale: {*at least*, *exactly*, *more than*}

[6] Closure under disjunction is also insufficient to exclude unwanted secondary implicatures across-the-board. Additional secondary implicatures are incorrectly expected to emerge in cases like *Every manager hired at least two cooks*, where a higher universal quantifier breaks symmetry and hence obviates ignorance implications (see previous footnote). Mayr ([10]) discusses a version of this problem in a related proposal.

Unfortunately, while this "one-scale analysis" analysis derives the correct set of implicatures for examples like (1), it fails to derive the intended ignorance implications in other cases with *at least*. As emphasized in [9] and [2], the syntactic distribution of *at least* resembles that of the focus sensitive particle *only*. One manifestation of this is the ability of *at least* to associate with a numeral at a distance. This is illustrated by (9), where *at least* can be read as associating with the numeral across a possibility modal.

(9) Al is at least allowed to hire two cooks.

In analogy to (1), an utterance of (9) can be read as conveying that the speaker is uncertain about whether Al is allowed to hire more than two cooks. Writing $\diamond$ to express deontic possibility, this ignorance implication can be captured under SR by assuming that (9) semantically conveys $\diamond\mathbf{[2,...)}$, the proposition that Al is allowed to hire more than one cook, and by positing the symmetric alternatives $\diamond\mathbf{[2]}\wedge\neg\diamond\mathbf{[3,...)}$, and $\diamond\mathbf{[3,...)}$, that is, the proposition that Al is only allowed to hire two cooks and the proposition that he is allowed to hire more than two. The problem for a one-scale analysis based on the Horn scale in (8) is that the distribution of comparative operators like *more than* differs greatly from that of *at least* (see [6]). In particular, the result of substituting *more than* for *at least* in (9), shown in (10), is ungrammatical. A one-scale analysis based on (8), then, seems to leave the $\diamond\mathbf{[3,...)}$ alternative unaccounted for.

(10) * Al is more than allowed to hire two cooks.

A two-scale analysis based on the Horn scales in (5) and (6) fares better in this respect. The alternative $\diamond\mathbf{[3,...)}$ for the interpretation of (9) can be taken to arise in the very same way as the alternative $\mathbf{[3,...)}$ for the interpretation (1), viz. by substituting the numeral *three* for the numeral *two*.[7]

This advantage of the two-scale analysis is a reason to explore a solution of the overgeneration problem that retains the two Horn scales (5) and (6) and locates the source of the problem elsewhere.

## 6 Strong Excludability

We are back to the assumption that the set of alternatives for the interpretation of (1) is ALT, the set of all meanings generated by the substitution method from the two Horn scales (5) and (6). The remaining analytical option is then to revise SR itself. This can be done by replacing weak excludability with a more restrictive property, call it *strong excludability*. The revised algorithm, call it the *Revised Standard Recipe* (RSR), posits strong excludability as a necessary condition for strengthening of a primary implicature to a secondary implicature.

A possible blueprint for the definition of strong excludability is given in [4] (see also [22, 16, 20]). If $\neg\Box\beta$ is a primary implicature, let us say that $\beta$ is *strongly excludable* just in case the potential secondary implicature $\Box\neg\beta$ is an element of every maximal set of potential secondary implicatures that is consistent with the implicature base.

Note first that by this definition, an alternative that is not weakly excludable cannot be strongly excludable, either. If a potential secondary implicature $\Box\neg\beta$ is itself inconsistent with the implicature base, then it will not be an element of any maximal set of potential secondary

[7]The sentence **Al is exactly allowed to hire two cooks* is actually no better better than (10), putting into question the viability of both Horn scales for *at least* under consideration. An amendment that suggests itself is to replace the Horn scale {*at least*,*exactly*} with {*at least*,*only*}. *Only* delivers the intended double-sided alternatives for all cases considered here, including (9). *Al is only allowed to hire two cooks* expresses the intended meaning $\diamond\mathbf{[2]}\wedge\neg\diamond\mathbf{[3,...)}$.

implicatures consistent with the implicature base, let alone every such set. For the case at hand, this guarantees that **[2]** and **[3,...)** are not strongly excludable. Just like SR, therefore, SRS preempts strengthening of the primary implicature $\neg\Box$**[2]** and $\neg\Box$**[3,...)**.

Consider now the set of potential secondary implicatures {$\Box\neg$**[4,...)**, $\Box\neg$**[4]**, $\Box\neg$**[5,...)**, $\Box\neg$**[5]**, **...**}. This set is consistent with the implicature base, but adding any further potential secondary implicature to it ($\Box\neg$**[2]**, $\Box\neg$**[3,...)**, or $\Box\neg$**[3]**) would render it inconsistent with the implicature base; hence it is a maximal set of potential secondary implicatures consistent with the implicature base; since it does not contain $\Box\neg$**[3]**, it shows that **[3]** is not strongly excludable. Next consider the set of potential secondary implicatures {$\Box\neg$**[3]**, $\Box\neg$**[5,...)**, $\Box\neg$**[5]**, $\Box\neg$**[6,...)**, $\Box\neg$**[6]**, **...**}. This set is again consistent with the implicature base, but adding any further potential secondary implicature to it ($\Box\neg$**[2]**, $\Box\neg$**[3,...)**, $\Box\neg$**[4,...)**, or $\Box\neg$**[4]**) would render it inconsistent with the implicature base; hence it is again a maximal set of potential secondary implicatures consistent with the implicature base; since it does not contain **[4,...)** or **[4]**, it shows that neither **[4,...)** nor **[4]** is not strongly excludable, either.

Extrapolating from these examples, it is apparent that in fact no alternative in **ALT** is strongly excludable. RSR therefore bars strengthening of any primary implicatures, hence correctly predicts that (1) does not trigger any Quantity inference beyond the ignorance implications $\neg\Box$**[2]**$\wedge\neg\Box\neg$**[2]** and $\neg\Box$**[3...)**$\wedge\neg\Box\neg$**[3...)**.

# 7 Conclusion

The main finding of this paper is that a neo-Gricean account of *at least* seems to call for a departure from the Standard Recipe: strengthening to secondary implicature requires strong excludability of the relevant alternative, rather than mere weak excludability. Under the resulting Revised Standard Recipe, unattested strengthening to secondary implicature may or may not be due to an ignorance implication. Failure of weak excludability results in ignorance implications, whereas mere failure of strong excludability does not.

The question that remains is whether strong excludability is indeed a plausible ingredient of a Gricean algorithm for the calculation of ignorance. It is also notable that strong excludability is transparently the neo-Gricean counterpart of Fox's ([4]) notion of *innocent exclusion*, an ingredient of his grammatical theory of scalar implicature. If strong excludability is deemed to be an unlikely component of Gricean reasoning, the results here could be taken to indicate that ignorance implications should be accommodates in a grammatical theory of Quantity implicature. In fact, the effects of the Revised Standard Recipe arrived at above are fully replicated in Meyer's ([11]) *Matrix K Theory*, a radically grammatical theory of Quantity implicature in that all kinds of implicatures, including ignorance implications, are derived in the grammar.

However, before adducing *at least* data as evidence for or against any theory of Quantity implicature, a good amount of work remains to be done to establish that an implicature account of the sort pioneered by Büring ([1]) is feasible in view of a broader set of data. It remains to be seen, in particular, whether the account successfully extends to *at most*, association of *at least* and *at most* with non-numerals, and in particular the bewildering interaction of *at least* and *at most* with modals and other operators.[8]

[8] See [1, 6, 19, 12, 14, 21, 13, 17, 18, 10, 2, 8].

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