On a Notion of Monotonic Support for Bipolar Argumentation Frameworks

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ABSTRACT

The bipolar argumentation framework (BAF) setting is an extension of Dung's setting for abstract argumentation, that considers an additional relation, called support relation. Several interpretations of such a support relation have been pointed out so far, including *deductive*, *necessity*, *general* and *backing* supports. These notions of support capture different kinds of interactions between arguments, that do not primarily correspond to attacks. In this paper, we propose a new notion of support, called monotonic support. Our approach is axiomatic: two postulates are introduced for capturing the intuition that underlies this notion of support in formal terms. The first postulate, monotony, prevents the support relation from downgrading the acceptance status of the supported argument. The second postulate, non-triviality, requires the existence of BAFs for which supporting an argument leads to increase its acceptance status. We present a general family of extension-based semantics for BAFs, called support score-based (SSB) semantics, that satisfy the two postulates and are parameterized by some aggregation functions. We prove a characterisation result linking the postulates that a SBB semantics satisfies with the properties of the aggregation functions used to define it. We also show that none of the previously introduced semantics for BAFs satisfies the monotony postulate.

KEYWORDS

Abstract Argumentation; Bipolarity; Support; Extension Selection

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1 INTRODUCTION

Deliberation is a fundamental issue for systems based on autonomous agents, that hold their own goals and beliefs. In these systems the elaboration of social goals and beliefs is needed for agents to coordinate and cooperate efficiently.

Typically, in a deliberation process, agents can exchange arguments for stating and explaining their respective positions, enabling them to envision the beliefs and the goals that are shared and those that are not shared, and from which the group of agents can try to make a collective decision.

In this perspective, argumentation-based deliberation systems appear as a cornerstone of future autonomous multi-agent systems. In fact, such deliberation systems based on argumentation already exist for decision aiding, allowing debates to be represented and recorded in an abstract way. Thus, several systems for online debates (e.g., D-BAS[1], Debate Graph[2], Argüman[3]) have been implemented and are commonly used for dealing with various deliberation situations, such as political debates (e.g., Debate Graph was used by the BBC), citizen consultation in local political decisions, or law elaboration.[4] These systems typically allow agents (individuals) to state arguments, to attack arguments [21], and to vote on arguments and/or attacks [18].

Argumentation is a topic that has been studied for a long time in philosophy and AI [5, 6, 8, 14, 16, 25, 26], and many semantics for arguments and attacks have been pointed out. However, the study of semantics that take into account supports between arguments is quite recent [1–3, 7, 9, 10, 12, 19, 22–24, 27], and there is still no clear consensus of what a semantics taking into account supports should look like. This is problematic since some deliberation systems also take advantage of support relations. To go a step further, having a clearer understanding of what supporting means is an important issue to be addressed.

In this paper, we focus on Dung's setting for abstract argumentation framework [16] which models argumentation frameworks (AFs) as graphs, where nodes represent arguments and arcs represent the attack relation between them. One can then define sets of arguments that can be accepted together. These sets represent coherent points of view (solutions), and are called extensions.

Recent studies in argumentation theory [3, 9, 10] have introduced the use of bipolarity in abstract argumentation and defined bipolar argumentation frameworks (BAFs). Then, several extension-based semantics generalizing to the BAF case the usual semantics for AFs have been pointed out. Let us mention: Bipolar Argumentation System (BAS) [9], Deductive and Defeasible Support (DDS) [7], Argumentation Framework with Necessities (AFN) [22, 23], and

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[1] https://dbas.cs.uni-duesseldorf.de/
[2] https://debategraph.org/
[3] https://en.arguman.org/
[4] https://www.republique-numerique.fr/

Backing-Undercutting Argumentation Framework (BUAF) [12]. All those semantics correspond to various (and somewhat conflicting) intuitions of what a support could be: *general*, *deductive*, *necessity* and *backing* supports.[5] Indeed, these semantics capture different kinds of interaction between arguments, that do not correspond to attacks. For example, a *deductive* support expresses a relation of implication between arguments, rather than a positive contribution (aid) from one argument to another.

In this work, we propose a new interpretation of the concept of support and make formal a notion of *monotonic support*. Two postulates for capturing this new interpretation are introduced. The first one, called monotony, prevents a support from downgrading the acceptance status of a supported argument. The second one, called non-triviality, requires the existence of BAFs for which supporting an argument leads to increase its acceptance status. Imposing this postulate prevents from considering as an acceptable semantics for BAFs any semantics that would simply ignore the support relation.

Here is an example from real life that illustrates the notion of *monotonic support* between arguments:

Example 1. *Let us consider the following statements coming from a conversation involving three agents:*
Agent 1: – "At the meeting, Alice was wearing pants". *(a)*
Agent 2: – "At the meeting, Alice was wearing a green skirt". *(b)*
Agent 3: – "At the meeting, Alice was dressed in green". *(c)*

Here, arguments a and b are in conflict, and argument c supports argument b. In our opinion, the fact that c supports b must not, in any way, degrade the acceptance degree of b, in the sense that, if b is accepted when the support from c to b is not taken into account, b must still be accepted when this support is considered. This is what we mean by *monotonic support*.

In the following, we will show that none of the BAS [9], DDS [7], AFN [22, 23] or BUAF [12] semantics for support satisfies the monotony postulate. This does not mean that there is something wrong with these semantics or that they are useless, but only that they capture other intuitions than the one on which the concept of monotonic support is based.

Later on in the paper, we present a general family of extension-based semantics for BAFs, called support score-based semantics. Within those semantics, supports are used for selecting extensions among those of the (classical) argumentation framework associated with the BAF under consideration. All the corresponding semantics for BAFs satisfy monotony and non-triviality, as expected.

The rest of the paper is organized as follows. Section 2 recalls basic background on Dung's abstract AFs and on BAFs. In Section 3, the two postulates, monotony and non-triviality, are presented. In Section 4, we review extension-based approaches for BAFs from the literature, and we show that none of them satisfies the monotony postulate. In Section 5, the family of support score-based (SSB) semantics for BAFs is introduced, and additional postulates satisfied by those semantics are identified. In Section 6, some properties connecting the support score-based semantics to the postulates are exhibited. Especially, we prove that each SSB semantics satisfies the monotony postulate and the non-triviality postulate. In Section 7, several semantics from the SSB family are exhibited; via an example, they are shown to lead to select distinct extensions in the general case. In Section 8, some related work is discussed. Finally, in Section 9, we conclude the paper and give some perspectives for future work.

2 BACKGROUND

Argumentation Frameworks (AFs)

An abstract argumentation framework [16] consists of a set of abstract arguments, and one type of interaction between them, given by an attack relation.

Definition 1 (Argumentation Framework). *An* argumentation framework *(AF) is a pair $\langle A, R\rangle$, where A is a finite and non-empty set of arguments, $R \subseteq A \times A$ is the attack relation between arguments (graphically represented by $\longrightarrow$).*

A central notion in Dung's setting is *admissibility*:

Definition 2 (Admissibility). *Let $\mathscr{F} = \langle A, R\rangle$ be an AF and $\mathscr{E} \subseteq A$ be a set of arguments. $\mathscr{E}$ is* conflict-free *if there are no arguments $\mathsf{a}, \mathsf{b} \in \mathscr{E}$ such that a attacks b ($\mathsf{a}R\mathsf{b}$). a is* acceptable *w.r.t $\mathscr{E}$ if and only if $\forall \mathsf{b} \in A$, if $\mathsf{b}R\mathsf{a}$ then there exists $\mathsf{c} \in \mathscr{E}$ such that $\mathsf{c}R\mathsf{b}$ (a is defended by c against b). $\mathscr{E}$ is an* admissible set *of $\mathscr{F}$ if and only if $\mathscr{E}$ is conflict-free and for all $\mathsf{a} \in \mathscr{E}$, a is acceptable w.r.t. $\mathscr{E}$.*

In what follows, we recall the definitions of *preferred*, *stable* and *complete* semantics from [16].

Definition 3 (Semantics). *Let $\mathscr{F} = \langle A, R\rangle$ be an AF and $\mathscr{E} \subseteq A$ be a set of arguments.*

- *$\mathscr{E}$ is a* preferred extension *of $\mathscr{F}$ (noted* pref*), if and only if $\mathscr{E}$ is an admissible set of $\mathscr{F}$, that is maximal w.r.t. set-inclusion.*
- *$\mathscr{E}$ is a* stable extension *of $\mathscr{F}$ (noted* stab*), if and only if $\mathscr{E}$ is conflict-free and $\forall \mathsf{a} \in A \setminus \mathscr{E}$, there exists $\mathsf{b} \in \mathscr{E}$ such that $\mathsf{b}R\mathsf{a}$.*
- *$\mathscr{E}$ is a* complete extension *of $\mathscr{F}$ (noted* comp*), if and only if $\mathscr{E}$ is an admissible set of $\mathscr{F}$ and for each argument a which is acceptable with respect to $\mathscr{E}$, $\mathsf{a} \in \mathscr{E}$.*

For a given AF $\mathscr{F}$, we denote by $Ext^{AF}_{\sigma}(\mathscr{F})$ the set of all extensions of $\mathscr{F}$ w.r.t. a given semantics σ.

Bipolar Argumentation Frameworks (BAFs)

Early studies [19, 27] suggested that in addition to the attack relation, which represents negative interactions between arguments, another kind of relation can be considered, namely a support relation. Such a relation aims to capture positive interactions between arguments. This leads to the notion of abstract BAF, defined as follows [9]:

Definition 4 (Bipolar Argumentation Framework). *A* bipolar argumentation framework *(BAF) is a triple $\langle A, R, S\rangle$, where A is a finite and non-empty set of arguments, $R \subseteq A \times A$ is the attack relation between arguments, and $S \subseteq A \times A$ is the support relation between arguments (graphically represented by $\longrightarrow$ and $\Longrightarrow$, respectively). We assume that $R \cap S = \emptyset$.*

Extension-based semantics for BAFs can be defined in the same way as for AFs:

Definition 5 (Extension-Based Semantics). *An* extension-based semantics *for BAFs σ is a mapping associating with every BAF $\mathscr{F} = \langle A, R, S\rangle$ a set of subsets of A, noted $Ext^{BAF}_{\sigma}(\mathscr{F})$.*

[5] A survey of these approaches is presented in [11] and [13].

Existing semantics for BAFs are presented in Section 4. Given a BAF $\mathscr{F} = \langle A, R, S\rangle$, $Ext^{AF}_{\sigma}(\mathscr{F})$ denotes the set of extensions of $\langle A, R\rangle$ with respect to the semantics σ, $\langle A, R\rangle$ being called the (classical) AF associated with $\mathscr{F}$.

Finally, we will need the following notation. Let $\mathscr{F} = \langle A, R, S\rangle$ be a BAF and a and b be two arguments of A. $\mathscr{F}^{S+(\mathsf{a},\mathsf{b})}$ denotes the BAF $\langle A, R, S \cup \{(\mathsf{a}, \mathsf{b})\}\rangle$.

An argument can have three different acceptance statuses with respect to an extension-based semantics. Indeed, an argument is *skeptically accepted* (Sk) if it belongs to every extension, it is *credulously accepted* (Cr) if it belongs to some but not all extensions,[6] and it is *rejected* (Rj) if it does not belong to any extension. An argument being considered as more acceptable when it belongs to all (resp. some) extensions than when it belongs only to some (resp. none) of them, acceptance statuses can be ordered so that $Sk > Cr > Rj$ and considered as (qualitative) acceptance degrees.

Definition 6 (Acceptance Degree). *Let $\mathscr{F} = \langle A, R, S\rangle$ be a BAF and σ be an extension-based semantics for BAFs. The* acceptance degree *of* $\mathsf{a} \in A$, *denoted* $Deg^{BAF}_{\sigma,\mathscr{F}}(\mathsf{a})$ *is an element of* $\{Sk, Cr, Rj\}$ *defined as follows:*

$$Deg^{BAF}_{\sigma,\mathscr{F}}(\mathsf{a}) = \begin{cases} Sk & \text{iff } \mathsf{a} \in \cap_{\mathscr{E} \in Ext^{BAF}_{\sigma}(\mathscr{F})} \mathscr{E} \\ Rj & \text{iff } \mathsf{a} \notin \cup_{\mathscr{E} \in Ext^{BAF}_{\sigma}(\mathscr{F})} \mathscr{E} \\ Cr & \text{otherwise} \end{cases}$$

3 MONOTONIC SUPPORT

In this section, we present two postulates that aim to be satisfied by any formal semantics for the notion of *monotonic support*. Providing such postulates is useful to circumscribe the semantics that matches (or not) the intuitions associated with this notion of support.

The idea underlying the notion of *monotonic support* is simple: support should reflect a positive interaction towards the supported argument. Especially, this implies that a support to an argument *should not in any way be negative for this argument*. Though the notion of support in abstract argumentation has been considered in some previous work, giving rise to several semantics for BAFs (recalled in the next section), there has not been any attempt at characterizing using axioms the intuitions underlying the concepts of support that have been considered. Our proposal clearly departs from such previous work by identifying the properties that the notion of support we want to capture must satisfy. This is done via two postulates, called *monotony* and *non-triviality*.

Thus, the following postulates focus on comparing the degree of a supported argument in two versions of the same BAF, one taking into account the received support by this argument and the other ignoring it. By this way, we can study the elementary impact of each support relation in a given BAF.

Our first postulate, *monotony*, requires that if an argument receives a support, then its acceptance degree must not decrease.

Definition 7 (Monotony). *A semantics σ for BAFs satisfies* monotony *if and only if for every BAF $\mathscr{F} = \langle A, R, S\rangle$, for every* $\mathsf{a}, \mathsf{b} \in A$, *we have* $Deg^{BAF}_{\sigma,\mathscr{F}}(\mathsf{a}) \leq Deg^{BAF}_{\sigma,\mathscr{F}^{S+(\mathsf{b},\mathsf{a})}}(\mathsf{a})$.

[6] This is not the usual notion of credulous acceptance, since the condition of not belonging to every extension is usually omitted, so that every skeptically accepted argument also is credulously accepted (provided that an extension exists). We slightly abuse words here but this is harmless; being credulously accepted should be considered as a short, yet more readable way of stating that the acceptance degree of the argument we consider is Cr.

This postulate captures the intuition that, just as attacking an argument should not increase its acceptance status, supporting it should not degrade its acceptance status.

The second postulate, *non-triviality*, just requires the support relation to be taken into account:

Definition 8 (Non-Triviality). *A semantics σ for BAFs satisfies* non-triviality *if and only if there exists a BAF $\mathscr{F} = \langle A, R, S\rangle$, and there exist* $\mathsf{a}, \mathsf{b} \in A$, *such that* $Deg^{BAF}_{\sigma,\mathscr{F}}(\mathsf{a}) < Deg^{BAF}_{\sigma,\mathscr{F}^{S+(\mathsf{b},\mathsf{a})}}(\mathsf{a})$.

This postulate states that there exists at least one BAF for which the acceptance degree increases for an argument that receives a support. Without this condition, any semantics for BAFs that do not take the support relation into account would be considered as acceptable, which is unexpected.

The two postulates above are the ones that we consider as mandatory for a monotonic support relation:

Definition 9 (Monotonic Support). *A semantics σ for BAFs is said to be based on* monotonic support *if and only if it satisfies monotony and non-triviality.*

Example 1 (**Cont.**). *This example is represented by the BAF $\mathscr{F}_1 = \langle\{a, b, c\}, \{(a, b), (b, a)\}, \{(c, b)\}\rangle$ and illustrated by Figure 1.*

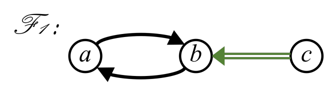


Figure 1: Graphical Representation of F_1

The acceptance degree of b is expected not to decrease when the support from c to b is considered. More precisely, when ignoring the support from c to b, under the preferred semantics, b is credulously accepted. Thus, if the support from c to b is now considered, assuming that this support is monotonic, the acceptance degree of b should not be degraded from Cr to Rj, even if c is attacked and rejected.

4 EXISTING SUPPORT RELATIONS

In this section, we recall different semantics for BAFs in abstract bipolar settings that have been pointed out so far in the literature. Those settings extend Dung's abstract framework with a support relation. We are especially interested in extension-based approaches: Bipolar Argumentation System (BAS) [9], Deductive and Defeasible Support (DDS) [7], Argumentation Framework with Necessities (AFN) [22, 23] and Backing-Undercutting Argumentation Framework (BUAF) [12], which take a BAF as input and produce a collection of sets of arguments as outputs.

Our study does not include the *evidential* interpretation of support (EAS) introduced in [24]. The special argument η considered in EAS cannot be taken into account in the other approaches, making a fair comparison impossible.

To define the acceptance of arguments, the semantics BAS, DDS, AFN, and BUAF consist in reducing BAFs to AFs: support relations in the original BAF are removed and extra-attacks resulting from the combination of existing support and attack relations are added (graphically represented by ⇢). This transformation process, called *flattening*, is based on a saturation principle that generates all possible extra-attacks. It produces an associated AF containing original and newly generated attacks (extra-attacks). The extensions of the input BAF are then defined as the extensions of the associated AF (w.r.t. a preset semantics for Dung's frameworks). Accordingly, in order to define the semantics for BAFs, presented in [7, 9, 12, 22, 23], it is enough to make precise how extra-attacks are generated.

Bipolar Argumentation System (BAS)

In the BAS semantics, supports are viewed as *confirmation* of arguments by other arguments. Formally, two kinds of extra-attacks are added during the flattening process. A *secondary attack* occurs when a attacks an argument c (aRc) that supports b (cSb). In this case, the attack of a is transmitted to b. The *supported attacks* convey the idea that if a supports an argument c (aSc) that attacks b (cRb), then this argument a also attacks b.

Let us state that a *support path* exists from argument a to argument b in a given BAF $\langle A, R, S\rangle$ if and only if there exists $a_1, \ldots, a_k \in A$ such that $a_1 = a$, $a_k = b$, and for all $i \in \{1, \ldots, k-1\}$, $a_i \, S \, a_{i+1}$ holds.

Definition 10 (BAS Semantics). *Let $\langle A, R, S\rangle$ be a BAF and $a_1, \ldots, a_n \in A$ with $n \geq 3$. There is an extra-attack from a_1 to a_n if and only if either $a_1 \, R \, a_2$ and there is a support path from a_2 to a_n, or there is a support path from a_1 to a_{n-1} and $a_{n-1} \, R \, a_n$.*

BAS is not a semantics for BAFs based on a monotonic support since monotony is not satisfied.[7]

Proposition 1. *BAS semantics for BAFs under preferred or stable semantics for AFs satisfies non-triviality but does not satisfy monotony.*

Sketch of Proof. Consider the following example, where the BAS semantics for BAFs is taken under the preferred (or the stable) semantics for AFs.

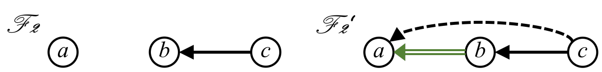


Figure 2: The BAFs $\mathscr{F}_2$ and $\mathscr{F}_2'$

In the initial BAF $\mathscr{F}_2$, represented by Figure 2, the acceptability degree of the argument a is Sk, after considering the support to a (BAF $\mathscr{F}_2'$) its degree becomes Rj. In this example, the support received from b played a degrading role for a. □

Deductive and Defeasible Support (DDS)

Deductive interpretation of the notion of support has also been considered. Roughly, for the DDS semantics, if an argument a supports an argument b and a is accepted, then b must be accepted too.

Formally, two kinds of extra-attacks are added: the first ones are "supported attacks" as defined in [9]. The second ones, called *mediated attacks*, take place when a attacks b (aRb) and an argument c supports (cSb). In this case, a also attacks c.

Definition 11 (DDS semantics). *Let $\langle A, R, S\rangle$ be a BAF and $a_1, \ldots, a_n \in A$ with $n \geq 3$. There is an extra-attack from a_1 to a_n if and only if either there is a support path from a_1 to a_{n-1} and $a_{n-1} \, R \, a_n$, or $a_1 \, R \, a_2$ and there is a support path from a_n to a_2.*

[7] Note that complete semantics has not been defined for BAS.

Here also, monotony is not satisfied:

Proposition 2. *DDS semantics for BAFs under preferred, stable or complete semantics for AFs satisfies non-triviality but does not satisfy monotony.*

Sketch of Proof. Consider the following example, where the DDS semantics for BAFs is taken under the preferred, the stable, or the complete semantics for AFs.

Figure 3: The BAFs $\mathscr{F}_3$ and $\mathscr{F}_3'$

The degree of argument a goes from Sk (in the initial BAF $\mathscr{F}_3$) to Cr (in BAF $\mathscr{F}_3'$), which shows that the support from b to a plays a degrading role. □

Argumentation Framework with Necessities (AFN)

In *necessity* supports, "necessity" means that if an argument a supports another argument b, then a is necessary to obtain b. In that way, if b is accepted, then a should also be accepted as well.

Formally, two types of extra-attacks must be added: extra-attacks from the first type are the same ones as the "secondary attacks" defined in [9]; extra-attacks from the second type are generated when an argument c supports a (cSa) and c attacks b (cRb). In this case, an extra-attack from a to b is generated.

Definition 12 (AFN Semantics). *Let $\langle A, R, S\rangle$ be a BAF and $a_1, \ldots, a_n \in A$ with $n \geq 3$. There is an extra-attack from a_1 to a_n if and only if either $a_1 \, R \, a_2$, and there is a support path from a_2 to a_n, or $a_{n-1} \, R \, a_n$, and there is a support path from a_{n-1} to a_1.*

Acceptance in AFNs follows the same principles as in Dung's semantics, using strong coherence instead of conflict-freeness. This limits our study of the necessity interpretation of support to support-acyclic BAFs, but does not impact acceptance of the associated Dung's AFs.

Like for the other semantics for BAFs considered above, monotony is not satisfied by AFN semantics:

Proposition 3. *AFN semantics for BAFs under preferred, stable or complete semantics for AFs satisfies non-triviality but does not satisfy monotony.*

Backing Argumentation Framework (BUAF)

In the BUAF semantics, supports are viewed as backings, strongly inspired from Toulmin's argumentation schemes [25]. BUAF semantics extend BAFs by considering an additional preference relation (formally, a partial order $\preceq$) over the arguments. Obviously, when no preference relation is available, every BAF can be viewed as a preference-based BAF, for which $\preceq \, = A \times A$ (all the arguments are equally preferred). Hence, the BUAF semantics also applies to BAFs.

Formally, within the BUAF semantics, two kinds of extra-attacks are added: the first ones, called *indirect attacks*, are the same ones as "secondary attacks" [9]; the second ones, called *implicit attacks*, are generated whenever a attacks b (aRb), and an argument c supports b (cSb). In this case, an attack from a to c and another one from c to a are added.

Definition 13 (BUAF Semantics). *Let $\langle A, R, S\rangle$ be a BAF (viewed as a BUAF $\langle A, R, S, \preceq\rangle$ where $\preceq = A \times A$). Let $a_1, \ldots, a_n \in A$ with $n \geq 3$. There is an extra-attack from a_1 to a_n if and only if $a_1\ R\ a_2$ and there is a support path from a_2 to a_n, and there is an extra-attack from a_1 to a_n and a_n to a_1 if and only if $a_1\ R\ a_2$ and there is a support path from a_n to a_2.*

Proposition 4. *BUAF semantics for BAFs under preferred, stable or complete semantics for AFs satisfies non-triviality but does not satisfy monotony.*

To sum up, none of the existing approaches for BAFs satisfies monotony; so it is interesting to look for new approaches that satisfy this postulate.

Before that, let us illustrate on an example the various semantics BAS, DDS, AFN and BUAF. The purpose is to show, using this example, that all of these semantics are pairwise distinct, in the sense that they define distinct sets of extensions. As such, they actually capture distinct intuitions about what "supporting" means.

Example 2. *Consider the BAF $\mathcal{F}_4$ represented by Figure 4.*

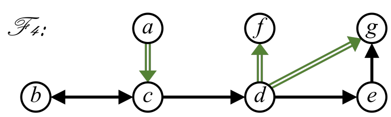


Figure 4: The BAF $\mathcal{F}_4$

Adhering to the preferred semantics $\sigma = pref$ for the corresponding AF leads to the following extensions: $Ext^{AF}_{pref}(\mathcal{F}_4) = \{\{a, b, d, f, g\}, \{a, c, e, f\}\}$. These two extensions are the ones obtained without taking into account the support relations.

*Consider the **BAS** semantics. In order to take into account the support relation, we apply the flattening process and we obtain the AF $\mathcal{F}_4^{BAS}$ represented by Figure 5. We get $Ext^{BAF}_{pref}(\mathcal{F}_4^{BAS}) = \{\{a, c, e\}\}$.*

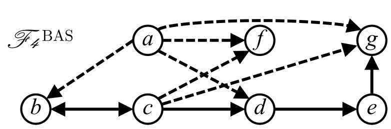


Figure 5: BAS semantics for BAFs applied to $\mathcal{F}_4$

*For the **DDS** semantics, we obtain the AF $\mathcal{F}_4^{DDS}$ represented by Figure 6, and we get $Ext^{BAF}_{pref}(\mathcal{F}_4^{DDS}) = \{\{b, d, f, g\}, \{a, c, e, f\}, \{b, e, f\}\}$.*

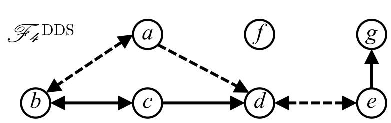


Figure 6: DDS semantics for BAFs applied to $\mathcal{F}_4$

*As to the **AFN** semantics, we obtain the AF $\mathcal{F}_4^{AFN}$ represented by Figure 7, and we get $Ext^{BAF}_{pref}(\mathcal{F}_4^{AFN}) = \{\{a, c, e\}, \{a, b, d, f, g\}\}$.*

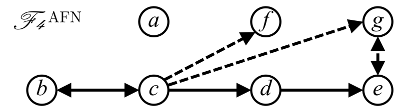


Figure 7: AFN semantics for BAFs applied to $\mathcal{F}_4$

*Finally, for the **BUAF** semantics, we obtain the AF $\mathcal{F}_4^{BUAF}$ represented by Figure 8. We get $Ext^{BAF}_{pref}(\mathcal{F}_4^{BUAF}) = \{\{b, d, f, g\}, \{a, c, e\}, \{b, e\}\}$.*

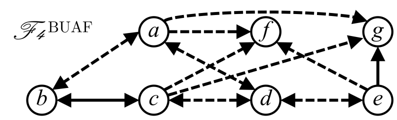


Figure 8: BUAF semantics for BAFs applied to $\mathcal{F}_4$

5 SUPPORT SCORE-BASED SEMANTICS

In this section, we define the *Support Score-Based* semantics SSB, a new family of semantics for BAFs where the notion of support is interpreted as a monotonic support.

The key idea of support score-based semantics is to keep separate the way in which attacks and supports are handled in the definition of extensions. Basically, extensions of the AF $\langle A, R\rangle$ associated with the input BAF $\mathcal{F} = \langle A, R, S\rangle$ are first computed. Then supports are exploited to make a selection between those extensions. The number of supports received by each extension is used to select the "best" extensions.

Note that as SSB semantics select extensions, considering semantics for AFs that characterize a single extension (e.g., the grounded one), would be meaningless (for such semantics for AFs the support relation is not taken into account within the SSB semantics). Thus, the SSB semantics for BAFs equipped with grounded semantics for AFs is not a monotonic support semantics since it violates the *non-triviality* postulate.

Let us start by providing a couple of definitions:

Definition 14 (Multi-Mapping Function). *A* multi-mapping function *is a family of mappings from $\mathbb{N}^n$ to $\mathbb{N}$, $\forall n > 0$.*

An aggregation function is then defined as follows:

Definition 15 (Aggregation Function Properties). *An aggregation function $\otimes$ is a multi-mapping function such that $\forall i \in \{1, \ldots, n\}$:*

- *if $x_i \leq x'_i$, then $\otimes(x_1, \ldots, x_i, \ldots, x_n) \leq \otimes(x_1, \ldots, x'_i, \ldots, x_n)$ (non-decreasingness)*
- $\otimes(x_1, \ldots, x_n) = 0$ *iff* $x_1 = \ldots = x_n = 0$ *(minimality)*
- $\otimes(0, x_1, \ldots, x_n) = \otimes(x_1, \ldots, x_n)$ *(neutral element)*
- $\otimes(x) = x$ *(identity)*

In addition to properties that an aggregation function must satisfy, a number of non-mandatory properties can also be considered to characterize subclasses of aggregation functions.

Definition 16 (Some Additional Properties). *Let $\otimes$ be a multi-mapping function.*

- *for any permutation π, $\otimes(x_1,\dots,x_n) = \otimes(\pi(x_1,\dots,x_n))$* *(symmetry)*
- $\otimes(x_1,\dots,x_i+1,x_{i+1},\dots,x_n) > \otimes(x_1,\dots,x_i,x_{i+1}+1,\dots,x_n)$ *(prioritization)*
- *if $\otimes(x_1,\dots,x_i,\dots,x_n) \geq \otimes(y_1,\dots,y_i,\dots,y_n)$ then $\otimes(x_1,\dots,x_i+1,\dots,x_n) \geq \otimes(y_1,\dots,y_i+1,\dots,y_n)$* *(co-monotony)*

Support score-based semantics take advantage of aggregation functions to determine how much each extension of the AF associated with the input BAF is supported. Though many aggregation functions can be exploited, for the sake of illustration, standard aggregation functions will be considered in the following. Thus, we focus in this paper on Σ (sum), and (more generally) $w\Sigma$ (weighted sum) aggregation functions, as well as lex (leximax).

lex associates with each vector $(x_1,\dots,x_n)$ of $\mathbb{N}^n$ a value $lex((x_1,\dots,x_n))$ in such a way that for any pair of vectors $(x_1,\dots,x_n)$ and $(x'_1,\dots,x'_n)$, we have $lex((x_1,\dots,x_n)) \leq lex((x'_1,\dots,x'_n))$ if and only if $(x_1,\dots,x_n)$ is lower than or equal to $(x'_1,\dots,x'_n)$ w.r.t. the lexicographic ordering. Assuming (wlog) that the vectors $(x_1,\dots,x_n)$ of $\mathbb{N}^n$ are such that $max_{i\in\{1,\dots,n\}} x_i < q$ where $q \in \mathbb{N}$ is a fixed integer, $lex((x_1,\dots,x_n))$ can be defined as $lex((x_1,\dots,x_n)) = \Sigma_{i=1}^{n} q^{n-i} \times x_i$. Thus, *lex* can be viewed as a specific weighted sum aggregator associated with the weight vector $(q^{n-1},\dots,1)$.

As evoked previously, each *support score-based* semantics takes into account the number of supports that are received by each extension, depending on the semantics σ for AFs that is used, to select the best supported extension(s). It is based on two multi-mapping functions ($\oplus$ and $\odot$) and it follows a three-step process. In the first step, a received support value is assigned to each argument $\mathsf{a} \in A$ for each acceptance degree in $\{Sk, Cr, Rj\}$.

Definition 17 (Received Support Value). *Let $\mathscr{F} = \langle A, R, S\rangle$ be a BAF and $\sigma \in \{pref, stab, comp\}$. For each acceptance degree $i \in \{Sk, Cr, Rj\}$, the* received support value *for an argument $\mathsf{a} \in A$ is defined as follows:*

$$\mathsf{SUPP}_i^{\sigma,\mathscr{F}}(\mathsf{a}) = |\ \{(\mathsf{b},\mathsf{a}) \in S \mid Deg_{\sigma,\mathscr{F}}^{BAF}(\mathsf{b}) = i\}\ |$$

In a second step, a score value is computed for each $\mathscr{E} \in Ext_\sigma^{AF}(\mathscr{F})$. For that purpose, a first multi-mapping function $\oplus$ evaluates the support level of an extension for each acceptance degree $\{Sk, Cr, Rj\}$ and a second multi-mapping function $\odot$ evaluates the overall support level of the extension.
Finally, in the third step, the set of selected extensions is computed, as the subset of extensions having a maximal overall support level.

Definition 18 (Support Score-Based Semantics). *Let $\mathscr{F}$ be a BAF, $\sigma \in \{pref, stab, comp\}$, $\odot$ and $\oplus$ be multi-mapping functions. W.r.t. the* support score-based semantics $\mathsf{SSB}_\sigma^{\oplus\odot}$*, the set of selected extensions of $\mathscr{F}$ given σ, $\odot$ and $\oplus$ is defined as:*

$$Ext_{\mathsf{SSB}_\sigma^{\oplus\odot}}^{BAF}(\mathscr{F}) = \{\mathscr{E} \in Ext_\sigma^{AF}(\mathscr{F}) \mid \forall \mathscr{E}' \in Ext_\sigma^{AF}(\mathscr{F}), \mathsf{SCORE}_{\mathscr{F}}^{\oplus\odot}(\mathscr{E}') \leq \mathsf{SCORE}_{\mathscr{F}}^{\oplus\odot}(\mathscr{E})\}$$

where for $\mathscr{E} \in Ext_\sigma^{AF}(\mathscr{F})$, the score value *of $\mathscr{E}$ is defined by:*

$$\mathsf{SCORE}_{\mathscr{F}}^{\oplus\odot}(\mathscr{E}) = \odot(\oplus_{\mathsf{a}\in\mathscr{E}}(\mathsf{SUPP}_{Sk}^{\sigma,\mathscr{F}}(\mathsf{a})), \oplus_{\mathsf{a}\in\mathscr{E}}(\mathsf{SUPP}_{Cr}^{\sigma,\mathscr{F}}(\mathsf{a})), \oplus_{\mathsf{a}\in\mathscr{E}}(\mathsf{SUPP}_{Rj}^{\sigma,\mathscr{F}}(\mathsf{a})))$$

Example 1 (**Cont.**). *Let us recall that $\mathscr{F}_1$ has two preferred extensions: $\mathscr{E}_1 = \{a, c\}$ and $\mathscr{E}_2 = \{b, c\}$. Let us consider $\oplus = \Sigma$ and $\odot = w\Sigma$ with $w_{Sk} = 4$, $w_{Cr} = 2$, and $w_{Rj} = 1$.*
We obtain $\mathsf{SCORE}_{\mathscr{F}_1}^{\oplus\odot}(\mathscr{E}_1) = \odot(0,0,0) = 0$ and $\mathsf{SCORE}_{\mathscr{F}_1}^{\oplus\odot}(\mathscr{E}_2) = \odot(1,0,0) = 4$. The result is $Ext_{\mathsf{SSB}_{pref}^{\oplus\odot}}^{BAF}(\mathscr{F}_1) = \{\{b, c\}\}$.
Thus, the extension $\{b, c\}$ is selected at the expense of $\{a, c\}$.

Example 2 (**Cont.**). *Let us recall that $\mathscr{F}_4$ has two preferred extensions: $\mathscr{E}_1 = \{a, b, d, f, g\}$ and $\mathscr{E}_2 = \{a, c, e, f\}$. Let us compute the set of selected extensions when $\oplus = \Sigma$ and $\odot = lex$. To compute lex as weighted sum, we set $q = |A|^2 + 1 = 50$. We obtain $\mathsf{SCORE}_{\mathscr{F}_4}^{\oplus\odot}(\mathscr{E}_1) = \odot(0,2,0) = 100$ and $\mathsf{SCORE}_{\mathscr{F}_4}^{\oplus\odot}(\mathscr{E}_2) = \odot(1,1,0) = 2550$.*
The result is $Ext_{\mathsf{SSB}_{pref}^{\oplus\odot}}^{BAF}(\mathscr{F}_4) = \{\{a, c, e, f\}\}$.

Let us now discuss some additional conditions on the multi-mapping function $\odot$ and $\oplus$, and their impact on the selection achieved by the corresponding SSB semantics. Imposing the symmetry condition on $\oplus$ is a way to comply with a notion of neutrality (it roughly means that no argument within an extension is considered as more important as any other argument of the extension), while the condition of prioritization on $\odot$ ensures that the support coming from an argument is as important as the acceptance degree of this argument is high. On this ground, we can state that:

Proposition 5. *If $\odot$ is an aggregation function that satisfies* prioritization *and $\oplus$ is an aggregation function that satisfies* symmetry*, then $\mathsf{SSB}_\sigma^{\oplus\odot}$ with $\sigma \in \{pref, stab, comp\}$, satisfies monotony and non-triviality.*

6 PROPERTIES OF $\mathsf{SSB}_\sigma^{\Sigma\odot}$

We start this section by presenting three additional postulates, that we do not consider mandatory for the notion of monotonic support, but that enable to delineate the family of *support score-based* semantics SSB. Those postulates are referred to as *Dung compatibility*, *irrelevance*, and *strength impact*.
The first one expresses some kind of compatibility with Dung's classical semantics:

Definition 19 (Dung Compatibility). *A semantics σ for BAFs satisfies* Dung compatibility *if and only if for every BAF $\mathscr{F}$, we have $Ext_\sigma^{BAF}(\mathscr{F}) \subseteq Ext_\sigma^{AF}(\mathscr{F})$.*

This property simply states that the extensions that result from the semantics of a BAF are among those of the corresponding AF. This property gives the insurance that the extensions that are considered are "true" extensions in the sense of Dung. Thus, when semantics for BAFs that are Dung compatible are considered, attacks are interpreted precisely as they are in Dung's setting for AFs.

The *irrelevance* postulate rules the impact of adding a support into a BAF. For any given BAF, it states that if an extension $\mathscr{E}$ of the corresponding AF is not an extension of the BAF (i.e., this extension is not selected) then adding a support to an argument not belonging to $\mathscr{E}$ is not enough make $\mathscr{E}$ become an extension of the BAF.

Definition 20 (Irrelevance). *A semantics σ for BAFs satisfies* irrelevance *if and only if for every BAF $\mathscr{F} = \langle A, R, S\rangle$, for every*

$\mathscr{E} \in Ext^{AF}_\sigma(\mathscr{F})$ such that $\mathscr{E} \notin Ext^{BAF}_\sigma(\mathscr{F})$, for every $\mathsf{a} \notin \mathscr{E}$ *and* $\mathsf{b} \in A$*, we have $\mathscr{E} \notin Ext^{BAF}_\sigma(\mathscr{F}^{S+(\mathsf{b},\mathsf{a})})$.*

Note that there is no direct link between Dung compatibility and irrelevance, as the first one deals with the origin of the processed extensions and the second one deals with the impact of the support on extensions.

Finally, in order to take into account the strength of supporting arguments, the *strength impact* postulate requires that if two extensions of a BAF are supported by one argument each, the one that receives support from the argument having the higher acceptance degree will be selected and the other will not be.

Definition 21 (Strength Impact). *A semantics σ for BAFs satisfies* strength impact *if and only if for every BAF $\mathscr{F} = \langle A, R, S\rangle$, $\forall \mathscr{E}_1, \mathscr{E}_2 \in Ext^{BAF}_\sigma(\mathscr{F})$, $\forall \mathsf{a} \in \mathscr{E}_1 \setminus \mathscr{E}_2$, $\forall \mathsf{b} \in \mathscr{E}_2 \setminus \mathscr{E}_1$, $\forall \mathsf{c}, \mathsf{d} \in A$ such that $(\mathsf{c}, \mathsf{a}) \notin S$, $(\mathsf{d}, \mathsf{b}) \notin S$, and $Deg^{BAF}_{\sigma,\mathscr{F}}(\mathsf{c}) > Deg^{BAF}_{\sigma,\mathscr{F}}(\mathsf{d})$, we have $\mathscr{E}_1 \in Ext^{BAF}_\sigma(\mathscr{F}^{S+(\mathsf{c},\mathsf{a})+(\mathsf{d},\mathsf{b})})$ and $\mathscr{E}_2 \notin Ext^{BAF}_\sigma(\mathscr{F}^{S+(\mathsf{c},\mathsf{a})+(\mathsf{d},\mathsf{b})})$.*

In the rest of this section, we focus on the case $\oplus = \Sigma$ (obviously it satisfies the symmetry condition) and we investigate the links between the properties satisfied by $\odot$ and the properties satisfied by the support score based-semantics induced by $\oplus$ and $\odot$. We consider the usual semantics for AFs (one just discards the grounded one, as explained before) and assume that $\odot$ is a single mapping of arity 3 since there are only three acceptance degrees. The connections between the properties satisfied by $\odot$ and the postulates satisfied by $\mathsf{SSB}^{\Sigma\odot}_\sigma$ are made precise by the following two propositions:

Proposition 6. *Let σ be any semantics for AFs among pref, stab, comp and let $\odot$ be any multi-mapping function.*

(1) $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* Dung compatibility.
(2) If $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* monotony *then $\odot$ satisfies* non-decreasingness.
(3) If $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* monotony *then $\odot$ satisfies* co-monotony.
(4) If $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* strength impact *then $\odot$ satisfies* prioritization.

Proposition 7. *Let σ be any semantics for AFs among pref, stab, comp, and let $\odot$ be any multi-mapping function.*

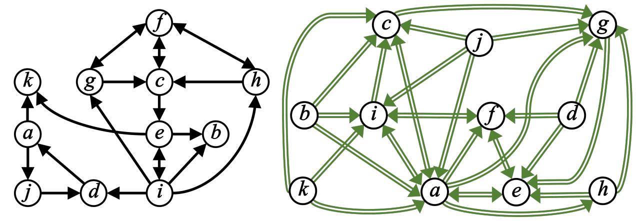
(1) If $\odot$ satisfies non-decreasingness *then* $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* monotony.
(2) If $\odot$ satisfies prioritization *and* non-decreasingness *then* $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* strength impact.
(3) If $\odot$ satisfies non-decreasingness *then* $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* irrelevance.
(4) If $\odot$ satisfies minimality *then* $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* non-triviality.

From the previous propositions we get the following theorem, matching the properties of the support score-based semantics and the postulates:

Theorem 1. *Let* $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *be a* support score-based *semantics where σ is any semantics for AFs among pref, stab, comp, and $\odot$ a multi-mapping function.* $\mathsf{SSB}^{\Sigma\odot}_\sigma$ *satisfies* monotony, strength impact *and* irrelevance *if and only if $\odot$ satisfies* prioritization *and* non-decreasingness.

7 ILLUSTRATION

In this section, an example of a BAF is provided in order to illustrate the behaviour of the support score-based semantics, depending on the aggregation functions that are used. This behaviour is characterized by the extensions that are selected.



(1) The attack graph of $\mathscr{F}_5$ (2) The support graph of $\mathscr{F}_5$

Figure 9: The BAF $\mathscr{F}_5$.

Consider the BAF $\mathscr{F}_5 = \langle A, R, S\rangle$ depicted on the two figures above. For the sake of clarity, $\mathscr{F}_5$ is represented graphically using two graphs: the first one gives the attacks (Figure 9(1)) and the second one gives the supports (Figure 9(2)).

Adhering to the preferred semantics $\sigma = pref$ for the corresponding AF leads to the following extensions:

$$Ext^{AF}_{pref}(\mathscr{F}_5) = \{\{a, c, i\}, \{e, g, h\}, \{e, f\}, \{a, f, i\}\}.$$

The columns of the next tables correspond to the four extensions of $Ext^{AF}_{pref}(\mathscr{F}_5)$. For each $\oplus \in \{\Sigma, max, min\}$ the values of the vectors $(\oplus_{x\in\mathscr{E}}(\mathsf{SUPP}^{\sigma,\mathscr{F}}_{Sk}(x)), \oplus_{x\in\mathscr{E}}(\mathsf{SUPP}^{\sigma,\mathscr{F}}_{Cr}(x)), \oplus_{x\in\mathscr{E}}(\mathsf{SUPP}^{\sigma,\mathscr{F}}_{Rj}(x)))$ are reported. Table 1 presents the value of $\mathsf{SCORE}^{\oplus lex}_{\mathscr{F}_5}(\mathscr{E})$ and Table 2 presents the value of $\mathsf{SCORE}^{\oplus w\Sigma}_{\mathscr{F}_5}(\mathscr{E})$ with $w_{Sk} = 4$, $w_{Cr} = 2$ and $w_{Rj} = 1$.

	$\{a, c, i\}$	$\{e, g, h\}$	$\{e, f\}$	$\{a, f, i\}$
$\oplus = \Sigma$	(0, 7, 9)	(0, 8, 3)	(0, 7, 2)	(0, 8, 7)
$\mathsf{SCORE}^{\Sigma,lex}_{\mathscr{F}_5}$	863	979	856	**983**
$\oplus = max$	(0, 3, 3)	(0, 4, 2)	(0, 4, 1)	(0, 3, 3)
$\mathsf{SCORE}^{max,lex}_{\mathscr{F}_5}$	369	**490**	489	369
$\oplus = min$	(0, 2, 3)	(0, 1, 0)	(0, 3, 1)	(0, 2, 1)
$\mathsf{SCORE}^{min,lex}_{\mathscr{F}_5}$	247	122	**367**	245

Table 1: $\mathsf{SSB}^{\oplus lex}_\sigma$ **with** $\oplus \in \{\Sigma, max, min\}$

	$\{a, c, i\}$	$\{e, g, h\}$	$\{e, f\}$	$\{a, f, i\}$
$\oplus = \Sigma$	(0, 7, 9)	(0, 8, 3)	(0, 7, 2)	(0, 8, 7)
$\mathsf{SCORE}^{\Sigma,w\Sigma}_{\mathscr{F}_5}$	**23**	19	16	**23**
$\oplus = max$	(0, 3, 3)	(0, 4, 2)	(0, 4, 1)	(0, 3, 3)
$\mathsf{SCORE}^{max,w\Sigma}_{\mathscr{F}_5}$	9	**10**	9	9
$\oplus = min$	(0, 2, 3)	(0, 1, 0)	(0, 3, 1)	(0, 2, 1)
$\mathsf{SCORE}^{min,w\Sigma}_{\mathscr{F}_5}$	**7**	2	**7**	5

Table 2: $\mathsf{SSB}^{\oplus w\Sigma}_\sigma$ **with** $\oplus \in \{\Sigma, max, min\}$

Since the number of supports received by an argument cannot exceed $|A|$ and an extension cannot contain more than $|A|$ arguments, the total number of supports received by an extension (when

Σ is used) cannot exceed $|A|^2$. Thus, we set q to $11^2 + 1 = 122$ for defining lex (see Section 5).

The extensions that are selected are the following ones:

$Ext^{BAF}_{\mathsf{SSB}^{\Sigma,lex}_{pref}}(\mathscr{F}_5) = \{\{a, f, i\}\}$,
$Ext^{BAF}_{\mathsf{SSB}^{max,lex}_{pref}}(\mathscr{F}_5) = \{\{e, g, h\}\}$,
$Ext^{BAF}_{\mathsf{SSB}^{min,lex}_{pref}}(\mathscr{F}_5) = \{\{e, f\}\}$,
$Ext^{BAF}_{\mathsf{SSB}^{\Sigma,w\Sigma}_{pref}}(\mathscr{F}_5) = \{\{a, c, i\}, \{a, f, i\}\}$,
$Ext^{BAF}_{\mathsf{SSB}^{max,w\Sigma}_{pref}}(\mathscr{F}_5) = \{\{e, g, h\}\}$,
$Ext^{BAF}_{\mathsf{SSB}^{min,w\Sigma}_{pref}}(\mathscr{F}_5) = \{\{a, c, i\}, \{e, f\}\}$.

This example illustrates the difference of behaviours achieved by letting the aggregation functions vary: most of the corresponding semantics lead to select distinct extensions (for space reasons, we do not provide additional examples, but it is easy to show that all these semantics are actually pairwise distinct).

8 RELATED WORK

This work can be related to previous work, in a number of directions.

Dung compatibility, strength impact and previous semantics for BAFs

As discussed previously, the semantics BAS, DDS, AFN, and BUAF reduce BAFs to AFs through a flattening process, by adding extra-attacks to the original AF. This leads to the inclusion or exclusion of certain arguments in initial extensions, and so the flattening process modifies the set of extensions. It turns out that none of the previous semantics for BAFs satisfies Dung compatibility:

Proposition 8. *Dung compatibility is not satisfied by BAS, DDS, AFN and BUAF semantics for BAFs under preferred or stable semantics for AFs, and BUAF semantics for BAFs under complete semantics for AFs.*

Strength impact is also not satisfied by those semantics.

Proposition 9. *Strength impact is not satisfied by BAS, DDS, AFN and BUAF semantics for BAFs under preferred or stable semantics for AFs, and BUAF semantics for BAFs under complete semantics for AFs.*

On Selecting Extensions

Our SSB semantics perform a selection, among the extensions associated with the corresponding classical Dung's framework, based on the supports received. Selecting Dung's extensions is an approach that has already been used in some previous work, but in different contexts and for different purposes.

Thus, in [20], the goal was to select extensions in Dung's framework, in order to increase the inference power (i.e., obtaining more skeptically inferred arguments and less credulously inferred arguments). To do so, extensions are compared with respect to the respective attacks that they received.

In [15], the aim was to select extensions in weighted argumentation frameworks [17], i.e., when attacks are labelled by a weight function. These weights are taken into account to compare extensions and select the best ones.

Let us also mention another approach [4] where preferences are used to select extensions.

In this work, extensions are also selected, but the selection process is driven by an additional relation, the support one. Note that the selection processes used in [4, 15, 20], and the one considered in this work could also be combined.

The Monotony Postulate

In the context of weighted BAFs, Amgoud and Ben-Naim presented several postulates that take into account attacks and supports to define acceptance semantics [2]. These authors defined some kind of monotony postulate, called bi-variate monotony. It turns out that there is no direct link between our notion of monotony and their bi-variate monotony. Indeed, there is a fundamental difference between how the support is taken into account in our approach and in their work. Indeed, in our monotony postulate, two different BAFs are compared, whereas bi-variate monotony compares two arguments from the same BAF. Furthermore, the bi-variate monotony postulate is concerned with attacks removal, whereas our monotony is only about adding supports. Finally, in their framework, all the information are encoded in the degrees of direct attackers and supporters; this is why the principles they point out have a local orientation, and concern arguments that are directly related. Contrastingly, the whole argumentation system is taken into account in our approach in order to define a collective acceptance (in the way of Dung).

In a previous work [1] by the same authors, another monotony postulate has been defined. Again, there is no direct link between the notion of monotony characterized by this postulate and the notion of monotony considered in our work (the way supports are taken into account differ).

9 CONCLUSION AND PERSPECTIVES

In this paper, we have modeled a new notion of support, called monotonic support, within an extension-based setting for abstract BAFs. We have pointed out two postulates that a monotonic support relation should satisfy, namely *monotony* and *non-triviality*. Then we have introduced a new family of *support score-based* semantics. We have explained how to select the best extensions, by considering the number of received supports for each extension. We have investigated the properties offered by the support score-based semantics and show, among other things, that they satisfy the monotony and non-triviality postulates. We have reviewed existing extension-based approaches for BAFs, based on different interpretations of what a support relation could be: *deductive*, *necessity*, *general* and *backing*. We have shown that none of these approaches satisfies *monotony*.

In our opinion, leveraging an axiomatic approach, as we did here by proposing postulates to characterize an interpretation of the support relation, is important to get a principled method for defining, studying and comparing on formal grounds different proposals for capturing the various intuitions about what "supporting" means. A perspective for further research is to develop full-axiomatic settings for argumentation, populated with representation theorems.

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