Solid Semantics and Extension Aggregation Using Quota Rules under Integrity Constraints

Extended Abstract

Xiaolong Liu
Institute of Logic and Cognition and Department of Philosophy
Sun Yat-sen University
liuxlong6@mail2.sysu.edu.cn

Weiwei Chen
Institute of Logic and Cognition and Department of Philosophy
Sun Yat-sen University
chenww26@mail2.sysu.edu.cn

ABSTRACT

In this paper, we propose *solid semantics* to obtain the most acceptable sets of arguments. Besides, we study the application of solid semantics in the field of judgement aggregation.

KEYWORDS

Abstract Argumentation; Solid Semantics; Social Choice Theory

ACM Reference Format:
Xiaolong Liu and Weiwei Chen. 2021. Solid Semantics and Extension Aggregation Using Quota Rules under Integrity Constraints: Extended Abstract. In *Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), Online, May 3–7, 2021*, IFAAMAS, 3 pages.

1 INTRODUCTION

An argumentation framework [10] is a directed graph, where nodes represent arguments and edges represent elements of a binary relation. One of the core notions of argumentation frameworks is *admissibility*. An *admissible extension* is a set of arguments that contains no internal conflict and defends against any attacker of their elements.

In this paper, we mainly focus on obtaining the most acceptable arguments in argumentation frameworks by strengthening Dung's admissibility. Before discussing this idea, let us first illustrate two drawbacks observed from the literature. The first one arises from the observation of graded acceptability [12] which provides an approach to rank arguments from the most acceptable to the weakest one(s) by parameterizing the numbers of attackers and counter-attackers. What if we want to find out precisely in an argumentation framework the sets of arguments such that they contain all the counter-attackers for each attacker of their elements? It is impossible to achieve this goal by tune the parameters, as different attackers may have different numbers of counter-attackers.

The other drawback arises from the observation of arguments which are attacked indirectly and defended indirectly by some argument. There are other semantics [1, 6] also provide approaches to rank arguments. But their approaches rely on assumptions regarding the processing of cycles. However, Dung [10] indicates that the presence of cycles could be a problem. An argument A is *controversial* w.r.t. B if A indirectly attacks and indirectly defends B. In the literature, there is no consensus on whether to accept or reject such arguments. There is a lot of discussion about this topic [2, 3, 5, 9]. Note that any argument in an odd-length cycle is controversial w.r.t. any argument in the odd-length cycle.

Motivated by the observations above, we argue that the most acceptable arguments should satisfy two criteria: (*i*) they should have defenders as many as possible, and (*ii*) they should avoid the undesirable interference of some arguments. Dung's admissibility only requires the weakest defense in the sense that only one mandatory defender is enough. An interesting fact is that some problematic situations disappear after Dung's admissibility is strengthened. In this paper, we propose *solid admissibility* which satisfies the two criteria. Roughly speaking, a *solidly admissible extension* is a set of arguments without internal conflict which defends against any attacker and contains all the defenders. We will show that if an argument A is controversial w.r.t. an argument B, then B will never occur in any solid extension based on solid admissibility. To sum up, such extensions not only contains all defenders of their elements, but also avoids the presence of any argument which is indirectly attacked and indirectly defended by some argument. This conforms to the intuition in practical reasoning or real life in the sense that if an argument has more defenders, then surely it has less controversies.

We also study the application of solid semantics in the field of *judgement aggregation*, a branch of social choice theory. Especially, we show that there are more possibility preservation results for solid semantics than for Dung's semantics.

2 PRELIMINARY

This part reviews some notions of abstract argumentation [10]. Some definitions are adopted from Grossi and Modgil [12].

Definition 2.1. An *argumentation framework AF* is a pair $\langle Arg, \rightharpoonup \rangle$, where Arg is a finite and non-empty set of arguments, and $\rightharpoonup$ is a binary relation on Arg. For $A, B, C \in Arg$ and $\Delta \subseteq Arg$. $A \rightharpoonup B$ (or A *attacks* B) denotes that $(A, B) \in \rightharpoonup$. $\overline{B}$ denotes the set of all the attackers of B. $\Delta \rightharpoonup B$ denotes that there exists an argument $A \in \Delta$ such that $A \rightharpoonup B$. $A \rightharpoonup \Delta$ denotes that there exists an argument $B \in \Delta$ such that $A \rightharpoonup B$. A is a *defender* of C iff there exists an argument $B \in Arg$ such that $A \rightharpoonup B$ and $B \rightharpoonup C$.

Definition 2.2. An argument A *indirectly attacks* an argument B iff there exists a finite sequence $A_0, \ldots, A_{2n+1}$ such that (*i*) $A = A_0$ and $B = A_{2n+1}$, and (*ii*) for each i, $0 \leqslant i \leqslant 2n$, A_{i+1} attacks A_i. An argument A *indirectly defends* an argument B iff there exists a finite sequence $A_0, \ldots, A_{2n}$ such that *(i)* $A = A_0$ and $B = A_{2n}$, and (*ii*) for each i, $0 \leqslant i < 2n$, A_{i+1} attacks A_i. A is *controversial* w.r.t. B iff A indirectly attacks and indirectly defends B.

Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), U. Endriss, A. Nowé, F. Dignum, A. Lomuscio (eds.), May 3–7, 2021, Online.

Definition 2.3. Given $AF = \langle Arg, \rightharpoonup \rangle$. $\Delta \subseteq Arg$ *defends* $C \in Arg$ iff for any $B \in Arg$, if $B \rightharpoonup C$ then $\Delta \rightharpoonup B$. The *defense function* $d^{AF}: 2^{Arg} \rightarrow 2^{Arg}$ of AF is defined as: for any $\Delta \subseteq Arg$, $d^{AF}(\Delta) = \{C \in Arg \mid \Delta \text{ defends } C\}$. The *neutrality function* $n^{AF}: 2^{Arg} \rightarrow 2^{Arg}$ of AF is defined as: for any $\Delta \subseteq Arg$, $n^{AF}(\Delta) = \{B \in Arg \mid \text{NOT } \Delta \rightharpoonup B\}$.

Definition 2.4. Given $AF = \langle Arg, \rightharpoonup \rangle$ and a set of arguments $\Delta \subseteq Arg$. Δ is a *conflict-free* extension of AF iff $\Delta \subseteq n(\Delta)$; Δ is a *self-defending* extension of AF iff $\Delta \subseteq d(\Delta)$; Δ is an *admissible* extension of AF iff $\Delta \subseteq n(\Delta)$ and $\Delta \subseteq d(\Delta)$; Δ is a *complete* extension of AF iff $\Delta \subseteq n(\Delta)$ and $\Delta = d(\Delta)$; Δ is a *preferred* extension of AF iff Δ is a maximal (w.r.t. set inclusion) admissible extension of AF; Δ is a *stable* extension of AF iff $\Delta = n(\Delta)$; Δ is the *grounded* extension of AF iff Δ is the least (w.r.t. set inclusion) fixed point of the defense function d^{AF}.

We next introduce a model for the aggregation of extensions [11]. Given $AF = \langle Arg, \rightharpoonup \rangle$. Let $N = \{1, \cdots, n\}$ be a finite set of *agents*. Suppose that each agent $i \in N$ reports an extension $\Delta_i \subseteq Arg$. Then $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_n)$ is referred to as a *profile* of extensions. An *aggregation rule* is a function $F: (2^{Arg})^n \rightarrow^{Arg}$, mapping any given profile of extensions to a subset of Arg.

Definition 2.5. Given $AF = \langle Arg, \rightharpoonup \rangle$. Let N be a finite set of n agents, and let $q \in \{1, \cdots, n\}$. The *quota rule* with quota q is defined as the aggregation rule mapping any given profile $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_1) \in (2^{Arg})^n$ of extensions to the set including exactly those arguments accepted by at least q agents:

$$F_q(\mathbf{\Delta}) = \{A \in Arg \mid \#\{i \in N \mid A \in \Delta_i\} \geqslant q\}. \tag{1}$$

The quota rule F_q for n agents with $q = \lceil \frac{n+1}{2} \rceil$ (resp., $q = 1, q = n$) for AF is called the *strict majority* (resp., *nomination*, *unanimity*) rule. In an argumentation framework, the set of all extensions under a semantics could be understood as a property.

Definition 2.6. Given $AF = \langle Arg, \rightharpoonup \rangle$. Let $\sigma \subseteq 2^{Arg}$ be a property of extensions of AF. Then an aggregation rule $F: (2^{Arg})^n \rightarrow 2^{Arg}$ for n agents is said to *preserve* σ if $F(\mathbf{\Delta}) \in \sigma$ for every profile $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_n) \in \sigma^n$.

3 SOLID SEMANTICS

In order to address the drawbacks in the introduction, we introduce solid admissibility by strengthening Dung's admissibility.

Definition 3.1. Given $AF = \langle Arg, \rightharpoonup \rangle$. $\Delta \subseteq Arg$ *solidly defends* (or *$\mathbb{S}$-defends*) $C \in Arg$ iff for any argument $B \in Arg$, if $B \rightharpoonup C$, then $\Delta \rightharpoonup B$ and $\overline{B} \subseteq \Delta$. The *solid defense function* $d_{\mathbb{S}}^{AF}: 2^{Arg} \rightarrow 2^{Arg}$ of AF is defined as follows. For any $\Delta \subseteq Arg$, $d_{\mathbb{S}}^{AF}(\Delta) = \{C \in Arg \mid \Delta\ \mathbb{S}\text{-defends } C\}$. $\Delta \subseteq Arg$ is *$\mathbb{S}$-self-defending* iff $\Delta \subseteq d_{\mathbb{S}}^{AF}(\Delta)$. $\Delta \subseteq Arg$ is a *$\mathbb{S}$-admissible* extension of AF iff $\Delta \subseteq n(\Delta)$ and $\Delta \subseteq d_{\mathbb{S}}(\Delta)$.

A $\mathbb{S}$-admissible extension is a set of arguments that is conflict-free and $\mathbb{S}$-self-defending.

Theorem 3.2. *Given $AF = \langle Arg, \rightharpoonup \rangle$. Let Δ be a $\mathbb{S}$-admissible extension Δ of AF. If an argument A is controversial w.r.t. an argument B, then $B \notin \Delta$.*

In the theorem above, A could be in a $\mathbb{S}$-admissible extension, because it could be an initial argument. It is not reasonable to reject an unattacked argument. However, any argument in an odd-length cycle will never occur in a $\mathbb{S}$-admissible extension. Next we develop *solid semantics* that strengthen Dung's semantics in the sense that given an argumentation framework AF, for any *solid extension* Δ of AF, there exists a Dung's extension Γ of AF such that $\Delta \subseteq \Gamma$. Besides, the solid extensions defined below are $\mathbb{S}$-admissible.

Definition 3.3. Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, Δ is a *$\mathbb{S}$-complete* extension of AF iff $\Delta \subseteq n(\Delta)$ and $\Delta = d_{\mathbb{S}}(\Delta)$; Δ is a *$\mathbb{S}$-preferred* extension of AF iff Δ is a maximal (w.r.t. set inclusion) $\mathbb{S}$-admissible extension of AF; Δ is a *$\mathbb{S}$-stable* extension of AF iff $\Delta = n(\Delta)$ and for any $A \notin \Delta$, $\overline{A} \subseteq \Delta$; Δ is the *$\mathbb{S}$-grounded* extension of AF iff Δ is the least (w.r.t. set inclusion) fixed point of $d_{\mathbb{S}}$.

4 PRESERVATION OF SOLID SEMANTIC PROPERTIES

Next, we analyze that in the scenarios where a set of agents each provides us with a set of arguments which satisfies a property, under what circumstance such a property will be preserved. We present our preservation results by using techniques in [4, 11].

Theorem 4.1. *Given $AF = \langle Arg, \rightharpoonup \rangle$. Any quota rule F_q for n agents preserves $\mathbb{S}$-self-defense for AF.*

Note that different from $\mathbb{S}$-self-defense, Dung's self-defense can not be preserved by some quota rule. One example is the strict majority rule, as Chen and Endriss [8] have demonstrated.

Theorem 4.2. *Given $AF = \langle Arg, \rightharpoonup \rangle$. Any quota rule F_q for n agents with $q > \frac{n}{2}$ preserves $\mathbb{S}$-admissibility for AF.*

No quota rule preserves Dung's admissibility for all argumentation frameworks [8]. However, we have obtained a positive result for $\mathbb{S}$-admissibility, i.e., there exist some quota rules (e.g., the strict majority rule) preserve $\mathbb{S}$-admissibility for all argumentation frameworks. Notably, a similar result [7] states that the majority rule guarantees admissibility on profiles of solid admissible sets during aggregation of extensions. Next, we let $\mathcal{M}(AF)$ denote the maximal number of the defenders of an argument in $\mathcal{E}(AF)$, where $\mathcal{E}(AF)$ denotes the set of arguments that are not initial arguments and whose attackers are not initial arguments either.

Theorem 4.3. *Given $AF = \langle Arg, \rightharpoonup \rangle$. A quota rule F_q for n agents preserves $\mathbb{S}$-completeness for AF if $q > \frac{n}{2}$ and $q \cdot (\mathcal{M}(AF) - 1) > n \cdot (\mathcal{M}(AF) - 1) - 1$.*

As the $\mathbb{S}$-grounded extension is unique, any quota rule preserves $\mathbb{S}$-groundedness for AF. We say that a property σ is *inclusion maximal* if for any $\Delta_1,\ \Delta_2 \in \sigma$, if $\Delta_1 \subseteq \Delta_2$ then $\Delta_1 = \Delta_2$. Both the $\mathbb{S}$-preferredness and $\mathbb{S}$-stability are inclusion maximal. Let $|\sigma| \geqslant 2$, and let n be the number of agents. If n is even, then no quota rule preserves σ for AF. If n is odd, then no quota rule different from the strict majority rule preserves σ for AF.

ACKNOWLEDGMENTS

We are grateful to Philippe Besnard, Sylvie Doutre and three anonymous reviewers for helpful suggestions. This work was supported by China Postdoctoral Science Foundation Grant, No. 2019M663352, the Fundamental Research Funds for the Central Universities, SYSU, No. 20wkpy94, and the MOE Project of Key Research Institute of Humanities and Social Sciences in Universities. No. 18JJD720005.

REFERENCES

[1] Leila Amgoud and Jonathan Ben-Naim. 2013. Ranking-based semantics for argumentation frameworks. In *International Conference on Scalable Uncertainty Management*. Springer, 134–147.

[2] Pietro Baroni, Massimiliano Giacomin, and Giovanni Guida. 2005. SCC-recursiveness: a general schema for argumentation semantics. *Artificial Intelligence* 168, 1-2 (2005), 162–210.

[3] Ringo Baumann, Gerhard Brewka, and Markus Ulbricht. 2020. Revisiting the Foundations of Abstract Argumentation–Semantics Based on Weak Admissibility and Weak Defense. In *Proceedings of the Thirty-Fourth AAAI Conference on Artificial Intelligence*.

[4] Philippe Besnard and Sylvie Doutre. 2004. Checking the acceptability of a set of arguments. In *NMR*, Vol. 4. Citeseer, 59–64.

[5] Gustavo A Bodanza and Fernando A Tohmé. 2009. Two approaches to the problems of self-attacking arguments and general odd-length cycles of attack. *Journal of Applied Logic* 7, 4 (2009), 403–420.

[6] Claudette Cayrol and Marie-Christine Lagasquie-Schiex. 2005. Graduality in argumentation. *Journal of Artificial Intelligence Research* 23 (2005), 245–297.

[7] Weiwei Chen. 2021. Guaranteeing Admissibility of Abstract Argumentation Frameworks with Rationality and Feasibility Constraints. (2021). Manuscript.

[8] Weiwei Chen and Ulle Endriss. 2018. Aggregating Alternative Extensions of Abstract Argumentation Frameworks: Preservation Results for Quota Rules. In *COMMA*. 425–436.

[9] Sylvie Coste-Marquis, Caroline Devred, and Pierre Marquis. 2005. Prudent semantics for argumentation frameworks. In *17th IEEE International Conference on Tools with Artificial Intelligence (ICTAI'05)*. IEEE, 5–pp.

[10] Phan Minh Dung. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial intelligence* 77, 2 (1995), 321–357.

[11] Umberto Grandi and Ulle Endriss. 2013. Lifting integrity constraints in binary aggregation. *Artificial Intelligence* 199 (2013), 45–66.

[12] Davide Grossi and Sanjay Modgil. 2019. On the graded acceptability of arguments in abstract and instantiated argumentation. *Artificial Intelligence* 275 (2019), 138–173.