Exploiting Hidden Convexities for Real-time and Reliable Optimization Algorithms for Challenging Motion Planning and Control Applications

Doctoral Consortium

Fatemeh Rastgar
University of Tartu
Tartu, Estonian
Fatemeh@ut.ee

ABSTRACT

Motion Planning and Control algorithms are often formulated as optimization problems as desired robot behaviors can be intuitively encoded in the form of cost and constraint functions. A fundamental challenge in robotics is to make optimization based motion planning reliable and real-time. This thesis aims to achieve this for motion planning and control problems encountered in a wide class of applications ranging from autonomous driving and object transportation to manipulation. In particular, we aim to develop optimization algorithms that identifies and leverages the underlying useful albeit limited convex structures in the problem through techniques like Alternating Direction Method of Multipliers and Bergman Iteration. During the first 1.5 years of the thesis, we have already obtained encouraging results that validated our core hypothesis and led to optimizer that surpasses the state of the art in many challenging problems.

KEYWORDS

Robots; Optimization Algorithms; Motion Planning; Multi-agent Systems; Collision Avoidance; Obstacles; Trajectory Optimization

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1 MOTIVATION

Robots are gradually transitioning from carefully instrumented factory floors to applications like autonomous driving, human-robot collaboration, object transportation etc, where they are required to operate in unstructured, dynamic and uncertain environments. To realize the full potential of robotics in these applications, we need motion planning and control algorithms that can reliably generate sophisticated maneuvers in real-time. Although, one can develop motion planning and control algorithms using a diverse set of tools from mathematics and control theory, approaches such as trajectory optimization theory and model predictive control based on optimization theory have become the preferred technique in the robotics community. The attractiveness of an optimization based approach is that one can encode very high level description of robot behaviors through carefully designed cost and constraint functions.

A fundamental challenge in robotics community is to make optimization based motion planning and control reliable and real-time. The algorithmic bottleneck stems from the fact most (or at least the most interesting ones) optimization problems in robotics are cursed with a property called non-convexity. Any optimizer designed for such a class of problems require the user to come up with a guess of what the solution might look like. A poor initial guess will invariably mean that optimizer will run for minutes of CPU time without converging to an feasible solution.

The over-arching aim of this study is to develop optimizer that are robust to poor initialization and at the same have computationally efficient numerical structure that allows them to run in real-time even on embedded boards like NIVIDIA Jetson TX2. We aim to do this by identifying and leveraging niche mathematical structures in the problem, specifically some hidden partial convexity.

2 HYPOTHESIS

The underlying hypothesis of this thesis is that a large class of optimization problems in robotics have some hidden convex structures that has been overlooked in most existing works. The basis of this assertion is rooted in recent works like [3] including some from our own group [13], [14] that has managed to induce and exploit some limited yet useful convex structures in problems that were erstwhile considered to be generic non-linear without any niche structures. All these cited works have a common link: they deal with optimization problems where cost and constraints are modeled as trigonometric functions. We also hypothesize that problems that do not naturally have this structure can be transformed or reformulated to have that particular form. For example, as shown in our recent work [12], the non-convex quadratic form of collision avoidance constraints can be reformulated in polar form to have the trigonometric structure similar to that encountered in [13], [14]. Finally, we also hypothesize that techniques like Alternating Direction Method of Multipliers have a very critical role in developing optimization algorithms that can exploit this trigonometric structure in the optimization problems.

3 ALREADY OBTAINED RESULTS

3.1 Improving Convex-Concave Procedure based trajectory optimization

The current method for solving trajectory optimization problem for affine systems is convex-concave procedure (CCP). However,

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this method has some critical limitations. Firstly, it needs a collision free initial guess of solution trajectory. Finding this collision free trajectory is too difficult and curb the CCP method. Moreover, CCP at each iteration involves solving constrained optimization problem which become restrictive for real-time computations. Our proposed algorithm not only does not need collision free initial guess, but also computationally is at least two orders of magnitude faster. This computationally efficiency is rooted in the interesting mathematical structure that we utilized in the paper. In most existing work, obstacle avoidance constraints are formulated as non-convex quadratic inequalities, (See Eq. 1 in [12]) that hinders solving the optimization problem. To have a convex optimization problem, CCP use the Conservative affine approximation of the mentioned inequalities. Despite having convex optimization, the problem is too conservative for the reasons mentioned above. We instead introduce new modelling for collision avoidance (see Eq. 2 in [12]) and take advantage of bi-convexity property in avoidance constraints. Then using this modelling, trajectory optimization problem has a computationally efficient convex approximation. Also, it should be mentioned that the constraints are relaxed in the optimization problem using augmented Lagrangian method. Finally, using Alternating Minimization Concept, reduces our problem into solving various convex problems (See Algorithm 1 and 2 in [12]). It should be mentioned that the results are validated by various benchmarks which have static and dynamic obstacles. At the end, to validate the performance of the trajectory optimization algorithm, it is implemented on quadrotors.

3.2 Improving Joint Multi-Agent trajectory optimization by GPU Accelerated Convex Approximations

Obtaining collision free trajectories connecting start and goal positions in multi-agent systems such as autonomous driving cars or quadrotors can be considered as one of the crucial problems. In addition to discussion in 3.1, in multi-Agent trajectory optimization, there are two core challenges when the number of agents grow. At first, the number of optimization variables has straightforward relationship with the number of agents. Secondly, the movement of each robot is considered as an obstacle for the other agents. Thus, by increasing the number of agents, the number of non-convex obstacle avoidance constraints grow by a factor $\binom{n}{2}$. Current methods [2, 5], not explicitly exploiting the cooperation between the agents, consider the other agents as non-responsive dynamic obstacles leading to smaller feasible space availability and thus conservative. [12]. In this work we proposed a novel method for inter-agent cooperation using set-up of sequential convex programming [1] which leads to all trajectories being computed using GPU jointly [9]. It should be mentioned that instead of modelling collision avoidance with non-convex quadratic constraints, we represent them in the polar form. Then the proposed optimizer converts to a quadratic programming (QP) problem based on our previous work [12] and it is accelerated by offline caching of matrix inverses and worked with trivial default parameters on dozens of examples.

In the proposed accelerated optimizer, we improved the computational tractability of our large scale optimization problem and it While trajectories have similar quality to [1], our proposed algorithm has less computation time. Moreover, computed trajectories are shorter in comparison with the current state of art [8] in all considered benchmarks (see section IV in [9]).

3.3 Distributed consensus-based robust Kalman filtering for state estimation in sensor networks

Estimation is one of the most challenging problems in networks. Among different mathematical approaches, Kalman filtering can be considered as one of the most powerful tools in estimation problems, if the parameters of the system model be exactly specified. However, as dynamic model in most real-world systems such as multi-robots, autonomous vehicles have uncertainties, the mentioned assumption is not guaranteed. Hence, to reduce the effect of non-satisfaction condition in the accuracy of estimation, different types of robust filtering schemes based on Kalman filtering are introduced [15]. Regarding to the source of uncertainties, they can be categorized in different groups including multiplicative noise, norm bounded uncertainty and failure in observation.

What is highly important in interactions among different networks is that how data fuses and how these interactions affect the measurements. Among different algorithms, distributed algorithms and specifically consensus based algorithms have been attracted much attention as these algorithms alleviate centralized and decentralized problems such as computational ability in sensor networks. In consensus-based algorithm, each sensor as a local estimator measures variables, communicates with its neighbors, exchanges information with them and do an appropriate estimation [6].

To the best of our knowledge, the number of studies on robust Kalman consensus filters as an important subject in sensor networks are limited for instance. In fact, most works did not consider uncertainties [4, 7]. Thus, we did a detailed study on this subject. First, we proposed a new distributed robust filtering approach based on consensus on estimation for stochastic discrete-time linear multisensor systems [10]. Then, using other types of consensus- consensus on information, consensus on measurements and Hybrid consensus- and also reformulating the problem, we modified the proposed robust Filter for stochastic uncertainties [11]. It should be mentioned that the stability for both designed algorithms are proved using Lyapanuv function and using numerical examples, we demonstrated the performabce of the proposed filter. Finally, it should be noted that in multi-agent trajectory optimization, agents reach an agreements on the shared information and find feasible collision free trajectories. Also, we are interested in extending the trajectory optimization algorithms for multi-agent systems under different types of uncertainties.

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